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LAGRANGIAN FORMULATION OF THE ZACHARIASEN MODEL
WITH A CDD POLE

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A B S T R A C T

The generalization of Thirring's formulation of the Zachariasen model in the presence of a CDD pole is given. The local field operator, describing an interacting CDD particle, is introduced. It is shown that the CDD pole can be interpreted as a particle only if the mass renormalization in the Zachariasen model is finite.

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1. INTRODUCTION

The singular integral equations, obtained by using the analyticity assumptions and elastic unitarity, do not have unique solutions. Firstly this property has been noted by Castillejo, Dalitz and Dyson¹⁾. They have shown that the ambiguity in the solution of the static Low equation²⁾ can be represented by addition to the inverse of scattering amplitude of some pole terms. These poles are the so-called CDD poles, and introduce zeros in the scattering amplitude. The existence of CDD poles implies, however, the appearance of poles in the scattering amplitude. If such a pole appears below the scattering threshold, it occurs on the real axis and describes some stable particle (bound state). Alternatively, the CDD pole appearing at the energy above the scattering threshold implies the presence of a pole in the scattering amplitude which is located off the real axis, on the second Riemann sheet. In the last case, the scattering amplitude develops a feature characteristic for a resonance³⁾. The resonances caused by the presence of CDD poles in the scattering amplitude are called the kinematic resonances⁴⁾. The CDD pole represents a dispersion theoretic way of introducing a resonance as an elementary object. To the contrary, a dynamical resonance arises solely from the nature of forces between the interacting particles, and the bump in the cross-section, characteristic for a resonant scattering, comes out from the dynamical equations without introducing any new degrees of freedom.

In this paper we shall introduce the CDD poles on the ground of relativistic Lagrangian model. We shall use the Lagrangian formalism especially adjusted to the description of S wave dynamics, based on Thirring's idea⁵⁾ of introduction of the field operator $\Psi(x;s)$, describing S wave pair of interacting scalar particles [see also Refs. 6)-8)]. For simplicity, we restrict our considerations to the case of Zachariasen model with one coupling constant⁹⁾.

In Thirring's formulation of the Zachariasen model, describing the S wave scattering of A particles (A neutral, scalar, mass $m_A = 1$) due to the interaction $AA \leftrightarrow B$ (B neutral, scalar, mass $m_B < 2$), there are introduced the following operators :

- a) the field operator $\Psi(x;s)$, describing interacting S wave pair of A particles with total mass square $S \geq 4$, and centre-of-mass co-ordinates x ;
- b) the field operator $B(x)$, describing interacting stable B particle.

It will be shown that the appearance of a CDD pole is achieved by the introduction of an independent field operator $C(x)$, coupled linearly to the field operator $B(x)$. Formally, the introduction of the CDD pole can be also described by the modification of the free Lagrangian for the field $B(x)$. In such an approach, the concept of the CDD pole can be expressed in the language of Feynman graphs as a modification of the free B particle propagators.

For the same of completeness, we present in Section 2 the formulation and one-parameter solution of the Zachariasen model in a dispersive form (Zachariasen model) and in a field theoretic form (Zachariasen-Thirring model). Further, the equivalence of the Zachariasen and the Zachariasen-Thirring models, defining the renormalization procedure, is discussed. In Section 3, we introduce the CDD poles as the arbitrariness in the choice of the denominator function $D(s)$ in the solution of the Zachariasen model, and we perform the "derrenormalization" procedure in order to be able to express this solution as a sum of Feynman diagrams. Our main result is contained in Section 4, where we generalize the Thirring formulation of the Zachariasen model in the presence of CDD poles. Furthermore, in Section 5, we discuss the particle interpretation of the CDD pole. We introduce the physical CDD particles in two steps :

- i) by considering firstly free CDD particles, obtained if the field $B(x)$ is not coupled with the field operator $\Psi(x;s)$ (Section 5A);
- ii) by switching in the interaction between the fields $B(x)$ and $\Psi(x;s)$ (Section 5B).

The mass renormalization for CDD quanta is performed, and one gets the result that the CDD pole describes a bound state or resonance, depending on the value of its physical mass, provided that the mass renormalization constant of the B particle is finite. If the mass renormalization of the B particle is infinite, the mass renormalization

for the CDD particle vanishes, and moreover because of occurring cancellations, the particle interpretation of the CDD pole disappears. The B particle propagator has only one pole, and the parameter, describing position of a CDD pole, determines only the subtraction point in the divergent mass renormalization integral.

