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FINAL STATE COULOMB CORRECTIONS TO NEUTRINO REACTIONS

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A B S T R A C T

Final state Coulomb corrections to reactions $\nu + \text{nucleus} \rightarrow \mu + \text{final state}$ are estimated for the differential cross-section $(\partial^2 \sigma) / (\partial q^2 \partial \nu)$ in a region of the kinematical variables where the nucleons inside the nucleus contribute incoherently. $[q^2 = (\text{four momentum transfer})^2, \nu = \text{energy transfer of leptons.}]$ Corrections of the order of 5% are found for heavy nuclei.

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1. INTRODUCTION

The assumption of single vector particle exchange -- one photon in electron scattering, a weak intermediate boson (even of infinite mass) in neutrino induced reactions -- leads to a restricted energy dependence of the cross-section¹⁻⁴⁾:

$$\frac{\partial^2 \sigma}{\partial q^2 \partial \nu} = \frac{1}{E_1^2} \left\{ A(q^2, \nu) E_1^2 + B(q^2, \nu) E_1 + C(q^2, \nu) \right\} \quad (1.1)$$

where q^2 is the four-momentum transfer squared, ν the energy transfer and E_1 the energy of the incident lepton in the laboratory system.

Here we will be concerned with the applicability of this formula to nuclear targets. At present, high statistics neutrino experiments require nuclei rather than nucleons as targets. Moreover, with both electrons and neutrinos, nuclei are perhaps of some interest in themselves. Now the reasoning leading to Eq. (1.1) is of course applicable to any target, but the assumption of single vector particle exchange becomes questionable for a nucleus because of the possibility of large Coulomb effects -- i.e. the exchange of extra photons. One could fear corrections of order $Z\alpha$, large in a nucleus. Here we investigate this problem (explicitly for neutrino reactions, but would expect similar results for the electromagnetic case) and find that the corrections are characterized rather by the parameter $Z\alpha/E_2 R$, where Z is the charge number of the nucleus, $\alpha = 1/137$ the fine structure constant, R the nuclear radius and E_2 the energy of the outgoing lepton.

Our investigation is based on the picture of a nucleus as an assembly of quasi-free nucleons scattering independently apart from the Coulomb effect. If this picture is good and if the Coulomb effects are small, then not only does the formula (1.1) apply to nuclei, but the structure functions are directly related to those for nucleons. In the conclusion we comment briefly on the kinematical region where this picture is expected to break down.

In the following we will restrict ourselves to high-energy reactions, i.e. we will assume that the wavelength of the leptons is much smaller than the nuclear radius. The lepton mass will be neglected.

Consider now a collision of the neutrino with a nucleon of momentum $\vec{p}$ at point $\vec{x}$ inside the nucleus (see Fig. 1). The Coulomb potential will be considered constant over the reaction region. The muon is created with a certain wave number, depending on $\vec{x}$. As it propagates out of the nucleus, its wave number will change, but the wave will also be defocused. If we neglect nuclear correlations, the cross-section for this process will be the cross-section for a free nucleon but with the appropriate change of μ wave number and normalization. Average over the nucleus and the Fermi motion of the nucleons leads to the final result. (We have to average cross-sections since we are assuming incoherent scattering).

The paper is organized as follows: In Section 2 we discuss the μ wave function, distorted by the Coulomb field. In Section 3 the cross-section for inelastic and quasi-elastic scattering is derived. Section 4 gives a comparison with other work on Coulomb corrections. Section 5 is devoted to discussion and conclusion. In an Appendix we give numerical values for our correction terms in some special cases.

We use units $\hbar = c = 1$, metric conventions of Bjorken-Drell⁵⁾ except for a change in sign in γ_5 . Our $\epsilon^{\mu\nu\lambda\rho}$ is normalized to $\epsilon^{0123} = -\epsilon_{0123} = 1$.

