A model for the parton distributions in hadrons

A. Edin\textsuperscript{a,1} and G. Ingelman\textsuperscript{a,b,2}

\textsuperscript{a} Dept. of Radiation Sciences, Uppsala University, Box 535, S-751 21 Uppsala, Sweden
\textsuperscript{b} Deutsches Elektronen-Synchrotron DESY, Notkestraße 85, D-22603 Hamburg, Germany

Abstract: A simple model is presented for the parton distributions in hadrons. The parton momenta in the hadron rest frame are derived from a spherically symmetric, Gaussian, distribution having a width motivated by the Heisenberg uncertainty relation applied to the hadron size. Valence quarks and gluons originate from the ‘bare’ hadron, while sea partons arise mainly from pions in hadronic fluctuations. Starting from a low $Q^2_0$ scale, the distributions are evolved with next-to-leading order DGLAP and give the proton structure function $F_2(x, Q^2)$ in good agreement with deep inelastic scattering data.

\textsuperscript{1}edin@tsl.uu.se
\textsuperscript{2}ingelman@tsl.uu.se
The parton distributions in hadrons play a very important role in particle physics. The factorization theorems of QCD show that they can be used to calculate the cross-section for hard processes with incoming hadrons by convoluting them with parton level cross-sections calculated using perturbation theory. The parton distributions are universal so that each hadron has a unique parton structure which can be used to calculate all hard processes involving that hadron. The parton distributions $f_i(x, Q^2)$ are interpreted as the probability to find a parton $i$ (quark of some flavour or gluon) with a fraction $x$ of the hadron momentum when probed by the momentum transfer $Q^2$. The $Q^2$-dependence is very successfully described by the DGLAP equations [1] in perturbative QCD (PQCD). Given the input distributions in $x$ at a scale $Q^2_0$ large enough for PQCD to be applicable, one can calculate the distributions at any higher $Q^2$.

However, this starting $x$-shape, which depends on non-perturbative QCD dynamics of the bound state hadron, has not yet been successfully derived from first principles. Instead they are obtained by fitting parameterizations to data, in particular structure function measurements in deep inelastic lepton-nucleon scattering (DIS), e.g. the GRV [2], CTEQ [3] and MRS [4] parameterizations.

In this Letter we present a simple theoretical model to derive the parton distributions from the non-perturbative dynamics confining the partons in hadrons. The basic idea is to define the parton momentum distributions in the hadron rest frame where they should be spherically symmetric. The shape of the momentum distributions should be close to a Gaussian as a result of many interactions binding the partons in the hadron. The typical width of this distribution is a few hundred MeV from the Heisenberg uncertainty relation applied to the hadron size. The Gaussian momentum distribution also has phenomenological support. The Fermi motion in the proton provides the ‘primordial transverse momentum’, which has been extracted from deep inelastic data and found to be well described by a Gaussian distribution of a few hundred MeV width [5]. However, this width depends on at what scale $Q^2_0$ it is extracted, since perturbative QCD effects from emission of partons in the initial state may also contribute.

This approach is not intended to provide the full wave function for the hadron, but only the four-momentum $k$ of a single parton which is probed. All other partons are treated collectively as a single remnant with four-momentum $r$, which corresponds to integrating out all other information in the hadron wave function.

The arguments above define only the three-momentum of the probed parton. The energy component does not have the same simple connection to the Heisenberg uncertainty relation. For simplicity, we assume it to be the current mass of the parton plus a Gaussian variation with the same width as for the three-momentum components. Thus, partons can be off-shell at this soft scale as expected from the soft binding interactions. This means a parton fluctuation life-time corresponding to the hadron radius. The parton is probed at the scale $Q^2_0$ supplied either by a virtual photon directly or indirectly through the starting point of a DGLAP evolution chain. The scale $Q^2_0$ must be sufficiently large, such that the parton can be considered ‘free’ in the interaction. Fig. 1 illustrates the basic process and defines the relevant four-momenta.

