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COMMUTATORS OF CURRENT DENSITY OPERATORS :
ASYMPTOTIC EXPANSION WITH THE PHOTON MASS TENDING TO INFINITY

W. Rühl
CERN - Geneva

A B S T R A C T

After a simple change of co-ordinates we can perform an $O(3,1)$ analysis for which the ratio of photon mass squared to photon energy in the rest system of the proton is fixed. Identifying the poles of the Fourier transforms with Lorentz poles leads us to a model in which the photon picks up part of the proton and scatters off the remainder of the proton by Lorentz pole exchange. Different Lorentz poles are linked together by current conservation. It is predicted that the ratio $\sigma_l : \sigma_t$ tends to zero at infinity.

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We consider the absorptive part of the amplitude for the forward scattering of a spacelike "charged" photon with momentum k

$$k^2 = -\mu^2, \quad \mu > 0$$

on a particle with mass M and spin s

$$\begin{aligned} M_{\mu\nu}^{ab}(k, p)_{q_2 q_1} = \\ = N^{-2} \int d^4x e^{ikx} \langle p, q_2 | [j_\mu^a(x), j_\nu^b(0)] | p, q_1 \rangle \end{aligned} \quad (1)$$

The notations are throughout the same as in ^{1), 2)}. Let us denote as usual

$$\nu = \frac{kp}{M}, \quad x = \frac{\mu^2}{2M\nu} \quad (2)$$

We introduce a timelike four-vector Q by

$$\begin{aligned} Q &= k + \alpha p \\ Q_0 &> 0 \\ Q^2 &= \ell^2 > 0, \quad \ell \text{ fixed} \end{aligned} \quad (3)$$

From (3) we deduce

$$\alpha M = -\nu + (\nu^2 + \mu^2 + \ell^2)^{\frac{1}{2}} \quad (4)$$

Instead of the four-vectors k and p which span the seven dimensional manifold

$$p^2 = M^2, k^2 = -\mu^2, 0 < \mu^2 < \infty$$

we can also use as co-ordinates the vectors Q and p and as seventh parameter the ratio $x^{-1}(2)$ varying over

$$-\infty < x^{-1} < \infty$$

We can define a new amplitude $M'_{\mu\nu}$ by

$$M'_{\mu\nu}(Q, p) q_2 q_1 = M_{\mu\nu}(k, p) q_2 q_1 \quad (5)$$

and submit it to an $O(3,1)$ analysis using the formalism displayed in 1) and 2) with minor adjustments only. If the particle with momentum p is at rest and the space component of Q has the direction of the positive third axis

$$Q = (\ell \cosh \vartheta, 0, 0, \ell \sinh \vartheta)$$

we obtain asymptotic expansions in powers of $e^{-\vartheta}$ for $\vartheta \rightarrow +\infty$ which leave x fixed. For $x > 0$ we have

$$\cosh \vartheta = \frac{1}{\ell} (\nu^2 + \mu^2 + \ell^2)^{\frac{1}{2}} \quad (6)$$

$$e^{\vartheta} = \frac{2\nu}{\ell} \left[1 + \frac{Mx}{\nu} + \frac{\ell^2 - 2M^2 x^2}{4\nu^2} + O(\nu^{-3}) \right] \quad (7)$$

Therefore $\nu \rightarrow +\infty$ implies $\nu \rightarrow +\infty$. In this limit we have for α

$$\alpha = x + \frac{\ell^2 - M^2 x^2}{2M\nu} + O(\nu^{-2}) \quad (8)$$

Harmonic analysis of the amplitude (5) on $O(3,1)$ is so far completely formal and cannot lead to any predictions. The situation changes, however, if we assume that the dominating singularities in the Fourier transforms are Lorentz poles, and if we accept the exchange philosophy usually connected with such poles. In order to justify this hypothesis we restrict x to the interval $0 < x < 1$. A close inspection into the kinematics (2) to (8) shows that the Lorentz poles are exchanged between an object of mass ℓ made of the photon and a fraction x of the timelike particle on one side, and the remainder of the timelike particle on the other side [note that replacing p by $(1-x)p$ on the left-hand side in (5) would not change our $O(3,1)$ analysis at all]. This picture interpolates between a pure resonance production at $x = 1$ and standard Lorentz pole exchange (Toller expansion) for $\mu^2 = \text{const.}$

In agreement with this model we change the definition (5) and make the ansatz

$$M_{\mu\nu}(k, p)_{q_2 q_1} = F(\mu^2, x) \tilde{M}_{\mu\nu}(Q, p)_{q_2 q_1} \quad (9)$$

where we assume that $F(\mu^2, x)$ takes the coupling and the propagation of the object of mass ℓ into account. We apply the Lorentz pole hypothesis to the amplitude $\tilde{M}_{\mu\nu}$. The $O(3,1)$ analysis of this amplitude is straightforward with exception of the constraints following from current conservation, because current conservation still makes use of the old momentum k and not of the new momentum Q . This leads not only to restrictions on the residues of each Lorentz pole, but relates different Lorentz poles forming "families" in addition.