2. THE DISTORTED μ WAVE FUNCTION

The μ wave function in the Coulomb field satisfies the following Dirac equation ($m_\mu = 0$):

$$\{-i\vec{\alpha} \cdot \vec{\nabla} - E + V(\vec{x})\} \varphi(\vec{x}) = 0 \quad (2.1)$$

$V(\vec{x})$ is the Coulomb potential. For a homogeneously charged sphere of radius R :

$$V(r) = \begin{cases} 2\alpha \frac{r^2 - 3R^2}{2R^3} & r < R \\ -2\alpha \frac{1}{r} & r > R \end{cases} \quad (2.2)$$

We want a solution of Eq. (2.1) which corresponds to an outgoing plane wave of momentum $\vec{k}$. This solution can be given in the eikonal approximation, and its general form is:

$$\psi(\vec{x}) = N(\vec{x}) u(\vec{x}) e^{iS(\vec{x})} \quad (2.3)$$

$$(\vec{\nabla} S(\vec{x}))^2 - (E - V(\vec{x}))^2 = 0 \quad (2.4)$$

$$\vec{k}(\vec{x}) \equiv \tilde{\vec{k}} = \begin{pmatrix} E - V(\vec{x}) \\ \vec{\nabla} S(\vec{x}) \end{pmatrix} \quad (2.5)$$

$$\tilde{\vec{k}} \cdot \vec{x} = 0 \quad (2.6)$$

$$\bar{u}(\vec{x}) u(\vec{x}) = 1 \quad (2.7)$$

$\vec{k}(\vec{x}) = \vec{\nabla} S(\vec{x})$ is the local wave number of the μ wave. $N(\vec{x})$ takes care of the change in normalization when the μ propagates out of the nucleus. The concrete values for $N(\vec{x})$, $S(\vec{x})$ will be taken from Yennie et al.⁶⁾

$$\vec{k}(\vec{x}) = \vec{k} + \delta\vec{k}(\vec{x})$$

$$\delta\vec{k}(\vec{x}) = -V(\vec{x}) \hat{\vec{k}} + a [\hat{\vec{k}}(\hat{\vec{k}}\vec{x}) - \vec{x}] (\vec{k}\vec{x}) k^2 + \text{terms odd in } \vec{x} \quad (2.8)$$

$$\hat{\vec{k}} = \vec{k} / |\vec{k}|$$

$$N^2(\vec{x}) = 1 - 2 \frac{V(0)}{k} - 2a k^2 \vec{x}^2 + 3a (\vec{k}\vec{x})^2 + \text{terms odd in } \vec{x} \quad (2.9)$$

$$a = \frac{7\alpha}{k^3 R^3} \quad (2.10)$$

3. DERIVATION OF THE CORRECTIONS TO THE DIFFERENTIAL CROSS-SECTION DUE TO THE COULOMB FIELD AND FERMI MOTION

The starting point is of course the usual current $\times$ current weak interaction Lagrangian:

$$\mathcal{L}(x) = -\frac{G}{\sqrt{2}} \left\{ \bar{\psi}_\mu(x) \gamma^\lambda (1 + \gamma_5) \psi_\nu(x) J_\lambda + h.c \right\} \quad (3.1)$$

J_λ is the hadron current.

The S-matrix element for the process described in Fig. 1 is:

$$\begin{aligned} \langle \mu \beta | S | \nu N(\vec{p}_1) \rangle &= -i \frac{G}{\sqrt{2}} 2\pi \delta(p_1^0 + E_1 - p_2^0 - E_2) \\ &\quad (2\pi)^3 \delta^3(\vec{p}_1 + \vec{k}_1 - \vec{p}_2 - \vec{k}_2(\vec{x})) \sqrt{\frac{m_\mu m_\nu}{E_1 E_2 V^2}} N(\vec{x}) \\ &\quad \bar{u}_\mu(\vec{x}) \gamma^\lambda (1 + \gamma_5) u_\nu(k_1) \langle \beta | J_\lambda | N(\vec{p}_1) \rangle \end{aligned} \quad (3.2)$$