The coordinate system is chosen with the negative $z$-direction along the probe. The momentum fraction $x$ of the parton is then defined as the light-cone fraction along the positive direction, i.e. $k_+ = xp_+$, which is equivalent to $k_z = xp_z$ in a frame where $p_z$ is
Figure 1: The probe \( q \), which can be either a photon or a parton initiating a DGLAP evolution chain, probes a parton \( k \), giving a scattered parton \( j \) and a remnant system \( r \).

large. The light-cone fraction \( x \) is invariant with respect to boosts along the \( z \)-axis. It is only possible to scatter on partons that give an allowed final state. The scattered parton must have a mass-squared in the range \( 0 < j^2 < W^2 \), where \( W \) is the invariant mass of the hadronic system. Furthermore, the hadron remnant must have a sufficient mass to contain the remaining partons, i.e. \( r^2 > \sum_i m_i^2 \) where the sum is over all partons in the remnant whose internal dynamics is neglected. At large energy \( j^2 = k^2 - q^2 \), so that \( j^2 > 0 \) is equivalent to \( x > 0 \), and \( j^2 < W^2 \) means that \( x < 1 \). For cases where there is only light quarks in the remnant it is enough to require \( r^2 > 0 \).

The parton density distributions are calculated numerically from the model using a Monte Carlo technique. The momentum components of the parton (to be probed) is chosen from a Gaussian distribution, as described above, which provides the vector \( k \). The four-momentum \( p \) of the hadron at rest is simply given by its mass. The four-momentum \( q \) of the probe is given by its virtuality \( Q_0^2 \) and \( q_- \) which must be large (but the exact value is not important). The latter is to ensure that the mass of the produced hadronic system is above the resonance region. The internal dynamics of the remnant (which is not measured) can then be neglected and the probability for hadronization is unity. The four-momenta \( j \) and \( r \) are then calculated from energy-momentum conservation and the exact kinematical constraints checked. If they are fulfilled, the light-cone fraction \( x \) of the parton is added to the parton distribution. Iterating this procedure gives the parton density distributions \( f_i(x) \) at \( Q_0^2 \).

As an example of how the parton distributions are obtained in the model we take the proton and first consider only the valence quarks. It is obvious that the distributions \( u_v(x) \) and \( d_v(x) \) must satisfy the normalization conditions \( \int_0^1 u_v(x) dx = 2 \) and \( \int_0^1 d_v(x) dx = 1 \), to get the correct quantum numbers for the proton. In addition, there must be a gluon distribution \( g(x) \) to represent the colour field and account for the fraction of the proton momentum carried by electrically neutral partons. Since the gluons are confined in essentially the same region (i.e. the proton) as the quarks, they are assumed to have the same basic Gaussian shape as the valence quarks. The normalization of the gluon density is given by the momentum sum rule \( \int_0^1 (xu_v(x) + xd_v(x) + xg(x)) \ dx = 1 \).

The parameters of the model for the valence partons are the widths (\( \sigma \)) of the Gaussian distributions for \( u_v, d_v \) and \( g \), whereas their normalization is given by the number and momentum sum rules. The widths are, as discussed, expected to be a few hundred MeV, but since they cannot be predicted accurately we treat them as free parameters and obtain their values by fitting to structure function data as described below. The resulting Gaussian
widths are $\sigma_u = 180$, $\sigma_d = 150$, $\sigma_g = 135$ MeV which are reasonable considering that the proton radius is $\sim 200$ MeV$^{-1}$. Since $\sigma$ applies in each dimension, one obtains a two-dimensional primordial transverse momentum with $\langle k^2_\perp \rangle = 2\sigma^2$ in basic agreement with data [5].

The momentum-weighted distributions $x f_i(x)$ for the proton as obtained from the model are displayed in Fig. 2a. The distributions look like conventional valence quark parameterizations at a low $Q^2$ scale and are similar to the GRV parameterization [2] which is also defined at a low scale $Q^2_0$. The proton momentum is in our case carried to 43%, 18%, 39% by $u$, $d$, and gluons, respectively, and the integrated gluon number density is $\int_0^1 g(x) dx = 2.4$.

