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A SIMULTANEOUS DESCRIPTION OF TWO AND THREE-BODY DATA
WITH THE GENERALIZED VENEZIANO MODEL

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A B S T R A C T

A generalized Veneziano model which describes the three-body reactions $K^+p \rightarrow K^0 \pi^+ p$, $K^-p \rightarrow \bar{K}^0 \pi^- p$, and $\pi^- p \rightarrow K^0 K^- p$ is applied to study the two-body reactions

$$K^-p \rightarrow \bar{K}^0 n$$

$$K^+n \rightarrow K^0 p$$

$$\pi^- p \rightarrow K^0 \Lambda$$

$$K^-n \rightarrow \pi^- \Lambda$$

which are given by the three-body amplitude at the nucleon and lambda pole. With no free parameters it is found that with one exception one can give a simultaneous description of these two and three-body reactions at energies between 2.5 and 13 GeV/c, and at all momentum transfers except the extreme forward ($-t < 0.2 \text{ GeV}/c^2$) and extreme backward ($-u < 0.2 \text{ GeV}/c^2$) directions. The one exception to this description is briefly discussed.

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1. INTRODUCTION

The generalized Veneziano formula ¹⁾ has suggested a new approach to multiparticle phenomenology ²⁾. In addition to being "dual" and crossing symmetric, the model has the attractive property of connecting in a consistent manner processes with different numbers of particles in the final state. The dual and crossing properties of the model have already been checked against experiment in several special cases of five-line reactions ^{3),4)}. The purpose of this paper is to check further the consistency of the model by relating four- and five-line processes.

In Ref. 4), the following reactions related by crossing :

$$K^+ p \rightarrow K^0 \pi^+ p \quad (1)$$

$$K^- p \rightarrow \bar{K}^0 \pi^- p \quad (2)$$

$$\pi^- p \rightarrow K^0 K^- p \quad (3)$$

were described using the same Veneziano formula with essentially only one free parameter. The formula contains respectively in the $K^- p$ and $\pi^- p$ channels the lambda and nucleon trajectories. Thus, by evaluating the residue either at a lambda or at a nucleon pole, one should obtain the amplitudes for the following four reactions with, in principle, no freedom whatsoever :

$$K^- p \rightarrow \bar{K}^0 n \quad (4)$$

$$K^+ n \rightarrow K^0 p \quad (5)$$

$$\pi^- p \rightarrow K^0 \Lambda \quad (6)$$

$$K^- n \rightarrow \pi^- \Lambda \quad (7)$$

However, since the three-body model does not treat fermion spins correctly, we cannot rigorously compute the normalization of the two-body amplitudes. In this paper, we have fixed the normalizations only by a

crude argument. The result is then compared with experiment and found to be reasonable for energies and momentum transfers in which the three-body model itself is valid, namely $P_L \gtrsim 2.5 \text{ GeV}/c$ and $-t \gtrsim 0.2 \text{ GeV}/c^2$.

We wish to emphasize that we are not aiming for a best fit to the two-body data, but only trying to show some consistency relations between two and three-body reactions.

2. MODEL

The amplitude in Ref. 4) for the reactions (1)-(3) takes the form

$$\eta = \beta \epsilon_{\mu\nu\rho\sigma} P_{\pi}^{\mu} P_{\rho}^{\nu} P_{K^0}^{\rho} P_{K^+}^{\sigma} \{ P + R + S \} \quad (8)$$

where P , R and S are B_5 functions. These functions are depicted in Fig. 1, with all particles incoming. The arguments of P , R and S are $J - \alpha_{ij}(s_{ij})$, where $\sqrt{s_{ij}}$ is the invariant mass in the two-body sub-system of particles i and j , and α_{ij} is the appropriate trajectory. For meson trajectories $J=1$; for baryon trajectories J is the spin of the lowest member on the trajectory (either $\frac{1}{2}$ or $\frac{3}{2}$) for $\sqrt{s_{ij}} > m_i + m_j$. Below threshold when $\sqrt{s_{ij}} < m_i + m_j$, J is given the value 1. This is made to ensure artificially the correct Regge behaviour for an exchanged baryon trajectory. The trajectory $\alpha(s)$ is taken to be of the form

$$\alpha(s) = \alpha_0 + \alpha' s + i \theta(s-s_0) c (s-s_0)$$

where α_0 , α' , c and s_0 are constants determined from the positions and widths of the resonances. For the actual numbers used, see Ref. 4). The kinematic factor in (8) describes the fact that in the processes investigated, the coupling of the lowest member on each trajectory is a p wave. The normalization constant β used in (8) was determined by Chan et al. to have the value $\beta = 1160 \text{ GeV}^{-5}$ for reaction (1). This gives the correctly normalized cross-section also for reaction (2), but is off for reaction (3) by a factor two.

