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VENEZIANO THEORY

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A B S T R A C T

We review recent progress in Veneziano theory. Topics discussed include the harmonic oscillator formalism, factorization properties, Olesen's cure of the loop divergence, precedents for dual models in nuclear physics, phenomenology with 5 point functions, experimental evidence against planar duality diagrams. "Classical" Veneziano, such as the theory of meson-meson scattering, is treated more briefly since it has been so much reviewed already.

Review paper at the Irvine Conference on Regge poles, 6 December 1969.

Uomini poi, a mal più ch'al bene usi,
Fuor mi rapirono del dolce inchiostro;
E Dio si sa qual poi mia vita fusi.

- Regge in the heaven of Ideas.

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1. FACTORIZATION OF N POINT FUNCTIONS

I want this talk to be different from anything you have heard before. I shall therefore play down "classical" Veneziano, i.e., everything more than six months old, in favour of recent developments. There have already been so many reviews (Chan 1969a, b, Fayyazuddin and Riazuddin 1969b, O.W. Greenberg 1969, Jacob 1969a, b, Lovelace 1969a) and introductions (Fubini 1969, Hirshfeld 1969, Horn 1969, Veneziano 1969a, b, Weinberg 1969) that you ought to survive a straight plunge into the deep end.

Consider the N point function with momenta $p_0, p_1, \dots, p_{N-1}$

and linear Regge trajectories

$$\alpha(i, j) = \alpha_{ij}^0 + \alpha' \left(\sum_{k=i}^j p_k \right)^2, \quad 0 \leq i \leq j \leq N-1. \quad (1.1)$$

The slope α' has to be universal, otherwise everything goes wrong^{*)}. We also suppose that the external particles have spin zero and are themselves on Regge trajectories, so

$$\alpha(i, i) = \alpha_{ii}^0 + \alpha' p_i^2 = 0. \quad (1.2)$$

The Veneziano N point function (Chan 1969a, and earlier references given there) is then

$$B_N(\alpha) = \int_0^1 dx_1 \dots \int_0^1 dx_{N-3} \cdot \varphi_N(x, \rho) \quad (1.3)$$

where

$$\begin{aligned} \varphi_N(x, \rho) = & \prod_{i=1}^{N-3} (x_i)^{-\alpha(0, i)-1} (1-x_i)^{-\alpha(i, i+1)-1} \\ & \times \prod_{i=1}^{N-4} \prod_{j=i+2}^{N-2} [1-u_{ij}]^{-\delta(i, j)} \end{aligned} \quad (1.4)$$

and

$$\begin{aligned} \delta(i, j) = & \alpha(i, j) + \alpha(i+1, j-1) \\ & - \alpha(i, j-1) - \alpha(i+1, j), \quad j \geq i+2, \end{aligned} \quad (1.5)$$

$$u_{ij} = \prod_{k=i}^{j-1} x_k, \quad j \geq i+1. \quad (1.6)$$

^{*)} If α' is not the same for all trajectories, then B_4 diverges exponentially at fixed angle (Mandelstam 1968), signature is impossible (Igi 1968), ghosts are inevitable (Wagner 1969a), chiral symmetry fails (Ademollo, Veneziano and Weinberg 1969), and levels are infinitely degenerate (Fubini and Veneziano 1969, Olesen 1969a).

α , x , p are collective notations. $B_N(\alpha)$ is invariant under cyclic or anticyclic permutations of the external lines. This is not obvious in the present formulation, but an equivalent expression due to Koba and Nielsen (1969b, c) makes it so.

Most papers on N point functions consider only the case when all intercepts α_{ij}^0 are equal. Unequal intercepts have been investigated only very recently (Mandelstam 1969d, Olesen 1969a) [special cases also by Nambu (1969), Bishari and Rittenberg (1969)]. However since this generalization has turned out crucial, I shall deal with it at once. Inserting (1.1) in (1.5) gives

$$\gamma(i, j) = -\varepsilon_{ij} + 2\alpha'(p_i p_j) \quad (1.7)$$

where

$$\varepsilon_{ij} = -\alpha_{ij}^0 - \alpha_{i+1, j-1}^0 + \alpha_{i, j-1}^0 + \alpha_{i+1, j}^0. \quad (1.8)$$

As we shall see, factorization will require that

$$\varepsilon_{ij} = 2\alpha' \sum_{m=1}^M d_i^m d_j^m \quad (1.9)$$

with some finite M independent of N . d_i^m will be constants, depending only on the quantum numbers of the external particle i . To solve (1.9) we define additive quantum numbers for the various channels (i, j)

$$C_{ij} = \sum_{k=i}^j c_k, \quad D_{ij}^m = \sum_{k=i}^j d_k^m, \quad (1.10)$$

and put

$$\alpha_{ij}^0 = b - C_{ij} + \sum_{m=1}^M \alpha' \cdot D_{ij}^m [D_{0,N-1}^m - D_{ij}^m], \quad (1.11)$$

where b is a universal constant. The trajectories thus depend quadratically or linearly on these additive quantum numbers. It is very important that they need not be conserved: $D_{0,N-1}^m \neq 0$. Also the sign of d_i^m may depend on where the line is in the diagram, since only (1.9) has a physical meaning. An example occurs in a model of Mandelstam (1969d), where the trajectories are quadratic in the total quark number (i.e., the minimum number of quarks plus antiquarks in the channel).

Now we define extended momentum vectors in an $M+4$ dimensional space

$$\hat{p}_i = (p_i^0, \underline{p}_i, d_i^m) \quad (1.12)$$

with scalar product

$$(\hat{p}_i \hat{p}_j) = p_i^0 p_j^0 - (\underline{p}_i \underline{p}_j) - \sum_{m=1}^M d_i^m d_j^m. \quad (1.13)$$

Then (1.4) reduces to

$$\begin{aligned} \varphi_N(x, \hat{p}) = & \prod_{i=1}^{N-3} (x_i)^{-\alpha(0,i)-1} (1-x_i)^{\beta-1} \times \\ & \times \prod_{i=1}^{N-3} \prod_{j=i+1}^{N-2} (1-u_{ij})^{-2\alpha'(\hat{p}_i \hat{p}_j)}. \end{aligned} \quad (1.14)$$

Note that the linear intercept contributions C_{ij} have disappeared except inside $\alpha(0,i)$.

We want to factorize this formula in the external momenta. To do so, we introduce an infinite set of harmonic oscillators (Fubini, Gordon and Veneziano 1969, Susskind 1969, Nambu 1969, Amati, Bouchiat and Gervais 1969)

$$[a_m^\mu, a_n^{\nu+}] = -g^{\mu\nu} \delta_{mn},$$

$$m, n = 1, \dots, \infty; \mu, \nu = 0, \dots, M+3, \quad (1.15)$$

where the upper indices are vector components in the $M+4$ dimensional space, and use the identities

$$\begin{aligned} (1 - u_{ij})^{-2\alpha'(\hat{p}_i \hat{p}_j)} &= \\ &= \exp \left\{ \sum_{n=1}^{\infty} 2\alpha'(\hat{p}_i \hat{p}_j) (u_{ij})^n / n \right\}, \\ e^{(\hat{f} \hat{a})} \cdot e^{(\hat{g} \hat{a}^+)} &= e^{(\hat{g} \hat{a}^+)} \cdot e^{(\hat{f} \hat{a})} \cdot e^{-(\hat{f} \hat{g})}. \end{aligned} \quad (1.16)$$