V is the normalization volume and should not be confused with the Coulomb potential. This is just the S-matrix element for collision of a neutrino with a free nucleon of momentum $\vec{p}_1$ in a constant Coulomb field with the value $V(\vec{x})$. Since charge is conserved in the process, we can replace in Eq. (3.2) energy conservation by kinetic energy conservation. Define the following four vectors; which have as zero component the kinetic energy of the corresponding particle

$$\tilde{p}_1 = \begin{pmatrix} \tilde{p}_1^0 = M + \frac{p_1^2}{2M} \\ \vec{p}_1 \end{pmatrix} \quad \tilde{p}_2 = \begin{pmatrix} \tilde{p}_2^0 \\ \vec{p}_2 \end{pmatrix} \quad (3.3)$$

$$\tilde{k}_2 = \begin{pmatrix} E_2 - V(\vec{x}) \\ \vec{k}_2 + \delta \vec{k}_2(\vec{x}) \end{pmatrix} \quad (3.4)$$

Substituting in formula (3.2) and squaring we find the cross-section:

$$\sigma = G^2 \sum (2\pi)^4 \delta(\tilde{p}_1 + k_1 - \tilde{p}_2 - \tilde{k}_2) \frac{1}{E_1 E_2} N^2(\vec{x}) \frac{m_\mu m_\nu}{2} \bar{u}_\mu(\vec{x}) \gamma^\lambda (1 + \gamma_5) u_\nu(k_1) \bar{u}_\nu(k_2) \gamma^\rho (1 + \gamma_5) u_\mu(\vec{x}) \langle N(\vec{p}_1) | \gamma_\rho^+ | \beta \rangle \langle \beta | \gamma_\lambda | N(\vec{p}_1) \rangle \quad (3.5)$$

$$d\sigma = \frac{G^2}{4\pi^2} \frac{E_2}{E_1} d\Omega dE_2 N^2(\vec{x}) \frac{M}{\tilde{p}_1^0} L^{\lambda\rho} \gamma_{\rho\lambda} \quad (3.6)$$

$$L^{\lambda\rho} = \tilde{k}_2^\lambda k_1^\rho + k_1^\lambda \tilde{k}_2^\rho - (k_1 \tilde{k}_2) g^{\lambda\rho} + i \varepsilon^{\alpha\lambda\rho\beta} \tilde{k}_{2\alpha} k_{1\beta} \quad (3.7)$$

$$\gamma_{\rho\lambda} = \frac{\tilde{p}_1^0}{M} \sum_\beta (2\pi)^3 V \delta(\tilde{p}_2 - \tilde{p}_1 - \tilde{q}) \langle N(\vec{p}_1) | \gamma_\rho^+ | \beta \rangle \langle \beta | \gamma_\lambda | N(\vec{p}_1) \rangle \quad (3.8)$$

where we have defined

$$\tilde{q} = k_1 - \tilde{k}_2 \quad (3.9)$$

In the following we will also use (M = nucleon mass)

$$\tilde{v} = (\tilde{p}_1 \tilde{q})/M \quad \tilde{E}_1 = (\tilde{p}_1 k_1)/M \quad (3.10)$$

where $\bar{V}$ is the Coulomb potential averaged over the nucleus and p_F the Fermi momentum. The average of the potential defined in Eq. (2.2) is:

$$\bar{V} = -\frac{6}{5} \frac{Z\alpha}{R} \quad (3.16)$$

Since $F(q^2, \nu, E_1)$ is a second order polynomial in E_1 only the third term in Eq. (3.15) violates the general structure described in Eq. (1.1).

The Fermi motion clearly cannot violate it since we could regard the whole nucleus as one target instead of an individual nucleon.