The two-body model is obtained from (8) by evaluating the three-body amplitude at a bound state pole. As an example, we evaluate (8) at the nucleon pole. At this pole the amplitude (8), for process (2), reduces to

$$\beta \varepsilon_{\mu\nu\rho\sigma} P_K^\mu P_K^\nu P_p^\rho P_p^\sigma 2 B_4\left(\frac{1}{2}-\alpha_N(s_{K-p}), 1-\alpha_{N_2}(s_{K-K_0})\right) \times \frac{1}{\frac{1}{2}-\alpha_N(s_{K-p})} P_{\pi-}^\sigma \quad (9)$$

where we have used the fact that $B_5(Z, X, U, V, Y)$ (see Fig. 2) reduces to ⁵⁾

$$B_4(U, V) \frac{1}{Z} = \frac{\Gamma(U)\Gamma(V)}{\Gamma(U+V)} \times \frac{1}{Z}$$

at the first pole in Z .

Factorizing the nucleon propagator and the π^-pn vertex (notice that our nucleon is formally a 1^- particle), we obtain from the model the two-body amplitude

$$T = \beta \sqrt{s} P_i P_f \sin\Theta \mathcal{B} \quad (10)$$

where for the process $K^-p \rightarrow \bar{K}^0 n$

$$\mathcal{B} = \frac{x}{g_{\pi^-pn} \alpha'_N} 2 B_4\left(\frac{1}{2}-\alpha_N(s), 1-\alpha_{N_2}(t)\right) \quad (11)$$

and for $K^+n \rightarrow K^0 p$

$$\mathcal{B} = \frac{x}{g_{\pi^+pn} \alpha'_N} 2 B_4(1-\alpha_N(u), 1-\alpha_{N_2}(t)) \quad (12)$$

Similarly, evaluating (8) at the lambda pole, we have for $\pi^-p \rightarrow K^0 \Lambda$:

$$\mathcal{B} = \frac{x'}{g_{\Lambda KN} \alpha'_\Lambda} \left\{ B_4 \left(\frac{1}{2} - \alpha_N(s), 1 - \alpha_{K^*}(t) \right) + B_4 \left(\frac{1}{2} - \alpha_N(s), 1 - \alpha_{Y^*}(u) \right) \right\} \quad (13)$$

and for $K^- n \rightarrow \pi^- \Lambda$

$$\mathcal{B} = \frac{x'}{g_{\Lambda KN} \alpha'_\Lambda} \left\{ B_4 (1 - \alpha_N(u), 1 - \alpha_{K^*}(t)) + B_4 (1 - \alpha_N(u), \frac{3}{2} - \alpha_{Y^*}(s)) \right\} \quad (14)$$

The kinematical variables s, t, u, p_i, p_f and θ , the slope of the trajectory α' , and the unknown normalization constants x and x' are the ones appropriate for each process. For later convenience, we include in the definitions of $\mathcal{B}$ the physical coupling constants $g_{\pi^{-}pn}$ and $g_{\Lambda^{0}KN}$. The predicted differential cross-section is thus ⁶⁾

$$\frac{d\sigma}{dt} = \frac{1}{64\pi p_i^2 s} \beta^2 s p_i^2 p_f^2 \sin^2 \theta |\mathcal{B}|^2 \quad (15)$$

In order to compare our model with the data, we choose to fix the unknown normalizations x and x' in (11)-(14) by a simple argument. We consider as an example reaction (2) at the nucleon pole, which is given by (9). We express (9) explicitly in terms of a product of the two-body amplitude (10), propagator and a vertex function $g P_\pi^\sigma$:

$$T \frac{1}{s_{\pi p} - m_\pi^2} g P_\pi^\sigma \quad (16)$$

where P_π^σ is the component along the normal $\hat{n}$ to the scattering plane, and g is the effective $\pi^{-}pn$ vertex coupling. Comparing (9) and (16) we relate the effective coupling g to the unknown two-body normalization x , obtaining

$$x g = g_{\pi^{-}pn} \quad (17)$$

Our estimate of x is based upon the guess that the vertex g_{π}^{σ} in (9) should be identified with the piece of the realistic vertex $g_{\pi-pn} \bar{u}(p) \gamma_5 u(n)$ having the same functional form. Formally, we evaluate the vertex g_{π}^{σ} in the rest frame of the neutron, obtaining

$$g_{\pi}^{\sigma} = g |P_{\pi}| \cos \xi$$

where ξ is the angle between the decay pion and the normal $\hat{n}$. The part of the realistic vertex which gives this angular dependence is

$$\begin{aligned} & g_{\pi-pn} \bar{u}^{\pm 1/2}(p) \gamma_5 u^{\pm 1/2}(n) \\ & = \pm g_{\pi-pn} \frac{2m_p}{m_p+m_n} \left(1 - \left(\frac{m_{\pi}}{m_p+m_n}\right)^2\right)^{-1/2} |P_{\pi}| \cos \xi \end{aligned} \quad (18)$$

where the spins are quantized along $\hat{n}$. Identifying $g |P_{\pi}| \cos \xi$ with one component of the vertex (18), we find

$$g = \frac{g_{\pi-pn} 2m_p}{m_p+m_n} \left(1 - \left(\frac{m_{\pi}}{m_p+m_n}\right)^2\right)^{-1/2}$$

Using (17) we obtain for x the value

$$x = \frac{m_p+m_n}{2m_p} \left(1 - \left(\frac{m_{\pi}}{m_p+m_n}\right)^2\right)^{1/2}$$

Evaluating x for reaction (4), and doing a similar calculation for (6), we find $x=1.0$ and $x'=0.84$.