Then we can rearrange (1.14) into

$$\begin{aligned} B_N(\alpha) &= \langle 0 | V(\hat{p}_{N-2}) P(\alpha_{0,N-3}) V(\hat{p}_{N-3}) \times \\ &\times P(\alpha_{0,N-4}) \dots V(\hat{p}_2) P(\alpha_{01}) V(\hat{p}_1) | 0 \rangle \end{aligned} \quad (1.17)$$

or diagrammatically

Here the two end lines $\hat{p}_0$ and $\hat{p}_{N-1}$ correspond to the ground state $|0\rangle$ of all the harmonic oscillators. The double lines are Reggeons with propagator

$$\begin{aligned}
 P(\alpha) &= \int_0^1 dx \cdot x^{-\alpha+H-1} \cdot (1-x)^{b-1} \\
 &= \sum_{j=0}^{\infty} \binom{j-b}{j} \frac{1}{j+H-\alpha},
 \end{aligned}
 \tag{1.18}$$

where

$$H = \sum_{n=1}^{\infty} n (\hat{a}_n^+ \hat{a}_n)
 \tag{1.19}$$

is the harmonic oscillator Hamiltonian. The blobs are vertex functions connecting two Reggeons and an external particle

$$\begin{aligned}
 V(\hat{p}) &= \exp \left\{ \sum_n \sqrt{2\alpha'/n} (\hat{a}_n^+ \hat{p}) \right\} \times \\
 &\times \exp \left\{ - \sum_n \sqrt{2\alpha'/n} (\hat{a}_n \hat{p}) \right\}.
 \end{aligned}$$

$$\tag{1.20}$$

To apply this result, we expand the Reggeon propagator in the occupation number states

$$|\lambda\rangle = \prod_{n=1}^{\infty} \prod_{\mu=0}^{M+3} (\lambda_n^{\mu}!)^{-\frac{1}{2}} [a_n^{\mu+}]^{\lambda_n^{\mu}} |0\rangle, \quad (1.21)$$

in which (1.19) and (1.18) are diagonal. The Reggeon propagator $P(\alpha)$ therefore has poles at $\alpha = 0, 1, 2, \dots$ as we would expect. However the residues at these poles become increasingly degenerate, as higher oscillators in (1.19) contribute. Thus at $\alpha = 0$ we have a single state

$$P(\alpha) \approx |0\rangle [-\alpha]^{-1} \langle 0|, \quad (1.22)$$

showing again that the harmonic oscillator vacuum describes the external particles. However the pole at $\alpha = 1$ is

$$P(\alpha) \approx [1-\alpha]^{-1} \left\{ \sum_{\mu=0}^{M+3} a_1^{\mu+} |0\rangle \langle 0| a_1^{\mu} + (1-b) |0\rangle \langle 0| \right\}, \quad (1.23)$$

which corresponds to a vector particle ($\mu=1,2,3$) and $M+2$ scalar particles, of which one ($\mu=0$) is a negative norm ghost. This arises from the timelike component of the harmonic oscillator [see (1.15)]. Higher residues can be analysed in a similar manner. At each $\alpha = j$, there is always just one particle of spin j on the leading trajectory, but it is accompanied by several first daughters with spin $j-1$ and an increasing tribe of further descendants. In fact there are so many states in this model that we need a second quantized field theory just to describe the internal

quantum numbers. That is the physical meaning of the infinite set of harmonic oscillators. However the factorization properties are entirely consistent - the residue of (1.17) when any trajectory $\alpha(0,i)=0$ will be the product of two Veneziano amplitudes with smaller numbers of particles, by (1.22). Higher residues give us the scattering of particles with spin. Since we can permute the external particles cyclically, all the channels (i,j) must have a similar pole structure, which is duality.

Because of the cyclic and anticyclic symmetry, B_N satisfies identities not manifest in the harmonic oscillator representation, which reduce somewhat the number of states. These have led recent authors to a profusion of twisting operators, gauge groups, Ward-like identities, pseudo-angular-momenta, etc., (Amati, Le Bellac and Olive 1969a, b, Caneschi et al. 1969a, b, Chiu et al. 1969, Fubini and Veneziano 1969, Gallardo, Galli and Susskind 1969, Gliozzi 1969, Gordon and Ramond 1969, Heimann 1969, Montvay 1969, Thorn 1969, Virasoro 1969). Physically, however, they are just investigating the signature properties of the Regge trajectories. These are incomplete in a single Veneziano term, but for full crossing symmetry $(N-1)!/2$ B_N terms must be added, corresponding to the non-cyclic permutations. The parent trajectory then has definite signature (Hopkinson and Chan 1969), and the ghost among the first daughters is cancelled. Extending this to lower daughters reduces the number of states by $\frac{1}{2}$, because certain combinations of (1.21) decouple from the vertex functions. If $b=1$ in (1.11), all ghosts decouple (Virasoro 1969). In any case ghosts are sparse since they arise only from the timelike oscillators.

The degeneracy of the level $\alpha = j$ can be expressed in terms of the partition function, and evaluated asymptotically by the Hardy-Ramanujan theorem (e.g., Erdelyi et al. 1955). This gives the Fubini-Veneziano (1969) level density formula

$$N(j) \sim \exp \left[\frac{2\pi}{\sqrt{6}} \cdot \sqrt{(M+4)j} \right]$$

(1.24)

(see also Bardakçi and Mandelstam 1969, Krzywicki 1969). The number of states thus increases exponentially with the mass.

One can multiply the integrands in (1.3) by suitably chosen functions of x , and still obtain N point functions which factorize along the leading trajectory (Gross 1969). This corresponds to the well-known satellite ambiguity in B_4 [see Eqs. (4.4-4.5) below]. However, the number of states then increases like a Gaussian in the mass (Olesen 1969a) instead of an exponential. So, to get minimal factorization properties, some basic particle must have scattering amplitudes free of satellites. The amplitudes for other particles, in particular those of higher spin, will then necessarily have satellites in well-determined proportions. This follows e.g., from (1.23). Consideration of isolated processes has led many people to exaggerate the satellite ambiguity.

2. DUAL LOOPS

The Veneziano formula for N mesons can be represented by a duality diagram (Imachi et al. 1968, Harari 1969, Rosner 1969, Neville 1969, Koba and Nielsen 1969a)

The lines here are quarks, and the pairs of neighbouring lines give the external mesons. The Regge trajectories are obtained by combining

any other pair of quark lines. We see that they occur in just those channels described by (1.4). To include $SU(3)$, we simply write a cyclic product of λ matrices describing the quark structure of each external meson, take its trace, multiply by the appropriate Veneziano term, and then sum over all non-cyclic permutations of the external lines (Paton and Chan 1969). Resonances then never occur in exotic channels.

Hoping to include unitarity, and thus eventually arrive at a complete dual field theory, Kikkawa, Sakita and Virasoro (1969a, b) and Frye and Susskind (1969a) looked at graphs with closed loops. Duality for such diagrams is quite amusing. The quark loop moves along inside like the boa constrictor's lunch, so that a meson box turns into a vertex part, then into a self-energy part, and finally to a tadpole:

However it was immediately pointed out (Veneziano and Virasoro 1969, Bardakçi, Halpern and Shapiro 1969) that their loop formula had wrong factorization properties. On correcting this, the integral was found to diverge exponentially, and for six months there the matter rested. The development which made me include loops in this talk, came just two weeks ago from Olesen (1969b) at CERN.

All previous work on loops considered only the simplified model with all intercepts equal. People were sure the loop divergence must be due to the exponential density of states, and since this is increased by intercept breaking, they supposed the divergence would get worse.