In this paper, we consider only the Zachariasen model with one CDD pole. The generalization to the case with several CDD poles does not present any new difficulties.

2. THE ZACHARIASEN AND THE ZACHARIASEN-THIRRING MODELS

In 1961, Zachariasen ⁹⁾ introduced the model with two neutral scalar particles A, B, having physical masses $m_A = 1$ and $m_B = \mu < 2$, respectively, which describes the S wave scattering of A particles. From standard analyticity assumptions and elastic unitarity, it follows that the S wave scattering amplitude $T(s)$ should satisfy the integral equation

$$T(s) = \frac{g^2}{s - \mu^2} - \frac{1}{\pi} \int_4^\infty \varrho(s') \frac{|T(s')|^2 ds'}{s' - s - i\epsilon} \quad (2.1)$$

where $\varrho(s)$ denotes the two-particle phase space factor. The solution of (2.1) can be written as the N/D fraction ¹⁰⁾:

$$T(s) = \frac{N(s)}{D(s)} \quad (2.2)$$

Assuming that for large s we have $T(s) \sim 1/s$, one gets

$$N(s) = \frac{g^2}{s - \mu^2} \quad (2.3a)$$

$$D(s) = 1 + \frac{(s - \mu^2)}{\pi} \int_4^\infty \varrho(s') \frac{N(s') ds'}{(s' - \mu^2)(s' - s - i\epsilon)} \quad (2.3b)$$

If we consider the Zachariasen model as a prescription for selecting the diagrams in perturbation theory with primary vertex $B \leftrightarrow AA$, the model describes the sum of AA bubbles, connected by means of the B propagator.

In 1962, Thirring ⁵⁾ introduced the Lagrangian formulation of the Zachariasen model. In Thirring's formulation the AA bubble is represented as the propagation of quanta of the generalized free field $\mathcal{A}_0(x)$, defined as follows :

$$\mathcal{A}_0(x) = \int_4^\infty g^{\frac{1}{2}}(s) \varphi_0(x;s) ds \quad (2.4a)$$

where the four-dimensional operator valued distribution $\varphi_0(x;s)$ is defined by means of the relations

$$(\square - s) \varphi_0(x;s) = 0 \quad (2.4b)$$

$$[\varphi_0(x;s), \varphi_0(x';s')] = i \Delta(x-x';s) \delta(s-s') \quad (2.4c)$$

and is known as the Licht field ⁶⁾.

The Lagrangian, proposed in Ref. 5), has the following form :

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int} \quad (2.5a)$$

where

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{2} \left\{ \partial^\mu B(x) \partial_\mu B(x) + \mu_0^2 B(x) B(x) \right\} - \\ & -\frac{1}{2} \int_4^\infty ds \left\{ \partial^\mu \varphi(x;s) \partial_\mu \varphi(x;s) + s \varphi(x;s) \varphi(x;s) \right\} \end{aligned} \quad (2.5b)$$

and

$$\mathcal{L}_{int} = -g_0 B(x) A(x) = -g_0 B(x) \int_4^\infty \rho^{\frac{1}{2}}(s) \psi(x; s) ds \quad (2.5c)$$

The mass μ_0 and coupling constant g_0 are the unrenormalized quantities. It can be shown using different techniques ^{5)-8), 11), 12)}, that the scattering amplitude $T_0(s)$, obtained from the Lagrangian (2.5a) is given by the following formula :

$$T_0(s) = \frac{N_0(s)}{D_0(s)} \quad (2.6a)$$

where

$$N_0(s) = \frac{g_0^2}{s - \mu_0^2} \quad (2.6b)$$

$$D_0(s) = 1 + \frac{N_0(s)}{\pi} \int_4^\infty \frac{\rho(s') ds'}{s' - s - i\epsilon} \quad (2.6c)$$

It should be stressed that both solutions (2.2) and (2.6a) satisfy elastic unitarity condition, but the formulae differ in their analytic form. The difference is twofold :

- a) the subtraction in $D_0(s)$ is not present;
- b) the Cauchy formula for $D_0(s)$ obtained by integration of the unitarity condition $\text{Im } D_0 = \rho N_0$ leaves the factor $N_0(s)$, holomorphic for $s \gg 4$, non-integrated.

