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COMPLEX LEE-WICK POLES AS CDD GHOSTS

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A B S T R A C T

The Zachariasen-Thirring model with intermediate unstable elementary objects, described by a pair of complex conjugate poles with complex conjugate residues, is solved. It is shown that the replacement of stable intermediate particle by the above mentioned pair of complex poles is equivalent to the introduction of a CDD ghost. Also, a modified renormalized solution is given leading to Lee-Wick regularization of the intermediate stable particle propagator.

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1. Recently Lee and Wick ¹⁾ considered the elementary unstable objects described by a pair of complex conjugate masses ²⁾

$$\kappa_{0\pm}^2 = \kappa_0^2 \pm i\Gamma \quad (1)$$

The propagator of this unstable object is given by the formula ³⁾

$$\Delta^F(p^2) = \frac{1}{2} \left[\frac{1+ia}{p^2-\kappa_0^2+i\Gamma} + \frac{1-ia}{p^2-\kappa_0^2-i\Gamma} \right] \quad (2)$$

If we have any Lagrangian $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$, describing two interacting scalar fields A and B with bare masses M_0 and μ_0 respectively

$$(\square - M_0^2) A(x) = - \frac{\delta \mathcal{L}_{\text{int}}(A, B)}{\delta A(x)} \quad (3a)$$

$$(\square - \mu_0^2) B(x) = - \frac{\delta \mathcal{L}_{\text{int}}(A, B)}{\delta B(x)} \quad (3b)$$

one obtains the replacement of the free propagator of the B particle in the perturbation diagrams

$$\frac{1}{p^2 - \mu_0^2 + i\epsilon} \rightarrow \frac{1}{2} \left[\frac{1+ia}{p^2 - \kappa_0^2 + i\Gamma} + \frac{1-ia}{p^2 - \kappa_0^2 - i\Gamma} \right] \quad (4)$$

if the Lagrangian $\mathcal{L}$ is modified as follows:

$$\begin{aligned} \mathcal{L} \rightarrow \mathcal{L} + \frac{1}{2} (\partial^\mu C \partial_\mu C + m_0^2 C C) - \\ - f_0 B C \end{aligned} \quad (5)$$

The parameters χ_o^2 , a and Γ , occurring in (4), are expressed by μ_o^2 , m_o^2 and f_o^2 as follows

$$\chi_o^2 = \frac{1}{2} (m_o^2 + \mu_o^2) \quad (6a)$$

$$\Gamma = \left[f_o^2 - \frac{1}{4} (\mu_o^2 - m_o^2)^2 \right]^{\frac{1}{2}} \quad (6b)$$

$$a = \frac{1}{2} (m_o^2 - \mu_o^2) \left[f_o^2 - \frac{1}{4} (\mu_o^2 - m_o^2)^2 \right]^{\frac{1}{2}} \quad (6c)$$

Because a and Γ are real, one gets the following restriction on the parameters, occurring in the substitution (5)

$$f_o^2 > \frac{1}{4} (\mu_o^2 - m_o^2)^2 \quad (7)$$

It should be observed that the free Lagrangian of the field C carries negative metric. On the other hand, the coupling (5) represents a way of introducing a CDD particle ⁴⁾. One can say, therefore, that the replacement (4) can be interpreted as an introduction of a CDD ghost in a renormalized solution.

2. In order to illustrate our conclusion we shall perform the substitution (5) in the Lagrangian, introduced by Thirring ⁵⁾ and describing the Zachariasen model ⁶⁾. The model describes the S wave scattering of A particles (A neutral, scalar, mass $m_A = 1$) due to the interaction $AA \leftrightarrow B$ (B neutral, scalar, mass $m_B < 2$). The interacting S wave pair of A particles with total mass square s ($s \geq 4$) and centre-of-mass co-ordinates x is represented by the field operator $\varphi(x; s)$ ⁷⁾. The modified field equations for Zachariasen model look as follows ⁸⁾:

$$\begin{aligned}
 (\square - m_0^2) C(x) &= -f_0 B(x) \\
 (\square - \mu_0^2) B(x) &= g_0 \int_4^\infty ds \, \varrho^{\frac{1}{2}}(s) \varphi(x; s) + f_0 C(x) \\
 (\square - s) \varphi(x; s) &= g_0 \varrho^{\frac{1}{2}}(s) B(x)
 \end{aligned}
 \tag{8}$$

The field $C(x)$ plays an auxiliary role. One can solve the model by calculating the causal Green functions

$$\begin{aligned}
 \tau(x; s, s') &= -i \langle 0 | T \{ \varphi(\frac{x}{2}; s) \varphi(-\frac{x}{2}; s') \} | 0 \rangle \\
 \tau(x; s) &= -i \langle 0 | T \{ B(\frac{x}{2}) \varphi(-\frac{x}{2}; s) \} | 0 \rangle
 \end{aligned}
 \tag{9}$$