The single loop can be evaluated rather easily in the harmonic oscillator formalism (Amati, Bouchiat and Gervais 1969, Gallardo and Susskind 1969). We just write down a complete cycle of Reggeon vertices and propagators, and take the trace over all harmonic oscillator levels

$$\text{Tr} [P(\alpha_N) V(\hat{p}_N) \dots P(\alpha_1) V(\hat{p}_1)] \quad (2.1)$$

Inserting (1.20) and the integral form of (1.18), the trace can be evaluated. We are left with an integral over the x 's, whose integrand behaves near $x_i = 1$ like

$$\exp \left\{ \frac{\pi^2}{6} \left(\frac{1}{1-u_{1N}} \right) [M+4 + 2\alpha' \sum_{i,j=1}^N \hat{p}_i \hat{p}_j] \right\} \quad (2.2)$$

For equal intercepts $M=0$. The second term in the exponent then vanishes by momentum conservation, and so we have an exponential divergence as

$$u_{1N} = \prod_{k=1}^N x_k \rightarrow 1.$$

We know from (1.12) that the effect of unequal intercept is to add M extra dimensions to the momentum. However, these do not have to be conserved. (2.2) then becomes

$$\exp \left\{ \frac{\pi^2}{6} \left(\frac{1}{1-u_{1N}} \right) [M+4 - 2\alpha' \sum_{m=1}^M (D_{1N}^m)^2] \right\} \quad (2.3)$$

which converges at $u_{1N} = 1$ provided

$$\sum_{m=1}^M (D_{1N}^m)^2 \geq \frac{M+4}{2\alpha'} . \quad (2.4)$$

Olesen also tried to define the integral by analytic continuation when (2.4) was violated. However, this adds a constant imaginary part to all trajectories, which is not very satisfactory. Let us therefore try to understand (2.4). It does not constrain any particular Regge trajectory, but only the quadratic contribution to them [see (1.11)], which has to be sufficiently negative for all trajectories. In Mandelstam's quark model (1969d), the trajectories were taken as quadratic in the total quark number (number of quarks plus anti-quarks). This gives a fifth momentum component, which is spacelike since the trajectories decrease with total quark number, and obviously not conserved. The spacelike sign is right for the cancellation in (2.3), and also means that the additional particles are real and not ghosts. So it seems that the loop converges because low-lying exotic trajectories compete with the normal ones in multi-peripheral diagrams.

For multiple loops or loops with twisted Reggeon propagators, the algebra is more complicated, and the divergence situation still unknown. Even for the simple loop, the adequacy of the harmonic oscillator trace formula (2.1), on which Olesen's result rests, has been questioned. Nevertheless, the prospects for a dual field theory are vastly brighter than a month ago.

3. INTERPRETATION OF DUAL MODELS

Since 1967 I have maintained an amateur interest in models of nuclear levels. I seem to be unfortunately unique in the Veneziano field in this respect. Some people call any unaccustomed idea "unphysical". Precedents in other branches of physics are a much better guide to what Nature likes.

My first analogy is that duality occurs in nuclear physics. It says (e.g., Vogt 1968) that if you plot the very narrow resonances which occur when a particle is scattered from a heavy nucleus and draw a smooth line through the tops, then this line

describes the scattering from an optical potential. The smooth line is obtained by going into the complex energy plane, thus smearing the resonances out. Recall what happens with Veneziano (1968). In the s channel the poles are on the real axis. If you smear them out by going off the real axis, then the formula is equivalent to exchange of a series of Regge poles in the t channel. Just to complete the analogy, Regge exchange has been interpreted as an optical potential by Arnold (1968) and his followers. Note that this is global duality not local ^{*}).

Though much narrower than ours, the resonances of nuclear physics are still not infinitely narrow. However, a model exists in which the poles are actually on the real axis. It is called the R matrix (e.g., Lane and Thomas 1958) and corresponds to enclosing the nucleus in a box, thus making all its resonances stable. To unitarize it, nuclear physicists simply remove the box by boundary conditions. The R matrix then gives the T matrix by a formula similar to the K matrix one [Eq. (4.8) below], but with some extra

^{*}) The real part of the nuclear optical potential can produce "size resonances" in the smoothed line. It is amusing to speculate that these might occur in the 100 GeV region and explain the peculiar Serpukhov total cross-sections. This is less silly than some other explanations we heard at this conference, and could easily be tested since size resonances have a definite angular momentum.

terms and factors. Unfortunately boxes are hopelessly non-relativistic. Indeed this is probably the ultimate reason why our resonances overlap so much more than nuclear ones. Nevertheless the K matrix is an obvious analogy for unitarizing Veneziano, and it seems to work rather well in practice (Lovelace 1969a). Ghosts are not allowed in either K matrix or R matrix. They would give poles on the physical sheet after unitarization. [This is contrary to optimism by Mandelstam (1969a) who hoped that the ghosts in his quark model would be exorcised by unitarization to remote nether regions.]

My third analogy is that the Fubini-Veneziano level density formula occurs in nuclear physics. It is called the Bethe fermion gas model (Bethe 1936, Ericson 1960), and says that the average distribution of excited nuclear states is like the levels of a free fermion gas, which are counted by the partition function and evaluated asymptotically by the Hardy-Ramanujan theorem (e.g., Mehta 1967, p.5), giving

$$N \sim e^{2\sqrt{cE}}$$

(3.1)

where E is the energy and c a constant related to the nuclear volume. Fubini-Veneziano is slightly different in that an infinite series of harmonic oscillators with linearly increasing frequencies replaces the fermion gas, and $E^{\frac{1}{2}}$ by its relativistic analogue $m = s^{\frac{1}{2}}$, giving

*) The relativistic change $E^{\frac{1}{2}} \rightarrow s^{\frac{1}{2}}$ is non-trivial thermodynamically, since instead of increasing slowly with energy as in nuclear physics where $\mathcal{T} \approx \sqrt{E/c}$, the particle temperature will tend to a constant upper limit (3.3) independent of the energy. Indeed this limiting temperature is a feature of Hagedorn's model (see below), and corresponds to resonances breaking up faster than they can be heated. Another unusual point is the occurrence of a fixed volume determined by α' . Here I am grateful to Professor C.N. Yang for comments.

$$N \sim e^{\sqrt{s}/\vartheta}$$

where ϑ is a limiting temperature, which for $\alpha' = 0.9 \text{ GeV}^{-2}$ and $M=1$ (as suggested by Mandelstam and Olesen) is (3.2)

$$\vartheta = \frac{1}{2\pi} [6/(M+4)\alpha']^{\frac{1}{2}} = 170 \text{ MeV}$$

(3.3)

However, nuclear physicists again have the big advantage of narrow resonances. To find whether their number increases exponentially no theologian is needed, you can count them. It does (Ericson 1960, Vogt 1968). This is important for judging the physical plausibility of the Fubini-Veneziano result. Many people seem to take it for granted that the number of resonances must be finite (it says so in the Rosenfeld tables), or at most classifiable by some finite set of quantum numbers. The idea that particle levels are as numerous as the excited states of a heavy nucleus may be distasteful, but to call such distributions unphysical merely displays ignorance of physics.