These two differences are removed by means of the mass and charge re-normalization. Using the identity :

$$D(s) = 1 + \frac{N(s)}{\pi} \int_4^\infty \frac{\rho(s') ds'}{s' - s - i\epsilon} - g^2 \frac{A(\mu^2)}{s - \mu^2} - g^2 B(\mu^2) \quad (2.7a)$$

where

$$A(\mu^2) = \frac{1}{\pi} \int_4^{\infty} \frac{\rho(s') ds'}{s' - \mu^2} \quad (2.7b)$$

$$B(\mu^2) = \frac{1}{\pi} \int_4^{\infty} \frac{\rho(s') ds'}{(s' - \mu^2)^2} \quad (2.7c)$$

it is easy to check the relation

$$T(s) = T_0(s) \quad (2.8)$$

provided that

$$\mu_0^2 = \mu^2 + \frac{g^2 A(\mu^2)}{1 - g^2 B(\mu^2)} \quad (2.9a)$$

$$g_0^2 = \frac{g^2}{1 - g^2 B(\mu^2)} \quad (2.9b)$$

Relations (2.9) connect the unrenormalized parameters u_0, g_0 , occurring in the Lagrangian model, with the physical values of the mass and coupling constants occurring in the dispersive formulation. Because the integral $A(u^2)$ is logarithmically divergent, the unrenormalized squared mass μ_0^2 is infinite. In order to have well-defined parameters in the theory, one can regularize the two-particle phase space factor by introducing $\rho_{\text{reg}}(s)$, satisfying the relation [see, e.g., Ref. 5]

$$\int_4^{\infty} \frac{\rho_{\text{reg}}(s) ds}{s} < \infty \quad (2.10)$$

The use of regularized two-particle phase space factor (corresponding to the modification of elastic unitarity for large s) is also needed in order to give a unique mathematical meaning to the unrenormalized D function (2.6c). On the other hand, the function $B(\mu^2)$ is finite

and positive without any regularization ¹³⁾, and if we write (2.9b) in the form

$$\frac{1}{g_0^2} = \frac{1}{g^2} - B(\mu^2) \quad (2.11)$$

we see that the Lagrangian formulation can be provided only if $g^2 < B^{-1}(\mu^2)$.

3. THE ZACHARIASEN MODEL WITH CDD POLES

The CDD poles are known in the N/D method as the following arbitrariness in the denominator function $D(s)$:

$$D(s) \rightarrow D^{CDD}(s) = D(s) - \sum_{i=1}^n \frac{f_i^2}{s - M_i^2} \quad (3.1)$$

where $f_i^2 > 0$, $M_i^2 > 0$. It is clear that the addition of the poles (3.1) does not change the absorptive part of the scattering amplitude, because

$$\text{Im } T^{CDD}(s) = -N^{-1}(s) \text{Im } D^{CDD}(s) |T^{CDD}(s)|^2 \quad (3.2)$$

and the additional contribution to the imaginary part, coming from the CDD poles, vanish on the ground of the identity $\delta(x)x^2 = 0$.

In order to express the substitution (3.1) as the modification of $D_0(s)$ and $N_0(s)$, we observe that the following relation is valid ¹⁴⁾ :

$$T^{CDD}(s) = \frac{N(s)}{D^{CDD}(s)} = \frac{N_0^{CDD}(s)}{D_0^{CDD}(s)} \quad (3.3)$$

where

$$N_0^{CDD}(s) = g_0^2 \frac{s - M_0^2}{(s - M_0^2)(s - \mu_0^2) - f_0^2} \quad (3.4a)$$

$$D_0^{\text{CDD}}(s) = 1 + \frac{N_0^{\text{CDD}}(s)}{\pi} \int_4^\infty \frac{\rho(s') ds'}{s' - s - i\epsilon} \quad (3.4b)$$

and (see the Appendix)

$$\mu_0^2 = \mu^2 + g^2 \frac{A(\mu^2)}{1 - g^2 B(\mu^2)} + \frac{f^2}{1 - g^2 B(\mu^2)} \quad (3.5a)$$

$$g_0^2 = \frac{g^2}{1 - g^2 B(\mu^2)} \quad (3.5b)$$

$$f_0^2 = f^2 \frac{M^2 - \mu^2}{1 - g^2 B(\mu^2)} \quad (3.5c)$$

$$M_0^2 = M^2 \quad (3.5d)$$

From formulae (3.5) we see that :

- a) the presence of a CDD pole does not modify the charge renormalization, and particularly the relation $g^2 < B^{-1}(\mu^2)$, allowing the applicability of the Lagrangian formalism, remains the same;
- b) the relation between the bare and physical masses of B particle is always modified;
- c) in the unrenormalized formula the position of the CDD pole is not changed;
- d) in order to have $f_0^2 > 0$ we should assume that $M^2 > \mu^2$.
It will be seen from the next Section that this condition is necessary, if we wish to set up a Lagrangian formulation.