3. COMPARISON WITH EXPERIMENT

In the following comparisons we use the two-body model with the normalizations x and x' obtained above, and the coupling constants $g_{\pi-pn}^2/4\pi = 28$ and $g_{\pi-KN}^2/4\pi = 16$ ⁷⁾. Since our aim is to show that

one can make a simultaneous description of both two and three-body data, we have normalized β in each case to the three-body reaction initiated by the same particles; i.e., for reaction (6), $\beta = 1160/\sqrt{2} \text{ GeV}^{-5}$, while for reactions (4), (5) and (7) $\beta = 1160 \text{ GeV}^{-5}$. We expect our result to be valid only for $P_L \gtrsim 2.5 \text{ GeV}/c$, $-t \gtrsim 0.2 \text{ GeV}/c^2$ and $-u \gtrsim 0.2 \text{ GeV}/c^2$, since it is in this region that the three-body model describes the data.

Our results for reactions (4)-(7) are given in Figs. 3-7 8)-14). Except for reaction (7), which we shall say more about below, we obtain a good over-all description of both the energy dependence of the cross-sections and the angular dependence of the differential cross-sections. We note in particular the following points :

- i) The prediction of the two-body normalization for reactions (4) and (6) at one energy we regard as a test of the property of the Veneziano amplitude which relates processes having different numbers of particles in the final state.
- ii) The known difficulty, namely a factor two discrepancy in cross-section between time reversed reactions due to exchange degenerate trajectories in the t channel ¹⁵⁾, is present also in our model. In reactions (4) and (5) the factor two may be present as can be seen in Figs. 3a and 3b. The factor two discrepancy between (6) and (7) is the same factor by which the three-body line reversed reactions (1) and (3) disagree ⁴⁾.
- iii) The reasonable description obtained for KN charge exchange we regard as a test of the hypothesis that the $N\alpha$ trajectory is dominant in the π^-p channel of the five-line process. From the three-body data, there is no strong evidence that the $N\alpha$ is or is not dominant.
- iv) Assuming the spin-flip amplitude is given entirely by our model, our results for $-t \lesssim 0.2 \text{ GeV}/c^2$ are compatible with the usual idea that in KN charge exchange the non-spin flip amplitude is small (Figs.4,5), whereas in reaction (6) and (7), this amplitude is large (Fig.6) ¹⁶⁾. One sees in Fig. 6 that the apparent discrepancies in cross-section for reaction (6) are due to these large non-spin flip contributions.

- v) The size of the backward peaks produced by baryon exchange in reactions (5) and (6) are predicted to have about the right magnitudes, although the detailed description of such peaks is still unclear (Fig.6) ¹⁷⁾.
- vi) The clear disagreement with experiment for reaction (7) is due to a known failure of Veneziano models having complex trajectories ¹⁸⁾. The difficulty occurs in the term in (14) containing s channel Y_1^* resonances and u channel nucleon exchange. This term contributes to the differential cross-section a large peak at small t which decreases exponentially at asymptotic energies. At the energies accessible, however, a numerical accident causes this peak to be quite large. With this term set to zero, we obtain both a reasonable description of the energy dependence of the cross-section (Fig.3), and the angular dependence of the differential cross-section (Fig.7).

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FIGURE CAPTIONS

- Figure 1 The B_5 functions used in Ref. 4) with all lines incoming. The label between each pair of adjacent lines represents the trajectory assumed to be dominant in that channel.
- Figure 2 The reduction of the B_5 amplitude at a pole in the Z channel.
- Figure 3 The energy dependence of the cross-section for
a) $K^-p \rightarrow \bar{K}^0 n$ 8), 9);
b) $K^+n \rightarrow K^0 p$ 10), 11);
c) $\pi^-p \rightarrow K^0 \Lambda$ 12), 13); and
d) $K^-n \rightarrow \pi^- \Lambda$ 12), 14).
The solid curves are the prediction of the model.
In d) the dotted curve is the prediction of the model with the s channel term in (14) set equal to zero [see point vi)].
- Figure 4 The angular distributions for $K^-p \rightarrow \bar{K}^0 n$ 9).
- Figure 5 The angular distributions for $K^+n \rightarrow K^0 p$ 11).
- Figure 6 The angular distributions for $\pi^-p \rightarrow K^0 \Lambda$ in the forward and backward direction 13).
- Figure 7 The angular distribution for $K^-n \rightarrow \pi^- \Lambda$ 14) compared with (14) when the s channel term is set equal to zero.

FIG.1

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FIG.4

FIG.5

FIG. 6

FIG. 7