However, there is still more to the analogy. The exponential level densities in nuclear physics are reflected via phase space in the spectra of secondaries (Vogt 1968). Now in particle physics at high energies, it is well known that secondary particles never have large transverse momenta. The distribution in the transverse momentum seems to be approximately constant and exponential (Orear 1964)

$$e^{-p_{\perp}/\vartheta}$$

(3.4)

with

$$\vartheta = 160 \pm 10 \text{ MeV.} \quad (3.5)$$

Before Veneziano was ever heard of, Hagedorn tried to explain this with a statistical model (Hagedorn 1965, 1968a, b, 1969, Hagedorn and Ranft 1968). He concluded that it required the number of particle levels to increase exponentially with mass

$$N \sim e^{m/\vartheta} \quad (3.6)$$

with the ϑ of (3.5). This is the same level density as found from the Veneziano model (1.24), and with the same numerical value of the limiting temperature (3.3). The old objection to statistical models in particle physics was that they should give isotropic cross-sections. Veneziano removes this objection, since statistical mechanics must be applied not to the physical momenta but to states of some underlying set of harmonic oscillators. Also it gives a clear distinction between longitudinal and transverse momenta - the former are coupled in (1.20) to the timelike oscillators with their negative metric. A future model for the Pomeron as an equilibrium state of the Reggeons, disintegrating randomly in the transverse momentum modes only, is therefore at least conceivable in the Veneziano framework. This would make the apparent numerical coincidence between (3.3) and (3.6) into a good analogue of the nuclear statistical model. Note that satellites would give a spectrum $\sim e^{cm^2}$ in total contradiction to Hagedorn (Olesen 1969a).

However, there is one feature in nuclear physics not analogous to Veneziano. Bethe's fermion gas model only describes the asymptotic distribution of levels. For a better local distribution, you look at eigenvalues of random matrices (Porter 1965, Mehta 1967). This leads to Wigner's eigenvalue repulsion phenomenon, which says that levels with the same spin and other quantum numbers very rarely occur close together. In nuclear physics this is

confirmed experimentally. However, for the Veneziano formula, as soon as we go to second or lower daughters, increasing numbers of levels will occur with the same mass and spin. Therefore, either eigenvalue repulsion fails completely relativistically, or else more probably it limits the validity of local duality.

My conclusion is that, whether true or not, N point Veneziano is a perfectly respectable model for the statistical distribution of energy levels. It is even better than nuclear ones in that the centrifugal barrier is correct (c.f., Vogt 1968) and the size optical potential built in. Provided it can be unitarized relativistically, the only suspicious features are the exact degeneracy of the lower daughters and the occasional ghosts.

4. VENEZIANO FOR REAL MESONS

Now let us see whether Veneziano is true. For four external lines (1.3) becomes the original formula (Veneziano 1968), which we write in various equivalent forms

$$B_4(\alpha_s, \alpha_t) = \int_0^1 dx \cdot x^{-\alpha_s-1} (1-x)^{-\alpha_t-1} \quad (4.1)$$

$$= \frac{\Gamma(-\alpha_s) \Gamma(-\alpha_t)}{\Gamma(-\alpha_s - \alpha_t)} \quad (4.2)$$

$$= \sum_{j=0}^{\infty} \binom{\alpha_t + j}{j} \frac{1}{j - \alpha_s} \quad (4.3)$$

(4.2) shows we have symmetry in s and t , so that (4.3) gives us the complete amplitude either as a sum of poles in s or as a sum of poles in t , which is duality. It has been suggested (Lichtenberg et al. 1969) that by splitting the range of integration in (4.1), the Veneziano formula can be written as poles in s plus poles in t , thus showing it does not satisfy duality. This is silly. Splitting the integral introduces spurious exponential singularities in s and t , which cancel in (4.1). Thus, though each term contains only poles in one variable, it is not a sum of poles, unlike (4.3). [For arguments against even the possibility of reasonable non-dual models, see Oehme (1969a, b) and Suzuki (1969).]

However a finite product of poles, obtained by approximating the gamma functions in (4.2), will always be equivalent by partial fractions to a finite sum of poles. By introducing a sufficient number of parameters, one can therefore always construct a non-dual model which simulates a dual one in any finite region (Boguta 1969, Jengo and Remiddi 1969a, Moffat 1969a, b, c, Aly et al. 1969, Johnson and Moffat 1969, Moen and Moffat 1969a, b, Barut and Moffat 1969) ^{*)}. To discriminate dual from non-dual, look therefore at the asymptotic behaviour and the number of parameters, rather than the agreement with data. "Resonances = Regge" is clearly more constrained and economical than "resonances + Regge". The relation is that of statue and dustcover. Dual models have a clear-cut shape like a statue. Subsequent non-dual alternatives have no shape of their own - they are moulded like a dustcover to the underlying dual model and would immediately fall flat without it.

The general satellite formula for 4 point functions is

$$\sum_{N=0}^{\infty} \sum_{R=N}^{2N} (-1)^R \beta_R^N \cdot \frac{\Gamma(N-\alpha_s) \Gamma(N-\alpha_t)}{\Gamma(R-\alpha_s-\alpha_t)} \quad (4.4)$$

^{*)} Of the ten papers mentioned, all but one (Moen and Moffat 1969b) imitated a pre-existing dual model.

for symmetry in s and t , and

$$\sum_{N=0}^{\infty} \sum_{R=N+1}^{2N} (-1)^R \beta_R^N \left\{ \frac{\Gamma(N-\alpha_s) \Gamma(N+1-\alpha_t) - (\alpha_s \neq \alpha_t)}{\Gamma(R-\alpha_s-\alpha_t)} \right\} \quad (4.5)$$

for antisymmetry. Other possible terms

$$\frac{\Gamma(N-\alpha_s) \Gamma(M-\alpha_t)}{\Gamma(R-\alpha_s-\alpha_t)}, \quad N, M \leq R \leq N+M, \quad (4.6)$$

are not independent (Whippman 1969, Kreps and Milgram 1969). We repeat that the satellite ambiguity essentially arises only because processes are considered in isolation (see end of Section 1).

The Veneziano theory of pseudoscalar meson-meson scattering (Lovelace 1969a, and earlier references given there) says simply that each duality diagram

contributes a term

$$- f^2 \frac{\Gamma(1-\alpha_s) \Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}. \quad (4.7)$$

The trajectories are to be chosen from three exchange-degenerate sets according to the number of λ quarks in the intermediate state:

$$\rho, \omega, f, A_2 : \alpha_s = \frac{1}{2} + \alpha' (s - m_\pi^2),$$

$$K^*, K^{**} : \alpha_s = \frac{1}{2} + \alpha' (s - m_K^2),$$

$$\varphi, f' : \alpha_s = \frac{1}{2} + \alpha' (s + m_\pi^2 - 2m_K^2).$$

(4.8)

The intercepts are determined from (4.12) below. Since they are split linearly, $SU(3)$ breaking will not increase the number of levels [see remark after (1.14)]. f^2 is the universal ρ coupling constant, α' the universal slope of all Regge trajectories, so there are no unknown parameters. (4.7) can be unitarized by the K matrix method (Lovelace 1969a), which says

$$T_\ell(s) = V_\ell(s) / [1 + \rho(s) V_\ell(s)] \quad (4.9)$$

where V_ℓ are the partial wave projections of (4.7) and $\rho(s)$ is the Chew-Mandelstam effective-range function, whose imaginary part on the right-hand cut is

$$\text{Im } \rho(s) = -2k/\sqrt{s}. \quad (4.10)$$

This imposes unitarity in exchange for the loss of low-energy crossing symmetry. The resulting meson-meson phase shifts have been compared with experiment in detail elsewhere (Lovelace 1969a). They explain the whole of the experimental data on $\pi\pi$ scattering, coming from several indirect sources, and a large part of that on $K\pi$ and $K\bar{K}$. The main defect is that the masses of the S wave daughters $\kappa(1100)$ and $\delta(960)$ are predicted 200 MeV too low. If the recent πA_1 bump near 1500 MeV (Davies et al. 1969) is identified with the ρ' (P wave first daughter of the f), the same splitting would occur here. By adding adjustable constants to $\rho(s)$ in the two S waves, a very good fit to $K\pi$ has been obtained (Bassompierre et al. 1969). This is still much more economic than traditional resonance fits, and also treats barrier penetration factors, CDD poles, etc., more reasonably. Similar K matrix fits to meson-baryon phase shifts would be interesting. Several people have also tried to restore low-energy crossing symmetry by adjusting the left-hand cut of $\rho(s)$ in the $\pi\pi$ case (Dobson 1969, Gensini and Saliani 1969, Lipinski 1969) [see also the crossing-symmetric K matrix equations of Humble (1969) and Ravenhall and Schult (1969)].