Following the same line of reasoning as for the proton, the model provides the parton distributions for other hadrons. Although different hadrons may have somewhat different sizes, we assume as an approximation the same widths of the Gaussian distributions for quarks and gluons as in the proton. However, if there are more than one quark of the same kind, e.g. $u$-quarks in the proton, it is a separate parameter which may reflect the slightly reduced available region due to the Pauli principle. The $u$-quark distribution in the proton is indeed found to have a $\sim 20\%$ larger width corresponding to a slightly smaller effective size.

The resulting valence quark and gluon distributions in the pion are shown in Fig. 2b and found to be similar to normal parameterizations of parton densities in the pion such as GRV [12]. Applying the model to hadrons containing heavier quarks we obtain the distributions shown in Figs. 2cd. In particular, the hard charm quark distribution is here a result of the charm quark mass in the applied kinematical constraints discussed above.

The reason why the quark distributions peak around $1/3$ in the proton and $1/2$ in the pion (see Fig. 2ab) has in this model nothing to do with having three or two valence quarks, which is anyhow dubious since one is then neglecting the substantial fraction of the hadron momentum carried by gluons. Instead the peak of the valence distributions depend on the ratio of the Gaussian width (or inverse hadron size) and the hadron mass, but is influenced by the kinematical contraints. A large quark mass has a substantial effect as illustrated in Figs. 2cd.

To illustrate the shapes of the valence quark and gluon distributions they are fitted to the functional form $f(x) = N x^a (1-x)^b$ that are often used to describe valence distributions. The results for the proton and the pion ($\pi^+$) are

$$p : \quad xu(x) = 13 x^{1.2} (1-x)^{3.4}, \quad xd(x) = 13 x^{1.4} (1-x)^{5.0}, \quad xg(x) = 47 x^{1.4} (1-x)^{6.2}$$

$$\pi^+ : \quad xu(x) = 2.3 x^{1.1} (1-x)^{1.0}, \quad xd(x) = 2.3 x^{1.1} (1-x)^{1.0}, \quad xg(x) = 1.9 x^{1.1} (1-x)^{1.0}$$

It is interesting to note that the powers of the $(1-x)$ factors are quite similar to the ones in parton parametrizations. Their values are often motivated by counting rules [6], although this does not generally work quite well since the $u$, and the $d$, in the proton have different powers and $g$ in the pion has the same power as the valence quarks.

Given the valence distributions of the proton, we apply QCD evolution starting from a low staring scale $Q_0 = 0.6 - 1.0$ GeV and evolve to higher $Q^2$ to make a comparison with data possible. The evolution is performed using the CTEQ program [3] which solves the
Figure 2: The valence quark and gluon distributions obtained from the model applied to (a) the proton, (b) the pion, (c) the strange meson $K^+$, (d) the charm meson $D^0$.

next-to-leading-order (NLO) DGLAP equations in the $\overline{MS}$ scheme. The varying effective number of quark flavours ($n_f$) with $Q^2$ is here taken into account using the standard procedure with $\Lambda^{(n_f)}$. For convenience, the starting distributions in Fig. 2a are fitted to the shape $xf(x, Q_0^2) = N x^a (1-x)^b (1+cx^d)$, that is also used by the CTEQ collaboration in their parton distribution fits. The evolution then gives the distributions $xf_i(x, Q^2)$ in NLO $\overline{MS}$ scheme.