From Eq. (3.15) it is easy to derive the correction to quasi-elastic scattering. All we have to do is to make the following replacement:

$$F(q^2, \nu, E_1) \rightarrow 2M \delta(2M\nu + q^2) f(q^2, E_1) \quad (3.17)$$

$f(q^2, E_1)$ is a polynomial of second order in E_1 , the coefficients are combinations of the usual vector and axial vector form factors. The differential cross-section per neutron for quasi-elastic scattering reads then:

$$\begin{aligned} \frac{d\sigma}{dq^2} \Big|_{\text{per neutron}} &= \frac{G^2}{4\pi E_1^2} \left\{ f(q^2, E_1) \right. \\ &\quad - \frac{\bar{V}}{E_2} \left[2 f(q^2, E_1) + q^2 \frac{\partial f}{\partial q^2}(q^2, E_1) \right] \\ &\quad \left. + \frac{p_F^2}{10M^2} \left[-5f + E_1 \frac{\partial f}{\partial E_1} + E_1^2 \frac{\partial^2 f}{\partial E_1^2} \right] \right\} \end{aligned} \quad (3.18)$$

Equations (3.15) and (3.18) are our results for the Coulomb and Fermi motion corrections to inelastic and quasi-elastic neutrino reactions. For anti-neutrino reactions we have to replace $\bar{V}$ by $-\bar{V}$ and the structure functions by the appropriate ones for anti-neutrinos.

For completeness we shall give our result for the inelastic case also in a form more directly related to the expression for the cross-section given in Ref. 7.

$$\left. \frac{\partial^2 \sigma}{\partial q^2 \partial \nu} \right|_{\text{per nucleon}} = \frac{G^2}{2\pi} \frac{E_2}{E_1} \left\{ \hat{W}_2 \cos^2 \frac{\theta}{2} + 2 \hat{W}_1 \sin^2 \frac{\theta}{2} - \hat{W}_3 \sin^2 \frac{\theta}{2} \frac{E_1 + E_2}{M} \right\} \quad (3.19)$$

$$\begin{aligned} \hat{W}_2 = & W_2(q^2, \nu) \left(1 - 3 \frac{\bar{V}}{E_2} \right) - \frac{\bar{V}}{E_2} q^2 \frac{\partial W_2}{\partial q^2} + \bar{V} \frac{\partial W_2}{\partial \nu} \\ & + \frac{P_F^2}{10M^2} \left[5 W_2 + 7 \nu \frac{\partial W_2}{\partial \nu} + (\nu^2 - q^2) \frac{\partial^2 W_2}{\partial \nu^2} \right] \end{aligned} \quad (3.20)$$

$$\begin{aligned} \hat{W}_1 = & W_1(q^2, \nu) \left(1 - 3 \frac{\bar{V}}{E_2} \right) - \frac{\bar{V}}{E_2} q^2 \frac{\partial W_1}{\partial q^2} + \bar{V} \frac{\partial W_1}{\partial \nu} \\ & + \frac{P_F^2}{10M^2} \left[-3 W_1 + 2 W_2 + 3 \nu \frac{\partial W_1}{\partial \nu} + (\nu^2 - q^2) \frac{\partial^2 W_1}{\partial \nu^2} \right] \end{aligned} \quad (3.21)$$

$$\begin{aligned} \hat{W}_3 = & W_3(q^2, \nu) \left(1 - 3 \frac{\bar{V}}{E_2} - \frac{\bar{V}}{E_1 + E_2} \right) - \frac{\bar{V}}{E_2} q^2 \frac{\partial W_3}{\partial q^2} + \bar{V} \frac{\partial W_3}{\partial \nu} \\ & + \frac{P_F^2}{10M^2} \left[5 \nu \frac{\partial W_3}{\partial \nu} + (\nu^2 - q^2) \frac{\partial^2 W_3}{\partial \nu^2} \right] \end{aligned} \quad (3.22)$$

The $\hat{W}_i$, $i = 1, 2, 3$ are no longer functions of q^2, ν only but depend also on the energies of the incoming and outgoing leptons.

4. COMPARISON WITH OTHER CALCULATIONS

Coulomb corrections to neutrino reactions have been treated by Kokkedee⁸⁾ and Piketty⁹⁾. However they give only corrections to the polarization of the μ transverse to the production plane, since they are interested in T violation problems. We can analyse their methods by comparing their effective μ wave functions. We shall also compare with a recipe given in de Forest et al.¹⁰⁾.