This corresponds to keeping just the CDD poles and their residues from Veneziano, and determining everything else from old-fashioned Chew-Mandelstam dynamics.

Since Regge trajectories cannot depend on the external masses, (4.7) can also be used to extrapolate off-shell, with only f^2 varying (Lovelace 1968). However, we recall that higher spin particles and their daughters will have satellites. Neglect of satellites in the extrapolation is therefore only plausible when

$$\Delta m^2 \ll (\alpha')^{-1} = 1.1 \text{ GeV}^2. \quad (4.11)$$

By taking $m_\pi \rightarrow 0$, or $m_K \rightarrow 0$ we can apply current algebra. Adler's self-consistency condition then gives

$$\frac{1}{2} = \alpha_\rho(m_\pi^2) = \alpha_{K^*}(m_K^2) = \alpha_\varphi(2m_K^2 - m_\pi^2), \quad (4.12)$$

already included in (4.8) and true in nature. Adler-Weisberger (AW) for soft pions gives

$$f^2 = \frac{2}{\pi \alpha' F_\pi^2} \approx \frac{m_\rho^2}{\pi F_\pi^2}, \quad (4.13)$$

which is a reggeized KSFR relation, better than the original one.

[For original references read Lovelace (1969a), and for further details Osborn (1969a).] For soft kaons, AW gives (Osborn 1969a, Riazuddin and Fayyazuddin 1969, Oyanagi and Tokuda 1969, Cronin and Kang 1969)

$$F_K / F_\pi = 1.24. \quad (4.14)$$

Osborn (1969a) has pointed out an apparent inconsistency in the $K\bar{K}$ AW relation, but I think the answer is that $s = m_K^2$ is above the threshold of the right-hand cut at $s = 4m_\pi^2$, so that unitarity effects must be considered.

A second application of off-shell extrapolation takes one pion to large positive masses, so it can decay. This gives correct zero-parameter descriptions of $K \rightarrow 3\pi$ and $\eta \rightarrow 3\pi$ spectra (Lovelace 1968), though because of small phase space the results do not differ much from other subsequent current algebra pole models (Arnowitt et al. 1969b, D.F. Greenberg 1969). If further assumptions are made, the rates can also be calculated (Hertel 1969, Jacob, Llewellyn Smith and Pokorski 1969, Sutherland 1969). The $\eta \rightarrow 3\pi$ rate agrees with experiment, but not the $K \rightarrow 3\pi$. A three-pion decay with much more phase space, and therefore stronger structure effects, is $\bar{p}n$ (at rest) $\rightarrow 3\pi$, where the initial 1S_0 state has the quantum numbers of a heavy pion. However in this last case

$$\Delta m^2 = 4m_N^2 - m_\pi^2 \approx 3/\alpha',$$

(4.15)

so by (4.11) we must expect satellites. Nevertheless the Dalitz plot of this decay is so strange (people used to infer an isospin 2 meson of mass 300 MeV from it!), and its strange features are so similar to Veneziano predictions, that there can be no doubt of the significance of the fit (Lovelace 1968, Berger 1969, Altarelli and Rubinstein 1969, Boldrighini and Pugliese 1969, Rubinstein et al. 1969). I shall not show it since it has become a stock feature of Veneziano reviews.

A third application of off-shell extrapolation is to the momentum transfer dependence of peripheral pion production (Wagner 1969b). For $\pi^- p \rightarrow \pi^+ \pi^- n$ a zero is predicted in the physical region of Δ^2 , with a particular dependence on dipion mass and this has been observed (Gutay et al. 1969, Loos et al. 1969).

This meson-meson theory was the first dynamical model in the history of strong interactions which really worked experimentally. However, I think it has had enough publicity already. The main obstacle to its further development is the regrettable fact that $m_\eta \neq m_\pi$ (Wallace 1969, Osborn 1969b). Nevertheless consistent and realistic models exist for five external mesons (Bardakçi and Ruegg 1969, Gunion and Yesian 1969), and there are some interesting fragments of a six-pion formula (Frye and Susskind 1969b, Susskind and Frye 1969, Taylor 1969, Osborn 1969c). In particular the Adler condition on the six-pion function restricts satellites for the four-pion function (Taylor 1969).

Many authors have tried to construct Veneziano representations for current algebra. Though a laudable activity, this is not at present a very successful one. The most obvious approach is to use the field current identity and suppose that the currents are dominated by π , A_1 and ρ poles (Schnitzer 1969, Arnowitt et al. 1969a, de Alwis et al. 1969, Nath et al. 1969, Brower and Weis 1969). This has led to some pretty formulae for meson electromagnetic form factors, and for the $K_{\ell 3}$ and $K_{\ell 4}$ decay form factors (Oyanagi 1969a, b, Fayyazuddin and Riazuddin 1969a, b, Toda 1969, Jengo and Remiddi 1969b, Hussain and Razmi 1969), but is almost certainly inconsistent (Rosner and Suura 1969, Geffen 1969). Either there must be subtractions or higher poles in the current masses, or else $A_1 \rightarrow \rho\pi$ has to be pure S wave, in contradiction to experiment (Deery et al. 1969), to get reasonable asymptotic behaviour of the form factors. This is not really very surprising: Veneziano predicts an infinite sequence of vector mesons, and why should all but one decouple from the currents? Recognizing this, other people have tried to construct amplitudes with external currents by mutilating higher N point functions (flattening some trajectories and identifying others) (Sugawara 1969, Ohba 1969, Bander 1969, Ademollo and Del Giudice 1969a, b, Cooper 1969a, b, Hara 1969a, b). Some and quite probably all of these schemes lead to serious trouble with factorization (Freedman 1969, also Brower, Rabl and Weis 1969).

There still remains the possibility of taking current amplitudes to be normal 4 point functions, multiplied by sufficiently convergent universal form factors in the masses, but this seems disjunct and arbitrary. Perhaps it would be wiser to wait for a more realistic theory of strong N point functions. However the double poles required in photoproduction by gauge invariance, fit easily into Veneziano schemes (Brower and Halpern 1969, Bender and Rothe 1969, Ahmad et al. 1969, Drago 1969, Roy 1969).

A Veneziano model for quarks has been constructed at Berkeley by Professor Mandelstam and an industrious team of typists (Mandelstam 1969a, b, c, d, e, Bardakçi, Halpern and Shapiro 1969, Halpern et al. 1969). Essentially this assumes that quarks rather than mesons satisfy basic Veneziano formulae, and applied duality diagrams to $SU(6)$ instead of $SU(3)$. The π and ρ must then be degenerate, which means discarding all experimental successes. In exchange we are offered $\tilde{U}(12)$, ghosts on leading trajectories, and doubling of both π and ρ . Though it contains interesting ideas, Mandelstam's model is too unphysical for me to swallow whole.