Finally, it should be mentioned that still the mass renormalization of the CDD particle remains to be performed ¹⁵⁾. We shall discuss this problem in Section 5.

4. LAGRANGIAN FORMULATION

Let us modify the Thirring Lagrangian (2.5a) as follows :

$$\mathcal{L}^{CDD} = \mathcal{L} + \mathcal{L}_0^{CDD} + \mathcal{L}_{int}^{CDD} \quad (4.1)$$

where

$$\mathcal{L}_0^{CDD} = -\frac{1}{2} (\partial^\mu C(x) \partial_\mu C(x) + M_0^2 C(x) C(x)) \quad (4.2a)$$

$$\mathcal{L}_{int} = f_0 B(x) C(x) \quad (4.2b)$$

The field equations are the following :

$$(\square - M_0^2) C(x) = -f_0 B(x) \quad (4.3a)$$

$$(\square - \mu_0^2) B(x) = g_0 \int_4^\infty ds \, \varrho^{\frac{1}{2}}(s) \psi(x;s) - f_0 C(x) \quad (4.3b)$$

$$(\square - s) \psi(x;s) = g_0 \varrho^{\frac{1}{2}}(s) B(x) \quad (4.3c)$$

Using the canonical commutation relations, particularly the following ones for the field $\psi(x;s)$:

$$\begin{aligned} [\dot{\psi}(\vec{x}, t; s), \psi(\vec{x}', t; s')] &= -i \delta(s-s') \delta(\vec{x}-\vec{x}') \\ [\dot{\psi}(\vec{x}, t; s), B(\vec{x}', t)] &= [\psi(\vec{x}, t; s), B(\vec{x}', t)] = 0 \end{aligned} \quad (4.4)$$

one gets, for the two-point propagators :

$$\left[(p^2 - \mu_0^2) - \frac{f_0^2}{p^2 - M_0^2} \right] \tau(p^2; s) = g_0 \int_4^\infty ds' \, \varrho^{\frac{1}{2}}(s') \tau(p^2; s, s') \quad (4.5a)$$

$$(p^2 - s) \tau(p^2; s, s') = \delta(s-s') + g_0 \varrho^{\frac{1}{2}}(s') \tau(p^2; s) \quad (4.5b)$$

where $\tau(p^2; s)$ and $\tau(p^2; s, s')$ denote the Fourier transforms of the following Green functions :

$$\begin{aligned}\tau(x; s) &= -i \langle 0 | T \{ B(\frac{x}{2}) \psi(-\frac{x}{2}; s) \} | 0 \rangle \\ \tau(x; s, s') &= -i \langle 0 | T \{ \psi(\frac{x}{2}; s) \psi(-\frac{x}{2}; s') \} | 0 \rangle\end{aligned}\quad (4.6)$$

From Eqs. (4.5) it follows after simple algebraic manipulations that

$$\begin{aligned}\tau(p^2; s, s') &= \frac{\delta(s-s')}{p^2-s'+i\epsilon} + g_0^2 \frac{s-M_0^2}{(s-\mu_0^2)(s-M_0^2)-f_0^2} \cdot \\ &\cdot \frac{\rho^{\frac{1}{2}}(s)\rho^{\frac{1}{2}}(s')}{(p^2-s+i\epsilon)(p^2-s'+i\epsilon)} \left\{ 1 - g_0^2 \frac{s-M_0^2}{(s-\mu_0^2)(s-M_0^2)-f_0^2} \cdot \frac{1}{\pi} \int_4^\infty \frac{\rho(s'') ds''}{p^2-s''+i\epsilon} \right\}^{-1}\end{aligned}\quad (4.7)$$

Let us introduce the one-particle asymptotic states ^{7),8)}

$$|\vec{p}; s\rangle_{\substack{\text{in} \\ \text{out}}} = a_{\substack{\text{in} \\ \text{out}}}(\vec{p}; s) | 0 \rangle \quad (4.8)$$

where

$$\psi_{\substack{\text{in} \\ \text{out}}}(x; s) = \frac{1}{(2\pi)^3} \int \frac{d^3\vec{p}}{(2\omega_s(\vec{p}))^{\frac{1}{2}}} \left\{ e^{i(\vec{p}\vec{x} - \omega_s(\vec{p})t)} a_{\substack{\text{in} \\ \text{out}}}(\vec{p}; s) + \text{H.c.} \right\} \quad (4.9)$$

and ^{8),12)}

$$\psi(x; s) \xrightarrow[t \rightarrow \mp\infty]{} \psi_{\substack{\text{in} \\ \text{out}}}(x; s) \quad (4.10)$$