Reference 8 starts with a Sommerfeld-Maue approximate wave function:

$$\varphi(\vec{x}) = \left[1 - \frac{i}{2E} \vec{\alpha} \cdot \vec{\nabla} \right] F(\vec{x}, \vec{k}) u(k) e^{i\vec{k} \cdot \vec{x}} \quad (4.1)$$

$F(\vec{x}, \vec{k})$ is a complicated function. However, $F(\vec{x}, \vec{k})$ and $\vec{\nabla} F(\vec{x}, \vec{k})$ are replaced by their average values over the nucleus

$$\begin{aligned} F(\vec{x}, \vec{k}) &\rightarrow \langle F(\vec{x}, \vec{k}) \rangle = G(k) \\ \vec{\nabla} F(\vec{x}, \vec{k}) &\rightarrow \langle \vec{\nabla} F(\vec{x}, \vec{k}) \rangle = i\vec{k} H(k) \end{aligned} \quad (4.2)$$

Substituting in Eq. (4.1) and remembering that for $m_\mu = 0$ we have

$$\frac{\vec{k} \cdot \vec{\alpha}}{E} u(k) = u(k) \quad (4.3)$$

we find for the effective wave function used in Ref. 8

$$\varphi(\vec{x}) = \bar{N} u(k) e^{i\vec{k} \cdot \vec{x}} \quad (4.4)$$

$$\bar{N} = G(k) + \frac{1}{2} H(k) \quad (4.5)$$

So Ref. 8 only allows for a change in normalization of the μ wave. Equation (4.4) is substituted into the Born approximation giving:

$$\frac{\partial^2 \sigma}{\partial q^2 \partial \nu} = (\bar{N})^2 \left(\frac{\partial^2 \sigma}{\partial q^2 \partial \nu} \right)^{(0)} \quad (4.6)$$

If we identify $\bar{N}$ with the average over the nucleus of our function $N(\vec{x})$

$$\bar{N} = \langle N(\vec{x}) \rangle \quad (4.7)$$

we find to first order in V/E_2

$$\bar{N}^2 = \langle N^2(\vec{x}) \rangle \quad (4.8)$$

Therefore, we can identify the correction given in Eq. (4.6) with that part of our correction coming from the change in normalization. The corrections coming from the change in μ wave number would be absent in this approach.

In Ref. 9 the opposite point of view is taken. The wave function used there has the general form (2.3) but with $N(\vec{x}) \equiv 1$. However, we cannot compare directly with Ref. 9 since there a more detailed non-relativistic model for the nucleons is used and the relevant approximations in the μ wave function are only made at a later stage. We think, however, that Ref. 9 is not far from the recipe given in Eq. (8.11) of Ref. 10, namely to use an effective wave number for the muon

$$\vec{k}'_2 = (E_2 - V(0)) \hat{k}_2 \quad (4.9)$$

and usual Born approximation. This leads to correction terms very similar to some we found, except that in our calculation the average $\bar{V}$ of the Coulomb potential and not its value at the origin appears. Of course, Refs. 9 and 10 cannot give the corrections coming from the change in normalization.

5. CONCLUSION

In Section 3 we have derived the Coulomb- and Fermi-motion correction to the cross-section for neutrino induced reactions on nuclei. Eqs. (3.15), (3.18) and (3.19) give these corrections for inelastic and quasi-elastic scattering. Since one of our basic assumptions was incoherent contribution of the individual nucleons, our inelastic cross-section is proportional to the number of nucleons, the quasi-elastic to the number of neutrons. The structure functions appearing are those for nucleons. It remains to see in what kinematic region these assumptions can be expected to hold.

At very small values of $|q^2|$ the exclusion principle will come into play as well as the coherent production of pions, i.e. the Adler PCAC effect^{11,12)}.

We would expect that for

$$|q^2| \geq 0,3 \text{ (GeV)}^2 \quad (5.1)$$

these effects may be neglected as we have done in our calculation.