5. DUALITY FOR BARYONS

The primary reason for the experimental success of the meson-meson theory is the exchange degeneracy (EXD) of Regge trajectories and residues. This follows from the absence of exotic states (Chiu and Finkelstein 1968), and in turn implies that Regge residues must always choose nonsense (Finkelstein 1969). Veneziano illustrates this prediction. Its $\pi\pi$ Regge limit for large s is

$$-\gamma(\alpha_t) \cdot \Gamma(1-\alpha_t) \cdot [-\alpha' s]^{-\alpha_t}$$

(5.1)

where $\gamma(\alpha_t)$ is a polynomial if the number of satellites is finite, and a constant in the usual $\pi\pi$ theory (4.7). Absence of exotics in either $\pi^+\pi^+$ or KN thus implies that the ρ residues in

these processes should choose nonsense at $\alpha_{\rho} = 0$. By factorization the ρ must also choose nonsense in πN for both spin states, even though there are here no exotic channels. Nevertheless a number of authors thought it worth while to construct πN Veneziano formulae in which the ρ did not choose nonsense at $\alpha_{\rho} = 0$. I have never been able to understand why.

If no other contribution were present, πN charge-exchange would vanish at $\alpha_{\rho} = 0$, whereas it merely dips experimentally. Moreover the phenomenological ρ' of Regge fits is only $\frac{1}{2}$ unit below the ρ (Barger and Phillips 1969a). That of Veneziano is 1 unit below, and any experimental ρ' is probably still lower (Davies et al. 1969). The conclusion seems clear - Veneziano requires Regge cuts (Lovelace 1969b). There are three traditional attitudes to Regge phenomenology, with convenient geographical labels:

Wisconsin: Regge cuts can be neglected, and residues choose whatever fancy mechanism best fits the data.

Argonne: Regge residues choose nonsense because of EXD, but this is lightly disguised by small Regge cuts (Arnold 1968).

Michigan: Regge residues are featureless, and are cancelled by huge cuts, producing dips by the Glauber effect.

A considerable number of papers have combined Veneziano with Wisconsin-style Regge fits, and one even postulated Michigan (Henyey 1969). So let me say it once again: all the enormous experimental success of Veneziano is direct evidence for the Argonne view on Regge phenomenology. If you disbelieve Argonne, then you should leave Veneziano alone. No dual model for Michigan exists, and it could not even conceivably resemble Veneziano. Any dual model for Wisconsin would require exotics and violate factorization. The only possible excuse for not putting in cuts is that Arnold et al. have frightened you with their Bessel transforms or convolution integrals, but absorptive model calculations for two-particle final states are really simple (Lovelace 1969b). You just project into partial waves (use high order Gaussian integration) and write (Cohen-Tannoudji et al. 1967)

$$T_{\ell} = T_{\ell}^P + T_{\ell}^R + 2ik T_{\ell}^P T_{\ell}^R \quad (5.2)$$

The first term on the right is Pomeron, the second is Regge pole (Veneziano), and the third gives the Regge cut. Notice that if the Pomeron is pure imaginary, then the third term has opposite sign to the second. The cancellation generates cross-over effects. Also a real pole will then give a real cut, which leads to EXD predictions (Krzywicki and Tran Than Van 1969a, b, Barger and Phillips 1969b, Michael 1969). Figure 1 is a typical absorbed Veneziano fit (Lovelace 1969b). It is good near the forward direction, but only qualitative at large t . In my opinion such deficiencies are more the fault of the absorptive model, in particular the fixed pole Pomeron, than of Veneziano.

To construct meson-baryon Veneziano formulae, we must first know the EXD classification of baryon trajectories. This does not follow from duality diagrams as easily as the mesons do. The direct analogue of the meson scheme would be baryon trajectories in all states of three quarks $(10+8+8+1)$, signature degenerate, and split only by the strangeness. This is certainly consistent with itself, but not with experiment. A number of authors have looked for smaller schemes (Capps 1969a, b, c, Barger and Michael 1969, Mandula et al. 1969a, b, c, C. Schmid, private communication, C. Lovelace, Appendix), but not always in my opinion with a full grasp of the problem. Firstly, for a complete theory it is not sufficient to forbid exotics in all channels - the trajectories present must also close under crossing, i.e., they must form complete eigenvectors of the $SU(3)$ crossing matrix. Secondly, the vector mesons certainly couple to the meson-baryon A' and B amplitudes with different F/D ratios (Sugawara and von Hippel 1966, Reeder and Sarma 1968). Spin cannot therefore be neglected. I found it quite easy to construct satisfactory meson-baryon (s,t) and (u,t) terms in $SU(3)$, though absorptive fits to hypercharge-exchange will be

needed to test them properly. (Since some people announce models in Phys.Rev.Letters without actually writing anything down, I have stated mine explicitly in the Appendix.) However if one insists on eigenvectors of the crossing matrix, the (s,u) term is very awkward. This can be understood from duality diagrams:

In the (s,t) diagram (left) two neighbouring quarks go straight through, and can therefore be symmetrized or antisymmetrized with complete consistency. This splits $[8_1] + [1]$ from $[10] + [8_{-1/3}]$ (subscripts are F/D). However in the (s,u) diagram (right), the symmetries are all mixed up. Experimentally, EXD in backward peaks is in far worse shape than forward (Barger 1969). Also there is considerable evidence for an exotic $Z_1(1900)$ about two levels below normal baryons (Andersson et al. 1969, Asbury et al. 1969, Kato et al. 1969). If you weaken the eigenvector requirement by allowing s channel parents to couple to u channel daughters, the whole case for baryon EXD is therefore undermined. For myself, I have not forgotten that the correct spin-parity of $Z_1(1900)$ were predicted by the strong-coupling model (Goebel 1966) three years before they were measured, and that the $SU(3)$ strong coupling model contains a consistent scheme for baryon (s,u) crossing, drastically different from EXD.

However baryon duality has one success. $SU(3)$ requires that $\Delta(1238)$ be EXD with $N_{\beta}(1680)$, and therefore that the Δ residue choose nonsense at $\alpha_{\Delta} = \frac{1}{2}$. This was earlier suggested by Igi et al. (1968) to help explain the surprisingly small π^-p backward cross-section.

After $SU(3)$ comes another problem. Linear fermion trajectories imply parity doublets. In Veneziano formulae, a finite number can be cancelled at the expense of increasing quantities of satellites, an infinite number cannot. There is no clear experimental evidence for parity doublets - even the usually cited $N_{\beta}(1680)-N_{\alpha}(1688)$ must be a coincidence since the F/D ratios are quite different (Tripp 1968). With a finite number of satellites, certain reduced baryon widths are polynomials in $s^{\frac{1}{2}}$ along both parity branches of the trajectory (Virasoro 1968). Experimentally, these reduced widths are indeed more smoothly varying than the unreduced ones, but the fits are not really quantitative (Berger and Fox 1969).

Having fixed up the leading baryons, still more satellites are needed to kill unwanted daughters, e.g., under $\Delta(1238)$. Comparing the eventual number of parameters to the number of predictions, I do not think elastic meson-baryon scattering gives any evidence for baryon Veneziano as a theory. It is just a parametrization for fulfilling EXD and FESR. Possibly we are not using the right formula for fermions.