The phase factor scattering of one-particle asymptotic states (4.8)

$$\langle \vec{p}; s | \vec{p}'; s' \rangle_{\text{in}}^{\text{out}} = e^{2i\eta(s)} \delta(\vec{p} - \vec{p}') \delta(s - s') \quad (4.11)$$

can be expressed, by using the reduction formula ^{7),8),11)}

$$\begin{aligned} \varrho(s) T^{\text{CDD}}(s) &= \frac{1}{2\pi i} (e^{2i\eta(s)} - 1) = \\ &= \lim_{\substack{s' \rightarrow s \\ s'' \rightarrow s}} [(s-s')(s-s'') \tau(s; s', s'')] \end{aligned} \quad (4.12)$$

by means of the residuum of the double pole in the propagator (4.7). One gets, using the relation (4.12), the formula (3.3) with unrenormalized parameters μ_0 , g_0 , f_0 and M_0 . If one introduces the renormalized parameters in such a way that the relations (3.5) are satisfied, one arrives at the result that the Lagrangian (4.1) describes the S wave scattering model with the following scattering amplitude

$$T(s) = \frac{\frac{g^2}{s - \mu^2}}{1 - \frac{f^2}{s - M^2} + \frac{g^2(s - \mu^2)}{\pi} \int_4^\infty \frac{\varrho(s') ds'}{(s' - \mu^2)^2 (s' - s - i\epsilon)}} \quad (4.13)$$

In the next Section we shall study the consequences of the modification (4.1) of the Zachariasen-Thirring model.

5. THE PARTICLE INTERPRETATION OF THE ZACHARIASEN MODEL WITH CDD POLE

A. Free CDD particle

We define a model with a free CDD particle as the one obtained by putting $g_0 = 0$ in (4.1). We have

$$\begin{aligned} \mathcal{L}_{g_0=0} &= \frac{1}{2} (\partial^\mu B \partial_\mu B + \mu_0^2 B^2) - \frac{1}{2} (\partial^\mu C \partial_\mu C + M_0^2 C^2) \\ &\quad + f_0 B C \end{aligned} \quad (5.1)$$

Such a model can be easily diagonalized. The equation for two mass square eigenvalues $\mu_{0\pm}^2$ has the following form :

$$(\mu^2 - \mu_0^2)(\mu^2 - M_0^2) - f_0^2 = 0 \quad (5.2)$$

One gets from (5.2) that

$$\mu_{0\pm}^2 = \frac{1}{2} (\mu_0^2 + M_0^2 \pm \sqrt{(\mu_0^2 - M_0^2)^2 + 4f_0^2}) \quad (5.3a)$$

In order to obtain real mass μ_{0-} , one should assume

$$\mu_0^2 M_0^2 > f_0^2 \quad (5.3b)$$

It is easy to see that if $f_0 \rightarrow 0$ we have

$$\begin{aligned} \mu_{0+}^2 &\rightarrow \text{Max}(\mu_0^2, M_0^2) \\ \mu_{0-}^2 &\rightarrow \text{Min}(\mu_0^2, M_0^2) \end{aligned} \quad (5.4)$$

We see, therefore, that if $M_0 > \mu_0$, μ_{0+} describes the unrenormalized mass of free CDD particle, and μ_{0-} the unrenormalized mass of the B particle, modified in the presence of a CDD pole. If $M_0 < \mu_0$, the roles of μ_{0+} and μ_{0-} are interchanged.