Using the field equations (8) and the canonical commutation relations we get the following set of equations for the Fourier transforms of (9):

$$\begin{aligned}
 (p^2 - \mu_0^2 + \frac{f_0^2}{p^2 - m_0^2}) \tau(p^2; s) &= g_0 \int_4^\infty ds' \, \varrho^{\frac{1}{2}}(s') \tau(p^2; s, s') \\
 (p^2 - s) \tau(p^2; s, s') &= \delta(s - s') + g_0 \varrho^{\frac{1}{2}}(s) \tau(p^2; s)
 \end{aligned}
 \tag{10}$$

Solving (10) one obtains

$$T(p^2; s, s') = \frac{\delta(s-s')}{p^2-s+i\epsilon} + g_0^2 \Delta^F(p^2).$$

$$\begin{aligned} & \cdot \frac{\varrho^{\frac{1}{2}}(s) \varrho^{\frac{1}{2}}(s')}{(p^2-s+i\epsilon)(p^2-s'+i\epsilon)} \left\{ 1 - g_0^2 \Delta^F(p^2) \cdot \right. \\ & \quad \left. \cdot \frac{1}{\pi} \int_4^\infty \frac{\varrho(s'') ds''}{p^2-s''+i\epsilon} \right\}^{-1} \end{aligned} \quad (11)$$

where $\Delta^F(p^2)$ is given by (2) and (6). The application of the reduction formula ⁹⁾

$$\varrho(s) T(s) = \lim_{\substack{s' \rightarrow s \\ s'' \rightarrow s}} [(s-s'')(s-s') T(s; s', s'')] \quad (12)$$

leads to the following result for the scattering amplitude

$$T(s) = \frac{g_0^2 \Delta^F(s)}{1 + \frac{g_0^2}{\pi} \Delta^F(s) \int_4^\infty \frac{\varrho(s') ds'}{s'-s-i\epsilon}} \quad (13)$$

One can express (13) in the renormalized form as follows:

$$T(s) = \frac{\frac{g^2}{s-\mu^2}}{1 + \frac{f^2}{s-m^2} + \frac{g^2(s-\mu^2)}{\pi} \int_4^\infty \frac{\varrho(s') ds'}{(s'-\mu^2)^2 (s'-s-i\epsilon)}} \quad (14)$$

where

$$\mu_0^2 = \mu^2 + \frac{g^2 A(\mu^2) - f^2}{1 - g^2 B(\mu^2)} \quad (15a)$$

$$g_0^2 = \frac{g^2}{1 - g^2 B(\mu^2)} \quad (15b)$$

$$f_0^2 = \frac{f^2(m^2 - \mu^2)}{1 - g^2 B(\mu^2)} \quad (15c)$$

$$m_0^2 = m^2 \quad (15d)$$

and

$$A(\mu^2) = \frac{1}{\pi} \int_4^\infty \frac{\varrho(s') ds'}{s' - \mu^2} \quad (16a)$$

$$B(\mu^2) = \frac{1}{\pi} \int_4^\infty \frac{\varrho(s') ds'}{(s' - \mu^2)^2} \quad (16b)$$

We see that the formula (14) represents the solution of Zachariasen model with the CDD ghost. Particularly, if we put $m_0^2 = \mu_0^2$ [the standard choice of Lee and Wick of $\Delta^F(p^2)$ with real residues], one gets the relation expressing the position m^2 of a CDD ghost in terms of renormalized parameters μ^2 , g^2 and f^2 :

$$m^2 = \mu^2 + \frac{g^2 A(\mu^2) - f^2}{1 - g^2 B(\mu^2)} \quad (17)$$

Besides, because $\lim_{m_o^2 \rightarrow \mu_o^2} \Gamma = f_0$,

the formula (15c) can be treated as the relation expressing the imaginary part of the complex mass of Lee-Wick poles in terms of renormalized parameters. The formula (15b) is identical to the one in the usual Zachariasen model, but the parameter g^2 in the presence of a CDD ghost does not describe the physical coupling constant g_{ph}^2 , defined as follows

$$g_{ph}^2 = \lim_{s \rightarrow \mu^2} (s - \mu^2) T(s) \quad (18)$$

We have instead of (15b)

$$g_{ph}^2 = \frac{g_o^2}{1 - g^2 B(\mu^2) - \frac{f_o^2}{(m^2 - \mu^2)^2}} \quad (19)$$

and the condition giving real value of physical coupling constant is modified in the presence of CDD ghost as follows:

$$1 > g^2 B(\mu^2) \Rightarrow 1 > g^2 B(\mu^2) + \frac{f_o^2}{(m^2 - \mu^2)^2} \quad (20)$$

3. The unstable elementary objects, described by the propagator (1.2), were introduced by Lee and Wick, mainly with the purpose

of regularizing the free propagators. We shall study now the Zachariasen model with the following replacement of the propagator of free B particle:

$$\frac{1}{p^2 - \mu_0^2 + i\epsilon} \rightarrow \frac{1}{p^2 - \tilde{\mu}_0^2 + i\epsilon} - \frac{1}{2} \left[\frac{1}{p^2 - \tilde{m}_0^2 + i f_0} + \frac{1}{p^2 - \tilde{m}_0^2 - i f_0} \right]$$