6. B₅ PHENOMENOLOGY

Let me end with a much happier story. N point Veneziano formulae give a new approach to multi-Regge phenomenology. They include resonances and Regge exchange in a dual manner, they ought to be valid for all values of the subenergies, and they have well-determined and theoretically plausible Regge residues. All the main drawbacks of the multi-Regge model are thereby removed. The new difficulties are the computation and what to do about the Pomeron. The former was removed for three-particle final states by J. Hopkinson,

who wrote a fast computer program for B_5 . The first application to $K^-p \rightarrow \pi^- \pi^+ \Lambda$, where Pomeron exchange is forbidden, was presented at the Lund conference by Petersson and Törnqvist (1969). It used complex trajectories adjusted to the known resonance widths. The entire world data on this process from 3 to 10 GeV/c was then fitted - spectra, angular distributions and all - with one free parameter, the B_5 normalization.

This success naturally encouraged extension to other processes. Last I heard, the Chan army had descended on the CERN bubble chamber groups like a swarm of locusts, and were eating every leaf in sight. The poor experimentalists are actually calculating B_5 themselves, to save some shreds of their data summary tapes from theoretical mandibles. However only five processes had been completely explained by the time I left.

They are doing reactions dominated by vector + tensor exchange first. At present, it is not clear how to include Pomerons in these models, while the lack of $\pi\eta$ EXD makes pion-exchange difficult also. However they are studying these problems. The baryons are put in with spin zero. They argue that because fermion trajectories are low-lying, this should not be crucial. One B_5 term is then written down for each duality diagram, and the relative coefficients chosen to give the known signatures, e.g., of the Δ trajectory. There remains one over-all normalization constant to be fitted to the data.

Note one important and very novel feature of these models: the legs of the B_5 formulae can be permuted by crossing symmetry to predict ten different $2 \rightarrow 3$ reaction channels, several of which are often observable. In each reaction channel a considerable number of charge combinations are possible. The Chan group are thus aiming at one parameter fits, not just to the entire world data on one process, but to as many as 20 processes simultaneously.

The first application was for two processes both observable and related by crossing

$$K^- p \rightarrow \pi^- \pi^+ \Lambda, \quad (6.1)$$

$$\pi^+ p \rightarrow K^+ \pi^+ \Lambda. \quad (6.2)$$

(6.1) has no Pomeron exchange, while the dominant $K^+ Y_1^*(1385)$ sub-channel of (6.2) is also Pomeron free. Figure 2 shows the two sets of duality diagrams they considered. (They could have taken any linear combination of the two, so it is slightly cheating to say they only have one parameter). The second are the standard Harari-Rosner ones (Imachi et al. 1968, Harari 1969, Rosner 1969), the first is an heretical model with four quarks and an antiquark in the $\bar{K}N$ channel. The fits seem conclusively in favour of the heretical model. This is basically because no resonances are observed in the $\pi^+ p \rightarrow K^+ Y_1^{*+}$ channel. Mandelstam's non-planar formula (Mandelstam 1969c, e, Burnett and Schwarz 1969), which has resonances in six channels instead of five, would not therefore help. The discrepancy (if it persists after they have done spin properly) requires either that $SU(3)$ break down in an unexpected way, or else exotics. Graphs of the fit to (6.1) are in their paper (Petersson and Törnqvist 1969) and in Chan's London talk (Chan 1969a). When they crossed to (6.2) (Törnqvist 1969), they found the normalization was too large by a factor two (Figure 3). To give a standard of comparison, a well-known backward $\pi^- p$ Regge fit when extrapolated to the Δ pole was out by a factor 2000. 2 may not be as good as 1, but it still ranks among the very best existing checks of crossing symmetry. In Fig. 3, which shows (6.2), the graph of the integrated cross-section displays the factor 2 discrepancy, but the other distributions have been renormalized. The solid line is the heretical PT duality diagram model, the dotted line the orthodox HR one. Note especially the $\pi^+ p$ angular distributions which should have a zero in the middle if trajectories of definite signature like the Δ dominate.

Next the Chan group attacked the four channels

$$KN \rightarrow K\pi N, \quad (6.3)$$

$$\bar{K}N \rightarrow \bar{K}\pi N, \quad (6.4)$$

$$\pi N \rightarrow K\bar{K}N, \quad (6.5)$$

$$\bar{N}N \rightarrow K\bar{K}\pi, \quad (6.6)$$

all observable and related by crossing symmetry. The duality diagrams are this time the orthodox Harari-Rosner ones. They could not think of any plausible alternative. 21 charge states have enough data to be worth studying, some being very well measured indeed. These were arranged in three groups, no doubt corresponding to the infrastructure of Chan's army. Previous multi-Regge fits suggest that group I goes by vector+Pomeron exchange, group II by vector exchange only, group III by pion+Pomeron exchange. Group II is therefore the immediate target, and my latest news is that three of its processes have just been conquered by a battalion consisting of Chan, Raitio, Thomas and Törnqvist (1969). These are

$$K^+\rho \rightarrow K^0\pi^+\rho, \quad (6.7)$$

$$\bar{K}^-\rho \rightarrow \bar{K}^0\pi^-\rho, \quad (6.8)$$

$$\pi^-\rho \rightarrow K^0\bar{K}^-\rho. \quad (6.9)$$

Once again, the normalization of (6.9) was predicted too large by a factor 2. This is shown in the integrated cross-sections (Fig. 4) but the other distributions of (6.9) have been renormalized. Note that

(6.7) and (6.9) have cross-sections differing by factors of 50. The quantity of data explained by this one parameter is quite considerable. Indeed it seemed to me to go on for nearly a hundred graphs. My remaining figures 4-12 are an attempt at a compact presentation. Occasional minor discrepancies are ascribed to pion exchange.

The fit is obviously significant, but it tests global duality rather than local. If the daughters were split wrong, you would never notice in such experiments. Veneziano always seems to look beautiful against tons of data accurate to 10%, but not nearly so impressive for a small quantity accurate to 1% (as in meson-baryon elastic scattering). This is what we might expect from the nuclear physics analogies discussed in Section 3.

Chan et al. ought to fit simultaneously the two-particle final states obtained by factorization at the first baryon pole. In particular this should enable spin to be included without extra parameters.

B_5 has also been successfully applied to $\bar{p}p$ (at rest) $\rightarrow 4\pi$ by Hopkinson and Roberts (1969). Pokorski and Satz (1969) tried fixed pole Pomeron exchange leading to B_4 , as a model for high energy $NN \rightarrow NN^*$ diffraction dissociation. They found the natural off-shell dependence of B_4 made any Pomeron vertex unnecessary. Thus perhaps one can include the Pomeron in multi-Veneziano fits without many more parameters.

The geographical distribution of this work is disturbing to anyone who believes in free enterprise. Apart from some beginnings at Urbana (Jones and Wyld 1969), there has been no B_5 phenomenology outside CERN. Traditional multi-Regge fits are about as competitive against B_5 as horses against a tank. I certainly hope that Chan's American competitors are not going to submit tamely to annihilation (because then Chan will never put in spin properly).

7. CONCLUSION

It seems to me there has been a radical change of style in particle theory in the last two years. The physical ideas are stronger and more vivid, the mathematics more transparent, the structures more daring and beautiful. It is very satisfying that this originality should have been rewarded by Nature.

I am grateful to Dr. G.L. Shaw and Professor R.H. Dalitz for invitations to give this review talk, and to audiences at CERN, Princeton and Stony Brook for comments on it.