From these we calculate the structure function $F_2(x, Q^2)$ in NLO and compare with experimental data from fixed target muon scattering (NMC [7]) and the HERA $ep$ collider (ZEUS [8]) as shown in Figs. 3 and 4. A fit to this data is made by varying the model parameters resulting in the numerical values for the Gaussian widths mentioned above. When varying $\Lambda^{(5)}$ between 150 and 300 MeV, we find that these widths are essentially constant (within errors), but the cut-off parameter $Q_0$ varies linearly with $\Lambda$ as expected since only their ratio enters in the QCD evolution. We take $\Lambda^{(5)} = 0.23 GeV$, corresponding
Figure 3: The DIS structure function $F_2$ versus $Q^2$ in bins of $x$. Fixed target NMC data [7] compared to the model starting from only valence quarks and gluons (dashed) and including also a sea quark component (full).

to the measured value of $\alpha_s(M_Z)$, and get $Q_0 = 0.85 GeV$, which is a quite reasonable value for the PQCD cut-off. The $F_2$ data for the proton is most sensitive to $Q_0^2$, the $u_v$ width and the sea distributions, whereas the $d_v$ and the gluon widths are less constrained. They could be determined better by including data from, e.g., neutrino DIS, Drell-Yan and prompt photon production.

Our simple model with only valence quarks and gluons gives a surprisingly good overall result, shown by the dashed lines in Figs. 3 and 4. However, the resulting structure function is too low at small $x$ and small $Q^2$; in particular compared to the NMC data in the region $0.008 < x < 0.1$. Small $x$ at large $Q^2$ is not that bad compared to HERA data, since it is strongly influenced by the QCD evolution. This deficiency at small $x$ is an indication for the need of a sea quark contribution already at the starting scale $Q_0^2$. This has also been noted by GRV [2] and they introduced sea quark parameterizations at their low $Q_0^2$ scale.
Figure 4: The DIS structure function $F_2$ versus $x$ in bins of $Q^2$. HERA $ep$ collider data from ZEUS [8] compared to the model starting from only valence quarks and gluons (dashed) and including also a sea quark component (full).

We do not want to introduce some ad hoc parameterization of sea quark distributions to solve this problem, but rather extend our model in a natural way to give a prescription for the sea distributions. Since our model is based on quantum fluctuations in the proton, we consider what fluctuations that are most important in the non-perturbative region at scales below $Q_0^2$. It seems appropriate that one should use a quantum mechanical basis of hadronic states and consider hadronic fluctuations. Since the pion mass is so small, fluctuation with virtual pions should dominate. These will have a life-times of the order $\sim 1/m_\pi$, which is similar to the widths of the parton momentum distributions and much
longer than the time-scale $1/Q_0$ of the probe.

Therefore, one should consider the proton wave function as an expansion in the hadronic Fock states containing a pion, i.e.

$$|p\rangle = a_0|p\rangle + a_1|p\pi^0\rangle + a_2|n\pi^+\rangle + a_3|\Delta\pi\rangle + \ldots,$$

(3)

The scattering of partons in such a pion will then generate sea quark and gluon distributions. Scattering on the baryons in these fluctuations can be neglected as a first approximation, since they give only smaller corrections to the valence distributions in the dominating ‘bare’ proton, i.e. $a_0|p\rangle$. The fluctuation into $|\Delta\pi\rangle$ states are less probable due to the higher $\Delta$ mass and can at first be neglected. We note that already the fluctuations into $|p\pi^0\rangle$ and $|n\pi^+\rangle$ leads to a breaking of the $u$-$d$ flavour symmetry in the sea, which might explain the observed difference of the $\bar{u}$ and $\bar{d}$ sea quark density parameterizations [9]. However, this is not taken into account in this first study, where a symmetric sea is obtained using for simplicity pion isospin symmetry or effectively considering only the $|p\pi^0\rangle$ term.

The parton distributions of the pion follow from the model as described above. In addition one must specify the momentum distribution of the pion fluctuations in the proton. This is derived using the same arguments as for the partons, i.e. using a spherically symmetric, Gaussian momentum distribution in the proton rest frame. However, the width is now expected to be of order tens of MeV based on the typical momenta of pions in the virtual pion cloud around a proton or in a nucleus. Again, we treat this width as a free parameter which is fitted to data.