(21)

The substitution (21) can be split into a sum of the modification of Pauli-Villars type:

$$\frac{1}{p^2 - \mu_0^2 + i\epsilon} \rightarrow \Delta_{PV}^F(p^2) = \frac{1}{p^2 - \tilde{\mu}_0^2 + i\epsilon} - \frac{1}{p^2 - \tilde{m}_0^2 + i\epsilon} = \frac{\tilde{\mu}_0^2 - \tilde{m}_0^2}{(p^2 - \tilde{\mu}_0^2 + i\epsilon)(p^2 - \tilde{m}_0^2 + i\epsilon)}$$

(22)

and the change (4) with $\mu_0^2 = m_0^2 = \tilde{m}_0^2$ [see formula (6)].

Let us introduce the Pauli-Villars modification of field equations of the Zachariasen model, leading to the replacement (22). We have

$$\begin{aligned}
 (\square - \tilde{\mu}_0^2)(\square - \tilde{m}_0^2) B(x) &= (\tilde{\mu}_0^2 - \tilde{m}_0^2) \cdot \\
 &\cdot g_0 \int_4^\infty ds \, s^{\frac{1}{2}}(s) \varphi(x; s) \\
 (\square - s) \varphi(x; s) &= g_0 s^{\frac{1}{2}}(s) B(x)
 \end{aligned}
 \tag{23}$$

The scattering amplitude, obtained from (23), has a form (13), where $\Delta^F(s)$ has been replaced by $\Delta_{PV}^F(s)$ [see (22)]. Performing the renormalization, we arrive at the following final result:

$$T_{PV}(s) = \frac{\frac{g^2}{s - \mu^2}}{1 + g^2(s - \mu^2) \left[\frac{1}{R} + \frac{1}{\pi} \int_4^\infty \frac{g(s') ds'}{(s' - \mu^2)^2 (s' - s - i\epsilon)} \right]}
 \tag{24}$$

where

$$R = g_0^2 (\tilde{\mu}_0^2 - \tilde{m}_0^2)
 \tag{25}$$

and $\tilde{\mu}_0^2, \tilde{m}_0^2$ are expressed by R, μ^2 and g^2 .

Let us observe that

$$T_{PV}(s) \xrightarrow{s \rightarrow \infty} \frac{R}{s^2}
 \tag{26}$$

One can conclude that, like in the case of two-point propagator, if $T(s)$ vanishes for large s stronger than $1/s$, this implies the presence of states with negative metric in underlying space of particle states.

4. We have shown that the introduction of Lee-Wick poles in Zachariasen model is equivalent to the introduction of a CDD ghost. Usually, the existence of such abnormal solutions is forbidden by the analyticity assumptions. Using the theory of Herglotz functions ¹⁰⁾ it can be shown that if the singularities of partial wave scattering amplitude in the complex energy plane occur only on the real axis, the CDD residua should be positive. Because, however, the Lee-Wick concept of unstable elementary object violates causality, this restriction on the sign of the residuum cannot be applied.

It should be mentioned that the substitution (5) is equivalent to the following one ¹¹⁾

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{1}{2} (\partial^\mu C \partial_\mu C + m_0^2 C C) - i f_0 B C \quad (27)$$

We see that there are the two following ways of introducing the modification (4)

- a) by introducing the auxiliary field with negative metric and real bilinear coupling (CDD coupling to a ghost);
- b) by introducing the auxiliary field with positive metric and imaginary bilinear coupling (Lee-Wick coupling).

After the elimination of the auxiliary field operator both couplings lead to identical modification of the field equations.

It should be mentioned that the coupling (5) introduces the CDD ghost in a scattering channel with B particle as intermediate state. If the interaction Lagrangian is trilinear, the modification (5) describes the CDD pole in suitable two-particle S wave scattering amplitude; if it is quadrilinear, the CDD particle appears in three-particle partial wave scattering amplitude with all spins equal to zero.

Our calculations do not meet the problem of modification of Feynman rules occurring if one deals with closed loops carrying complex mass particles. The Zachariasen model, apart from the relativistic kinematics, is dynamically identical with $N-\theta$ sector in Lee model¹²⁾. It is known that in this sector the usual Feynman rules in Lee-Wick theory need not be modified.

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- 3) See 1b), formula (2.15), where the propagator (2) is introduced with $a=0$. The case with arbitrary complex conjugate residues represents the most general way of introducing a pair of complex mass particles. See
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- 7) See 5a); also
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- 8) It should be mentioned that the modification of Zachariasen model
 - a) $g_0 \rightarrow ig_0$;
 - b) change of a sign of the free Lagrangian for the field B; has been studied by O. Nachtmann (unpublished). The imaginary coupling, introduced by Nachtmann, modifies the physical interaction, and in our approach we get the elementary unstable objects by adding a coupling of CDD type. It should be stressed that these two modifications are essentially different: only in the approach presented here one gets the perturbation theory with virtual lines describing complex mass particles.
- 9) See 4) and 7b).
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