Let us now interpret the introduction of a CDD pole as the modification of the field equations for the field operator $B(x)$. From (4.3a)-(4.3b) one gets

$$\left[\square - \mu_0^2 - \frac{f_0^2}{\square - M_0^2} \right] B(x) = g_0 \int_4^\infty ds \rho^{\frac{1}{2}}(s) \psi(x; s) \quad (5.5)$$

The difference in comparison with the usual Zachariasen-Thirring model consists in the following replacement of the free propagator of a B particle :

$$\begin{aligned} \frac{1}{p^2 - \mu_0^2 + i\epsilon} &\rightarrow \frac{1}{p^2 - \mu_0^2 - \frac{f_0^2}{p^2 - M_0^2}} = \frac{p^2 - M_0^2}{(p^2 - \mu_{0+}^2 + i\epsilon)(p^2 - \mu_{0-}^2 + i\epsilon)} = \\ &= \frac{Z_{0+}}{p^2 - \mu_{0+}^2 + i\epsilon} + \frac{Z_{0-}}{p^2 - \mu_{0-}^2 + i\epsilon} \end{aligned} \quad (5.6)$$

where

$$Z_{0+} = \frac{\mu_{0+}^2 - M_0^2}{\mu_{0+}^2 - \mu_{0-}^2} \quad Z_{0-} = \frac{M_0^2 - \mu_{0-}^2}{\mu_{0+}^2 - \mu_{0-}^2} \quad (5.7)$$

Because $\mu_{0+} > M_0 > \mu_{0-}$ one gets

$$Z_{0+} > 0 \quad Z_{0-} > 0 \quad (5.8)$$

and both particles, with masses μ_{0+} and μ_{0-} , carry positive definite metric.

One can introduce the four-dimensional commutator function $\Delta(x; \mu_{0+}^2, \mu_{0-}^2)$ for a modified free B field. We have

$$\Delta(x; \mu_{0+}^2, \mu_{0-}^2) = Z_{0+} \Delta(x; \mu_{0+}^2) + Z_{0-} \Delta(x; \mu_{0-}^2) \quad (5.9)$$

Because

$$Z_{0+} + Z_{0-} = 1 \quad (5.10)$$

the presence of a CDD pole does not modify the first four time derivatives of the commutator function at $t=0$. Particularly the canonical equal time limit is not modified :

$$\dot{\Delta}(\vec{x}, 0; \mu_{0+}^2, \mu_{0-}^2) = \dot{\Delta}(\vec{x}, 0; \mu_0^2) = i \delta(\vec{x}) \quad (5.11)$$

and, similarly, the third derivative

$$\ddot{\Delta}(\vec{x}, 0; \mu_{0+}^2, \mu_{0-}^2) = \ddot{\Delta}(\vec{x}, 0; \mu_0^2) = i(\Delta - \mu_0^2) \delta(\vec{x}) \quad (5.12)$$

The last equality is valid due to the relation

$$Z_{0+} \mu_{0+}^2 + Z_{0-} \mu_{0-}^2 = \mu_0^2 \quad (5.13)$$

The difference shows up if we calculate the fifth derivative of the commutator function (5.9).

B. Interacting CDD particle

Let us write the equation, following from (5.5) and (4.3c), for the B particle propagator $\tau(p^2)$:

$$\begin{aligned} \frac{(p^2 - \mu_{0+}^2)(p^2 - \mu_{0-}^2)}{p^2 - M_0^2} \tau(p^2) = 1 + \\ + \frac{g_0^2}{\pi} \int_4^\infty ds \frac{\rho(s)}{p^2 - s + i\epsilon} \tau(p^2) \end{aligned} \quad (5.14)$$

The renormalized masses of the B particle and CDD particle are given by the following non-linear relation :

$$(\mu^2 - \mu_{0+}^2)(\mu^2 - \mu_{0-}^2) - g_0^2(\mu^2 - M_0^2) A(\mu^2) = 0 \quad (5.15)$$

where $A(\mu^2)$ is defined by (2.7b). Introducing the subtraction

$$A(\mu^2) = A(M_0^2) + (\mu^2 - M_0^2) B(\mu^2) \quad (5.16)$$

one can perform the mass renormalization procedure in two steps :

a) one diagonalizes the equation

$$(\mu^2 - \mu_{0+}^2)(\mu^2 - \mu_{0-}^2) - g_0^2(\mu^2 - M_0^2)A(M_0^2) = 0 \quad (5.17)$$

b) one looks for the solutions of the following equation :

$$(\mu^2 - \tilde{\mu}_{0+}^2)(\mu^2 - \tilde{\mu}_{0-}^2) - g_0^2(\mu^2 - M_0^2)^2 B(\mu^2) = 0 \quad (5.18)$$

where $\tilde{\mu}_{0\pm}^2$ represent two solutions of the quadratic equation (5.17)

$$\begin{aligned} \tilde{\mu}_{0\pm}^2 = & \frac{1}{2} \{ \mu_{0+}^2 + \mu_{0-}^2 + g_0^2 A(M_0^2) \pm \\ & \pm [(\mu_{0+}^2 + \mu_{0-}^2 + g_0^2 A(M_0^2))^2 - 4\mu_{0+}^2 \mu_{0-}^2 - 4g_0^2 M_0^2 A(M_0^2)]^{\frac{1}{2}} \} \end{aligned} \quad (5.19)$$