The normalization is in principle given by the probability amplitude coefficients $a_i$ in eq. (3). These are partly given by Clebsch-Gordan coefficients, but also depend on non-perturbative dynamics that cannot be calculated from first principles in QCD. Hence, we represent them by a free parameter for the total amount of the proton momentum that is carried by the sea partons generated from the pions.

Applying the kinematical constraints as above, one obtains the pion momentum distribution as shown in Fig. 5a, where $x_\pi$ is the light-cone fraction of the proton momentum carried by the pion. One should note that this distribution is softer than the pion flux factor in Regge phenomenology which has a broad maximum at $x_\pi = 0.2 - 0.5$ [10]. There is no reason they should be the same, since the first case relates to a low energy quantum fluctuation whereas the Regge approach is for high energy hadron interactions, in particular diffractive-like interactions with a high energy leading particle produced.

The proton sea quark distributions are then obtained by folding the pion momentum distribution in the proton with the valence quark and gluon distributions in the pion. Applying the same fitting procedure of the model to the data in Figs. 3 and 4, we obtain the pion parameters $\sigma_\pi = 52$ MeV and that 7.7% of the proton momentum is carried by the sea partons. This width is of the expected magnitude and the amount of sea is similar to the GRV parameterization [11]. The resulting parton distributions, including the sea, in the proton at $Q_0 = 0.85$ GeV are shown in Fig. 5b.

The evolved parton distributions, including the sea from pion fluctuations, provide a quite good description of the structure function data in Figs. 3 and 4. The parameter values of the model, which are collected in Table 1, are correlated and the minimum not
very well-defined, especially for $\sigma_d$ and $\sigma_g$. Therefore, some variations of the parameter values can result in essentially equally good fits. The resulting $\chi^2$ is about 2 per degree of freedom, which is not as good as in standard parton density parameterizations, such as GRV [2], CTEQ [3] and MRS [4]. However, these have many more parameters and do not provide any physical model for the non-perturbative input parameterizations.

<table>
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<th>$Q_0$</th>
<th>$\sigma_u$</th>
<th>$\sigma_d$</th>
<th>$\sigma_g$</th>
<th>$\sigma_\pi$</th>
<th>$N_{\text{sea}}$</th>
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<tbody>
<tr>
<td>850 MeV</td>
<td>180 MeV</td>
<td>150 MeV</td>
<td>135 MeV</td>
<td>52 MeV</td>
<td>7.7 %</td>
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Table 1: The model’s parameter values obtained from the fit using $\Lambda^{(5)}_{\overline{MS}} = 230$ MeV.

The virtue of our parton distributions is that they are derived from a simple physical model with few assumptions and few free parameters. Although we have determined the parameter values from data, it is very comforting to note that their values agree well with the definite expectations based on the model.

In this Letter we have presented a model to describe the parton distributions in hadrons, in an attempt to understand the soft interactions which confine the quarks and gluons to hadrons. Normally the parton distributions are obtained by fitting a parametrization to experimental data, giving little insight into the physical processes involved.
Our model has two main contributions to the parton distributions. Valence quarks and gluons come directly from the ‘bare’ hadron, \( e.g. \langle p \rangle \) for the proton, while sea quarks and ‘sea’ gluons come from hadronic fluctuations, mainly with pions such as \( \langle p \pi^0 \rangle \). This provides a one-to-one correspondence between the Fock state expansion of the hadronic wave-function of the proton and the partonic structure in terms of valence and sea partons. Although the model is quite simple in this first attempt, it provides parton densities which, when evolved with standard PQCD, successfully fits structure function data over a wide range in \( x \) and \( Q^2 \).

The model straightforwardly gives the parton densities in other hadrons. Of course, this must be tested by comparing to measurements, \( e.g. \) the parton distributions for the pion. Considering also proton fluctuations into charm, \( e.g. \langle \Lambda_c^+ D^0 \rangle \), would lead to a charm quark component in the proton derived in a different way compared to the intrinsic charm model [13]. Thus, our simple model not only describes the parton distributions in the proton well, it also has interesting features that deserve further investigations.

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References

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