As in 2) we study the specific example of a spin $\frac{1}{2}$ particle (proton) with momentum p and two currents of equal intrinsic parities. We find that scaling of both W_1 and W_2 is equivalent with the single relation

$$\lim_{\substack{\nu \rightarrow \infty \\ x \text{ fixed}}} \frac{2\nu}{\ell} F(\mu^2, x) = f(x) < \infty \quad (10)$$

provided the leading pole of class I is the Pommeranchuk pole. Since the ratio of νW_2 over W_1 does not contain $F(\mu^2, x)$ its value can be predicted. Independently of what the leading singularity of class I is, we get

$$\lim_{\substack{\nu \rightarrow \infty \\ x \text{ fixed}}} \frac{\nu W_2}{W_1} = 2Mx \quad (11)$$

With the cross-sections for photoproduction by longitudinal or transverse spacelike photons, σ_ℓ respectively σ_t , we can express (11) as

$$\lim_{\substack{\nu \rightarrow \infty \\ x \text{ fixed}}} \frac{\sigma_\ell}{\sigma_t} = 0 \quad (12)$$

We present the results in detail now. The amplitude $\tilde{M}_{\mu\nu}$ is decomposed into seven invariant functions $\tilde{\varphi}_i$, $i = 1, 2, 3, \dots, 7$ [see 2), Eq. (56)] such that

$$\varphi_i(\nu, \mu^2) = F(\mu^2, x) \tilde{\varphi}_i(\nu, \mu^2)$$

The symmetric tensor part of $\tilde{M}_{\mu\nu}$ possesses four ($i = 1, 2, 3, 4$) invariant amplitudes and correspondingly four Fourier transforms, namely

$$\chi' = [1, 1] : F_{(0,0)}(\lambda), F_{(0,2)}(\lambda), F_{(0,-2)}(\lambda)$$

$$\chi' = [0, 0] : F(\lambda)$$

Their singularities (say Lorentz poles) are all of class I. A Lorentz pole with quantum number λ_I appears at the positions [see 2), Eq. (71)]

$$\begin{aligned} F_{(0,0)}(\lambda) &\approx \frac{A}{\lambda - \lambda_I}, \quad F_{(0,2)}(\lambda) \approx \frac{B}{\lambda - \lambda_I - 2} \\ F_{(0,-2)}(\lambda) &\approx \frac{C}{\lambda - \lambda_I + 2}, \quad F(\lambda) \approx \frac{D}{\lambda - \lambda_I} \end{aligned} \quad (13)$$

and gives the asymptotic contributions

$$\begin{aligned} M^2 \tilde{\varphi}_1 &\approx 2\pi 2^{\frac{1}{2}} \frac{B}{\lambda_I + 2} \left(\frac{Mx}{e}\right)^2 \left(\frac{2\nu}{e}\right)^{\lambda_I - 1} \\ e^2 \tilde{\varphi}_2 &\approx \left(\frac{Mx}{e}\right)^{-2} M^2 \tilde{\varphi}_1 \\ M e \tilde{\varphi}_3 &\approx 2 \left(\frac{Mx}{e}\right)^{-1} M^2 \tilde{\varphi}_1 \\ 4 \tilde{\varphi}_4 &\approx 2\pi 2^{\frac{1}{2}} \left[-\frac{1}{2} A \frac{(\lambda_I - 1)^2}{\lambda_I} - 2B \frac{1}{(\lambda_I + 1)(\lambda_I + 2)} \right. \\ &\quad \left. + \frac{1}{\lambda_I} D \right] \left(\frac{2\nu}{e}\right)^{\lambda_I - 1} \end{aligned} \quad (14)$$