We also have ignored possible coherent effects arising from the propagation of heavier mesons (ρ , A1-meson, for instance). This limits our considerations to the following kinematic region [see Bell¹³⁾, Eq. (23)]

$$m_\rho^2 - q^2 \gg 2\nu \sigma_{\rho N} \cdot \rho \quad (5.2)$$

$\sigma_{\rho N}$: ρ -nucleon cross-section ≈ 30 mb

ρ : nuclear density $\approx 2 \times 10^{38} \text{ cm}^{-3}$.

The kinematic region which remains and where our calculations are perhaps applicable is shown in Fig. 2.

In conclusion we comment on the magnitude of our corrections. As shown in the Appendix they do not exceed 5% in typical cases for reactions on uranium if we restrict the outgoing meson to energies greater than 1 GeV. For lower energies we expect the corrections still to be governed by the ratio potential energy/kinetic energy of the muon; however our formulae should no longer be used for quantitative comparison in this region.

APPENDIX

NUMERICAL ESTIMATES

To give a numerical example we will choose the extreme case of a reaction on uranium:

$$U^{238}, \quad Z = 92, \quad A = 238 \quad (A.1)$$

For the nuclear radius and the Fermi momentum we take (see Ref. 10):

$$R = 1.07 A^{1/3} f \quad (A.2)$$

$$p_F = 0.27 \text{ GeV} \quad (A.3)$$

a) Quasi-elastic scattering

Write Eq. (3.18) as follows:

$$\frac{d\sigma}{dq^2} = \left(\frac{d\sigma}{dq^2} \right)^{(0)} (1 + \delta_C + \delta_F) \quad (A.4)$$

$$\delta_C = -\frac{\bar{V}}{E_2} \left[2 + \frac{q^2 \frac{\partial f}{\partial q^2}(q^2, E_1)}{f(q^2, E_1)} \right]$$

$$\delta_F = \frac{p_F^2}{10M^2} \left[-5 + \frac{1}{f} E_1 \frac{\partial f}{\partial E_1} + \frac{1}{f} E_1^2 \frac{\partial^2 f}{\partial E_1^2} \right]$$

δ_C and δ_F are the Coulomb- and Fermi-motion correction. For the sake of simplicity we only consider one vector and one axial vector form factor:

$$\begin{aligned} \langle p(p_2) | J_2 | u(p_1) \rangle = & \frac{M}{\sqrt{p_2^0 p_2^0 V^2}} \bar{u}(p_2) \left[F_V(q^2) \gamma_2 \right. \\ & \left. + F_A(q^2) \gamma_2 \gamma_5 \right] u(p_1) \end{aligned} \quad (A.5)$$

This gives for $f(q^2, E_1)$:

$$\begin{aligned}
 f(q^2, E_1) = & 2 (F_V^2 + F_A^2) E_1^2 \\
 & - \frac{q^2}{M} (F_V + F_A)^2 E_1 \\
 & + \frac{q^2}{2} \left[\left(1 + \frac{q^2}{2M^2}\right) (F_V^2 + F_A^2) - 2F_A^2 \right. \\
 & \left. + \frac{q^2}{M^2} F_V F_A \right]
 \end{aligned}
 \tag{A.6}$$

Parametrize the form factors as follows [see Perkins¹⁴⁾]

$$F_V(q^2) = \frac{1}{(1 - q^2/M_V^2)^2} \tag{A.7}$$

$$F_A(q^2) = \frac{\lambda}{(1 - q^2/M_A^2)^2} \tag{A.8}$$

$$\lambda = 1, 2 \tag{A.9}$$

$$M_V^2 = M_A^2 = 0,71 \text{ (GeV)}^2 \tag{A.10}$$

The resultant corrections δ_C and δ_F for an incident neutrino energy $E_1 = 2 \text{ GeV}$ have been calculated with the interactive computer Gamma and are shown in Fig. 3.