The first step allows to distinguish two cases :

$$B1) \quad A(M_0^2) < \infty$$

Such an assumption means that we have introduced regularized two-particle phase space [see (2.10)]. If the subtraction constant $A(M_0^2)$ is finite, Eq. (5.18) has two solutions $\mu_{\pm}^2$. If $\mu_{\pm}^2 < 4$, both solutions are real, and one can construct the free asymptotic fields B_{out}^{in} , describing stable asymptotic particles. The wave renormalization constants $Z_{\pm}$ are given by the formula

$$Z_{\pm}^{-1} = 1 + \frac{f_0^2}{(\mu_{\pm}^2 - M_0^2)^2} + g_0^2 B(\mu_{\pm}^2) \quad (5.20)$$

and describe the normalization of one-particle contributions to the full B propagator. It can be checked that in the limit $g_0 \rightarrow 0$ the formulae (5.7) and (5.20) are consistent.

If $\text{Re} \mu_{\pm}^2 > 4$, it follows that necessarily $\text{Im} \mu_{\pm}^2 < 0$ and the solutions of Eq. (5.18) describe unstable particles, decaying into AA pairs.

$$B2) \quad A(M_0^2) = \infty, \quad B(M_0^2) < \infty$$

Using Eq. (5.19), it is easy to see that if $A(M_0^2) \rightarrow \infty$ we obtain

$$\begin{aligned} \tilde{\mu}_{0+}^2 &\longrightarrow \mu_{0+}^2 + g_0^2 A(M_0^2) \\ \tilde{\mu}_{0-}^2 &\longrightarrow M_0^2 \end{aligned} \quad (5.21)$$

and Eq. (5.18) can be written as follows :

$$\mu^2 - \mu_{0+}^2 - g_0^2 A(M_0^2) - g_0^2 (\mu^2 - M_0^2) B(\mu^2) = 0 \quad (5.22)$$

We see that Eq. (5.22) is the conventional relation for the renormalized mass μ of the B particle, where M_0 plays the role of a subtraction point. One can conclude, therefore, that if the mass renormalization for the B particle is infinite, the particle interpretation of the CDD pole disappears, and the B particle propagator has only one pole, determined by the relation (5.22).

6. FINAL REMARKS

We have studied here the field theoretic model with a CDD pole introduced in the simplest case of elastic S wave scattering without the left-hand discontinuities. It has been shown that the N/D solution can be represented as the on-shell result in a bilinear Lagrangian model with the field operator having continuous spectrum of asymptotic masses. One can ask how large a class of N/D solutions can be represented by a similar field theoretic scheme. In order to make the Zachariasen model more realistic, one should :

- a) introduce the left-hand cut;
- b) take into consideration the inelastic effects.

In formulating the Lagrangian theory with these two improvements included, the basic difficulty is related to the description of the left-hand discontinuity. Formally, such a discontinuity can be represented by

a field operator with a purely imaginary continuum spectrum of asymptotic fields (a kind of generalized tachyon field). On the other hand, the Lagrangian description of inelastic effects - whether by considering the multichannel N/D method ¹⁶⁾ or the inelastic one-channel N/D formalism ¹⁷⁾ - does not present any particular difficulty. It may be interesting to study in the frame of a field theoretical description the relation between the one-channel inelastic and multichannel N/D methods. It has been shown ¹⁸⁾ that the equivalence of these two approaches implies the necessary presence of CDD poles in the one-channel formulation.

Another application of the above formalism is the description of non-CDD poles, leading to the complex conjugate pair of poles on the physical sheet ¹⁹⁾. Such poles have been advertised as the ones which may resolve the divergence problems in present local quantum field theory ²⁰⁾. We shall consider their relation to CDD ghosts in our subsequent publication.