Current conservation necessitates $B = 0$ and spoils therefore the asymptotic formulae (14) completely. In order to satisfy current conservation to all orders we assume the existence of two families of Lorentz poles of class I at the points $\lambda_I - 2n$, respectively $\lambda_I - 2n - 1$, $n = 0, 1, 2, \dots$

$$\begin{aligned}
 F_{(0,0)}(\lambda) &= \sum_{n=0}^N \left[\frac{A_n}{\lambda - \lambda_I + 2n} + \frac{A'_n}{\lambda - \lambda_I + 2n + 1} \right] + R_{(0,0)}^N(\lambda) \\
 F_{(0,2)}(\lambda) &= \sum_{n=0}^N \left[\frac{B_n}{\lambda - \lambda_I + 2n - 2} + \frac{B'_n}{\lambda - \lambda_I + 2n - 1} \right] + R_{(0,2)}^N(\lambda) \\
 F_{(0,-2)}(\lambda) &= \sum_{n=0}^N \left[\frac{C_n}{\lambda - \lambda_I + 2n + 2} + \frac{C'_n}{\lambda - \lambda_I + 2n + 3} \right] + R_{(0,-2)}^N(\lambda) \\
 F(\lambda) &= \sum_{n=0}^N \left[\frac{D_n}{\lambda - \lambda_I + 2n} + \frac{D'_n}{\lambda - \lambda_I + 2n + 1} \right] + R^N(\lambda)
 \end{aligned}
 \tag{15}$$

The constraint of current conservation [see 2), Eq. (58)]

$$\tilde{\vartheta}_1 - x \tilde{\vartheta}_3 = 0$$

implies relations within each family only. We find

$$\begin{aligned}
 \left(\frac{Mx}{e} \right)^2 B_0 &= 0 \\
 \left[1 - \left(\frac{Mx}{e} \right)^2 \right] A_0 - \frac{1}{\lambda_I - 1} \left(\frac{Mx}{e} \right)^2 B_1 \\
 + \lambda_I (\lambda_I - 1) (\lambda_I - 2) C_0 &= 0
 \end{aligned}
 \tag{16}$$

and further relations between A_n , B_{n+1} and C_n . Replacing A_n , B_n , C_n by A'_n , B'_n , C'_n and λ_I by $\lambda_I - 1$ yields the corresponding constraints for the other family. The second constraint of current conservation

$$-8Mx\nu\tilde{\vartheta}_2 + 2M\nu\tilde{\vartheta}_3 + 4\tilde{\vartheta}_4 = 0$$

gives in addition cross relations between the two families

$$\begin{aligned} D_0 + \frac{1}{2} (\lambda_I - 1)^2 A_0 &= 0 \\ D'_0 + \frac{1}{2} (\lambda_I - 2)^2 A'_0 &= \\ &= 2 \frac{Mx}{e} \frac{\lambda_I - 1}{\lambda_I} [(\lambda_I - 1)A_0 + B_1] \end{aligned} \quad (17)$$

The asymptotic formulae for the invariant functions following from (15) to (17) is

$$\begin{aligned} M^2 \tilde{\vartheta}_1 &\simeq 2\pi 2^{\frac{1}{2}} \frac{1}{\lambda_I} (\lambda_I - 1)^2 A_0 \frac{Mx}{e} \left(\frac{2\nu}{e}\right)^{\lambda_I - 2} \\ e^2 \tilde{\vartheta}_2 &\simeq 2\pi 2^{\frac{1}{2}} \frac{1}{\lambda_I} [-A_0(\lambda_I - 1)(\lambda_I - 2) + B_1] \\ &\quad \times \left(\frac{2\nu}{e}\right)^{\lambda_I - 3} \end{aligned} \quad (18)$$

For the functions W_1 and νW_2 we find from (18)

$$2\pi W_1 \simeq \frac{\pi}{2} 2^{\frac{1}{2}} \frac{1}{\lambda_I} (\lambda_I - 1)^2 A_0 F(\mu^2, x) \left(\frac{2\nu}{e}\right)^{\lambda_I - 1} \quad (19)$$

and the ratio (11).

A particular solution of the current conservation constraint is

$$A'_n = B'_n = C'_n = D'_n = 0 \quad \text{for all } n$$

so that only the first family in (15) survives. In this case current conservation entails that the contributions $\tilde{\varphi}_1^*$, $\tilde{\varphi}_2^*$, of this family to $\tilde{\varphi}_1$, $\tilde{\varphi}_2$ are such that to any order

$$M^2 \tilde{\varphi}_1^* = -2M \times v \tilde{\varphi}_2^* \quad (20)$$

The results (10) to (12) remain valid.