b) Highly inelastic scattering

The cross-section for highly inelastic neutrino scattering has been measured at CERN¹⁵⁾. The following experimental fit is found:

$$\frac{\partial^2 \sigma}{\partial q^2 \partial \nu} = \frac{G^2}{4\pi} \frac{1}{E_1^2} \mathcal{F}(q^2, \nu, E_1) \quad (\text{A.11})$$

$$\begin{aligned} \mathcal{F}(q^2, \nu, E_1) = & \frac{2x_2}{\nu} E_1^2 - 2(x_2 - x_3) E_1 \\ & + (x_1 - x_3) \nu + \frac{q^2}{2\nu} x_2 \end{aligned} \quad (\text{A.12})$$

$$x_1 = x_2 = x = 0,9 \pm 0,3 \quad (\text{A.13})$$

$$x_3 = 0 \quad (\text{A.14})$$

The limits of validity of Eq. (A.12) are

$$\nu > 1.5 \frac{|q^2|}{M} \quad |q^2| > 1 \text{ GeV}^2 \quad (\text{A.15})$$

Since we have considered only highly relativistic muons we will also restrict ourselves to

$$E_2 = E_1 - \nu > 1 \text{ GeV} \quad (\text{A.16})$$

Write Eq. (3.15) in the following form:

$$\frac{\partial^2 \sigma}{\partial q^2 \partial \nu} = \left(\frac{\partial^2 \sigma}{\partial q^2 \partial \nu} \right)^{(0)} \left\{ 1 + \delta_c^{(1)} + \delta_c^{(2)} + \delta_F \right\} \quad (\text{A.17})$$

$$\delta_c^{(1)} = -\frac{\bar{V}}{E_2} \left[2 + \frac{1}{\mathcal{F}} q^2 \frac{\partial \mathcal{F}}{\partial q^2} \right] \quad (\text{A.18})$$

$$\delta_c^{(2)} = \bar{V} \frac{1}{\mathcal{F}} \frac{\partial \mathcal{F}}{\partial \nu} \quad (\text{A.19})$$

$$\delta_c = \delta_c^{(1)} + \delta_c^{(2)} \quad (\text{A.20})$$

$$\delta_F = \frac{p_F^2}{10M^2} \left[-3 + \frac{1}{\mathcal{F}} \left(3\nu \frac{\partial \mathcal{F}}{\partial \nu} + 3E_1 \frac{\partial \mathcal{F}}{\partial E_1} + (\nu^2 - q^2) \frac{\partial^2 \mathcal{F}}{\partial \nu^2} + E_1^2 \frac{\partial^2 \mathcal{F}}{\partial E_1^2} + (2E_1 \nu - q^2) \frac{\partial^2 \mathcal{F}}{\partial \nu \partial E_1} \right) \right] \quad (\text{A.21})$$

$\delta_C^{(1)}$ is that part of the Coulomb correction which violates the general structure of Eq. (1.1). The numerical results δ_C , δ_F are shown in Fig. 4.

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Figure captions

- Fig. 1 : Scattering of ν from an individual nucleon. In brackets are the momenta of the particles.
- Fig. 2 : Kinematic region (shaded) where our formulae are expected to hold.
- a) quasi-elastic line: $-q^2 = 2M\nu$
 - b) $-q^2 = 0.3$ [see Eq. (5.1)]
 - c) $-q^2 = 2\nu\sigma_{\rho N} \cdot \rho - m_\rho^2$ [see Eq. 5.2)].
- Fig. 3 : Corrections for Coulomb effect (δ_C) and Fermi motion (δ_F) for quasi-elastic scattering of neutrinos of incident energy $E_1 = 2$ GeV on uranium.
- Fig. 4 : Coulomb corrections [δ_C , $\delta_C^{(1)}$, $\delta_C^{(2)}$] and Fermi motion (δ_F) for incident neutrino energy $E_1 = 5$ GeV, momentum transfer $q^2 = -1$ (GeV)² as function of the energy transfer ν on uranium.

FIG. 1

FIG. 2

FIG. 3

FIG.4