A P P E N D I X

DUALITY FOR BARYONS IN SU(3)

Here we consider the SU(3) structure of Veneziano terms for meson-baryon scattering. The (s,t) crossing matrix has been given explicitly by de Swart (1964), and the (s,u) crossing matrix is easily obtained by multiplying it by phase factors. We use his notations except for $[8_s]$ and $[8_a]$ instead of $[8_1]$ and $[8_2]$. The labels of SU(3) states will be put in square brackets, and their numerical coefficients outside. In the s channel we define

$$[8_F] = 5[8_{ss}] + 3\sqrt{5}F\{[8_{sa}] + [8_{as}]\} + 9F^2[8_{aa}], \quad (A.1)$$

where F is the value of the F/D ratio. In the t channel, coupling to tensor mesons is

$$\sqrt{5}D[8_{ss}] + 3F[8_{sa}] \quad (A.2)$$

and to vector mesons

$$\sqrt{5}D[8_{as}] + 3F[8_{aa}]. \quad (A.3)$$

First we consider the Veneziano (s,t) term. Define in the s channel [using (A.1)]

$$[S] = [8_1] + 16[1], \quad (A.4)$$

$$[D] = 8[10] + [8_{-1/3}], \quad (A.5)$$

take their sum and difference, and then multiply by the (s,t) crossing matrix. This gives the t channel decompositions (no exotics)

$$[S] + [D] = -4\sqrt{5} [8_{sa}] + 12 [8_{aa}] + 32 [1], \quad (A.6)$$

$$[S] - [D] = 10 [8_{ss}] - 2\sqrt{5} [8_{sa}] - 6\sqrt{5} [8_{as}] + 6 [8_{aa}]. \quad (A.7)$$

Comparing (A.6) to (A.3), we see that it gives pure F coupling to the vector mesons. The electric A' amplitude at the vector meson poles must therefore have this structure. Comparing (A.7) with (A.2), we see that it gives $F/D = -1/3$ for the tensor mesons. This will decouple the f from πN , as is required for the A amplitude according to Regge fits (Lovelace 1969b). Therefore, in the (s,t) Veneziano term we take $[S]$ for the nucleon $[8; \frac{1}{2}^+] - [1; \frac{3}{2}^-]$ trajectory, thus decoupling the Σ 's from $\bar{K}N$ in agreement with experiment. For the $[10; \frac{3}{2}^+] - [8; \frac{5}{2}^-]$ trajectory, we take $[D]$, thus decoupling $\Lambda(\frac{5}{2}^-)$ from $\bar{K}N$, again in approximate agreement with experiment. The relative coefficients of $[S]$ and $[D]$ will be different in the A and B amplitudes. They are determined by requiring the leading t channel trajectory to couple only to (A.7) in the A amplitude, and only to (A.6) in the A' amplitude at the vector meson mass. This distinction can also be used to cancel the $\frac{3}{2}^-$ on the $[D]$ trajectory and the $\frac{3}{2}^+$ on the $[S]$ trajectory (Lovelace 1969b). The (u,t) terms follow immediately from crossing, which must be even in A and odd in B .

Next we have to construct (s,u) terms which will give our trajectories the required signatures. In particular we want the $[8_{-1/3}]$ to be present only in one signature state of $[D]$, and the $[1]$ only in one signature state of $[S]$. At first sight this is quite easy - the combinations

$$[\bar{S}] = \frac{1}{2} [8_1] - 16 [1], \quad (A.8)$$

$$[\bar{D}] = 16 [10] - [8_{-1/3}] \quad (A.9)$$

cross into u channel amplitudes with no exotics. Thus we would get the $[S]$ trajectory structure

$$\frac{3}{2} [8_1] \quad \text{at} \quad j = \frac{1}{2}, \frac{5}{2}, \text{etc.} \quad (A.10)$$

$$\frac{1}{2} [8_1] + 32 [1] \quad \text{at} \quad j = \frac{3}{2}, \frac{7}{2}, \text{etc.} \quad (A.11)$$

and the $[D]$ trajectory structure

$$24 [10] \quad \text{at} \quad j = \frac{3}{2}, \frac{7}{2}, \text{etc.} \quad (A.12)$$

$$-8 [10] + 2 [8_{-1/3}] \quad \text{at} \quad j = \frac{5}{2}, \frac{11}{2}, \text{etc.} \quad (A.13)$$

by adding the (s,u) terms (A.8) and (A.9) to the (s,t) terms (A.4) and (A.5) respectively. (A.13) would give a ghost $[10]$ for the simplest Veneziano formula but this can be remedied by introducing opposite signature satellite to make the $[10]$ residue vanish at the $5/2^-$ position and change sign higher up. More serious is that $[\bar{S}]$ and $[\bar{D}]$ in the s channel do not cross into $[\bar{S}]$ and $[\bar{D}]$ in the u channel. To see whether there is any way out, we calculate the eigenvectors of the (s,u) crossing matrix. (They correspond to pure states in the t channel.) If we require no exotics, then there are two even eigenvectors

$$\begin{aligned} [10] : [8_{ss}] : [8_{sa}] &= [8_{as}] : [8_{aa}] : [1] = \\ -2x + 4y : -\frac{15}{2}x - \frac{5}{2}y : -\frac{\sqrt{5}}{2}x + \sqrt{5}y : \frac{25}{2}x + \frac{13}{2}y : -20x - 16y. \end{aligned} \quad (A.14)$$

where x, y are arbitrary, and one odd eigenvector

$$- 2 [10] + \frac{1}{2} [8_{1/3}] - 4 [1].$$

(A.15)

$x = 2y$ in (A.14) eliminates $[10]$ and gives the usual meson-meson formula. Otherwise (A.14) is not simple in terms of F/D ratios. (A.15) is $F = +1/3$, not $-1/3$ as in (A.5). We might suppose that (A.14) would be suitable for the Veneziano (s,u) term in the A amplitude, and (A.15) for B . However, it is clear that no combination of them and the (s,t) terms will give acceptable signature and F/D structures.

The obvious conclusion is that s channel parents must couple to u channel daughters which have different F/D ratios. However, in view of the strong evidence for an exotic $Z_1(1900)$ resonance at granddaughter level, there is then no justification for excluding u channel exotics.

Of the material in this Appendix, general EXD schemes which would eliminate exotics have been given by several groups (see Section 5). The points about vector meson F/D ratios, and especially about crossing matrix eigenvectors have not been noticed before. I am grateful to Dr. C. Michael for discussion of this Appendix.

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FIGURE CAPTIONS

- Figure 1 : πN charge-exchange at 10 GeV/c, compared with an absorbed Veneziano fit.
- Figure 2 : Duality diagrams for $K^-p \rightarrow \pi^- \pi^+ \Lambda$. Top line Petersson-Törnqvist model, bottom line Harari-Rosner model.
- Figure 3 : Törnqvist fit to $\pi^+ p \rightarrow K^+ Y_1^*(1385)^+$.
- Figure 4 : Chan et al. fit to (6.7), (6.8), (6.9): total cross-sections.
- Figure 5 : Chan et al. fit: mass spectra of (6.7).
- Figure 6 : Chan et al. fit: mass spectra of (6.8).
- Figure 7 : Chan et al. fit: mass spectra of (6.9).
- Figure 8 : Chan et al. fit: cross-sections in resonance bands.
- Figure 9 : Chan et al. fit: momentum transfer distributions of (6.7).
- Figure 10 : Chan et al. fit: longitudinal momentum and π^+ angular distributions of (6.7).
- Figure 11 : Chan et al. fit: momentum transfer distributions in resonance bands.
- Figure 12 : Chan et al. fit: resonance decay distributions.

FIG. 1

FIG. 1

FIG. 2.

FIG. 3

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7a.

FIG. 7b.

FIG. 8

FIG. 9

FIG. 10

FIG.11

FIG. 11

FIG.12

FIG. 12a.

FIG. 12 b.

A_2

FIG. 12c.