The antisymmetric tensor part of $\tilde{M}_{\mu\nu}$ possesses three invariant functions $\tilde{\varphi}_5$, $\tilde{\varphi}_6$, $\tilde{\varphi}_7$ and correspondingly three Fourier transforms, namely $\chi' = [1,0] : F_{(0,0)}(\lambda)$, $F_{(-1,1)}(\lambda)$, and $F_{(1,-1)}(\lambda)$. The singularities of the Fourier transform $F_{(0,0)}$ are of class II, those of the two other Fourier transforms are class III. A single pole of class II [see 2), Eq. (72)]

$$2^{\frac{1}{2}} F_{(0,0)}(\lambda) \approx \frac{E}{\lambda - \lambda_{II}} \quad (21)$$

implies

$$\begin{aligned} M \tilde{\varphi}_5 &\approx 4\pi 3^{\frac{1}{2}} E \frac{2\lambda_{II}-3}{\lambda_{II}} \frac{M \times}{e} \left(\frac{2\nu}{e}\right)^{\lambda_{II}-2} \\ \ell \tilde{\varphi}_6 &\approx 4\pi 3^{\frac{1}{2}} E \frac{\lambda_{II}-1}{\lambda_{II}} \left(\frac{2\nu}{e}\right)^{\lambda_{II}-2} \\ M e^2 \tilde{\varphi}_7 &\approx -8\pi 3^{\frac{1}{2}} E \frac{\lambda_{II}-2}{\lambda_{II}} \left(\frac{2\nu}{e}\right)^{\lambda_{II}-3} \end{aligned} \quad (22)$$

A single pole of class III [see 2), Eq. (73)]

$$F_{(-1,1)}(\lambda) \approx \frac{F}{\lambda - \lambda_{\text{III}} - 1}, \quad F_{(1,-1)}(\lambda) \approx \frac{G}{\lambda - \lambda_{\text{III}} + 1} \quad (23)$$

yields

$$\begin{aligned} M\tilde{\rho}_5 &\approx -4\pi 3^{\frac{1}{2}} \frac{F}{\lambda_{\text{III}} + 1} \frac{Mx}{e} \left(\frac{2\nu}{e}\right)^{\lambda_{\text{III}} - 1} \\ e\tilde{\rho}_6 &\approx +4\pi 3^{\frac{1}{2}} \frac{F}{\lambda_{\text{III}} + 1} \left(\frac{2\nu}{e}\right)^{\lambda_{\text{III}} - 1} \\ M e^2 \tilde{\rho}_7 &\approx +8\pi 3^{\frac{1}{2}} \frac{F}{\lambda_{\text{III}} + 1} \left(\frac{2\nu}{e}\right)^{\lambda_{\text{III}} - 2} \end{aligned} \quad (24)$$

Current conservation forces the class II and class III poles to conspire. There is only a single constraint of current conservation [see 2), Eq. (58)]

$$\tilde{\rho}_5 + 2Mx\nu \tilde{\rho}_7 = 0$$

which can be satisfied to any order by the ansatz

$$\begin{aligned} 2^{\frac{1}{2}} F_{(0,0)}(\lambda) &= \sum_{n=0}^N \left[\frac{E_n}{\lambda - \lambda_{\text{II}} + 2n} + \frac{E'_n}{\lambda - \lambda_{\text{II}} + 2n+1} \right] + R_{(0,0)}^N(\lambda) \\ F_{(-1,1)}(\lambda) &= \sum_{n=0}^N \left[\frac{F_n}{\lambda - \lambda_{\text{III}} + 2n-1} + \frac{F'_n}{\lambda - \lambda_{\text{III}} + 2n} \right] + R_{(-1,1)}^N(\lambda) \\ F_{(1,-1)}(\lambda) &= \sum_{n=0}^N \left[\frac{G_n}{\lambda - \lambda_{\text{III}} + 2n+1} + \frac{G'_n}{\lambda - \lambda_{\text{III}} + 2n+2} \right] + R_{(1,-1)}^N(\lambda) \end{aligned}$$

$$\lambda_{\text{II}} = \lambda_{\text{III}} + 1 \quad (25)$$

describing two families of classes II and III. Current conservation implies

$$\begin{aligned} \frac{M \times}{e} \frac{1}{\lambda_{II}} (E_0 + F_0) &= 0 \\ \frac{M \times}{e} \frac{1}{\lambda_{II}-1} (E'_0 + F'_0) + \frac{1}{\lambda_{II}} E_0 + (\lambda_{II}-1) G_0 &= 0 \end{aligned} \quad (26)$$

If $E_0 = -F_0 \neq 0$ we may simply add up (22) and (24) since no complete cancellations occur. We note that a particular solution exists for which the primed members of both families are absent. In this case the contributions $\tilde{\varphi}_6^*$ and $\tilde{\varphi}_7^*$ to $\tilde{\varphi}_6$ and $\tilde{\varphi}_7$ due to the family considered satisfy to any order

$$e \tilde{\varphi}_6^* = - M \nu e \tilde{\varphi}_7^* \quad (27)$$

A single family of class II does never satisfy current conservation but needs a compensating family of class III. On the other hand it is possible to have a class III family without a conspiring class II family.

REFERENCES

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