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A P P E N D I X

In this Appendix we shall derive the formulae for "derrenormalization" of physical values of μ^2 , g^2 , f^2 and M^2 . Using the formulae (2.7a) and (3.1), one gets :

$$T^{CDD}(s) = \frac{\frac{g^2}{s-\mu^2}}{1 + \frac{g^2}{s-\mu^2} \cdot \frac{1}{\pi} \int_4^\infty \frac{\rho(s') ds'}{s'-s-i\epsilon} - g^2 \frac{A(\mu^2)}{s-\mu^2} - g^2 B(\mu^2) - \frac{f^2}{s-M^2}} =$$

$$= \frac{g^2}{(1 - g^2 B(\mu^2)) (s - \mu^2 - \frac{g^2 A(\mu^2)}{1 - g^2 B(\mu^2)}) - f^2 \frac{s - \mu^2}{s - M^2} - \frac{g^2}{\pi} \int_4^\infty \frac{\rho(s') ds'}{s'-s-i\epsilon}}$$

Introducing g_0^2 and f_0^2 [see Eqs. (3.5b)-(3.5c)] we have

$$T^{CDD}(s) = \frac{g_0^2}{s - \mu^2 - \frac{g_0^2 A(\mu^2) + f_0^2}{1 - g^2 B(\mu^2)} - \frac{f_0^2}{s - M^2} - \frac{g_0^2}{\pi} \int_4^\infty \frac{\rho(s') ds'}{s'-s-i\epsilon}}$$

If we use the definition of unrenormalized mass μ_0^2 [see (3.5a)], and divide the numerator and denominator by the expression

$$s - \mu_0^2 - \frac{f_0^2}{s - M^2} = \left[\frac{s - M^2}{(s - M^2)(s - \mu_0^2) - f_0^2} \right]^{-1}$$

one gets the result (3.3) and (3.4). Besides we see that $M_0 = M$.

REFERENCES AND FOOTNOTES

- 1) L. Castillejo, R.H. Dalitz and F. Dyson, Phys.Rev. 101, 453 (1956).
- 2) F.E. Low, Phys.Rev. 97, 1392 (1955).
- 3) See, e.g., R.J. Eden, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, "The Analytic S Matrix", Cambridge University Press (1966).
- 4) A.O. Barut and K.H. Ruei, Phys.Rev. 122, 1340 (1961).
- 5) W. Thirring, Phys.Rev. 126, 1209 (1962). See also, W. Thirring in "Theoretical Physics", Trieste seminar 1962, Ed. Vienna (1962).
- 6) A.L. Licht, Ann.Phys.(N.Y.) 34, 161 (1965).
- 7) J. Lukierski, Nuovo Cimento 60A, 353 (1969).
- 8) W. Karwowski, J. Lukierski and N. Sznajder, Nuovo Cimento 63A, 509 (1969).
- 9) F. Zachariasen, Phys.Rev. 121, 1851 (1961).
- 10) G.F. Chew and S. Mandelstam, Phys.Rev. 119, 467 (1960).
- 11) J. Lukierski and L. Turko, Bull.Acad.Soc.Polon. 16, 905 (1968).
- 12) W. Karwowski and N. Sznajder, Acta Phys.Polon. 34, 1057 (1968).
- 13) After performing the integral (2.7c) one gets ⁵⁾

$$B(\mu^2) = \frac{1}{16\pi^2\mu^2} \left\{ \frac{\arctg \sqrt{\frac{\mu^2}{1-\mu^2}}}{\sqrt{\mu^2(1-\mu^2)}} - 1 \right\}$$

and for example if $\mu \rightarrow 0$ one gets

$$B(\mu^2) \rightarrow \frac{2}{3} \frac{1}{16\pi^2}$$

- 14) For simplicity we shall assume that $n=1$ in (3.1), $M_1 \equiv M$ and $f_1 \equiv f$. The generalization to arbitrary finite n does not introduce any qualitative difficulties.

- 15) We recall that the CDD pole determines the position of the real zero of the scattering amplitude, and the CDD particle is described by a pole term, implied by the presence of such a zero.
- 16) J.D. Bjorken, Phys.Rev.Letters 4, 473 (1960).
- 17) The best for the Lagrangian description is the inelastic N/D method of Chew and Mandelstam ³⁾, where the input information about inelasticity is given by the ratio σ_{el}/σ_{tot} .
- 18) See, e.g., J.B. Hartle and C.E. Jones, Phys.Rev. 140, B90 (1965); D. Atkinson, K. Dietz and D. Morgan, Ann.Phys.(N.Y.) 37, 77 (1966).
- 19) T.D. Lee and G.C. Wick, Nuclear Phys. B9, 209 (1969).
- 20) T.D. Lee, Proceedings of the 1969 Topical Conference on Weak Interactions, CERN, January 1969, report CERN 69-7, p.427.