A DUAL RESONANCE ANALYSIS OF DIFRACTIVE TWO-PION PHOTOPRODUCTION

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ABSTRACT

A recently proposed dual resonance model for diffraction dissociation is extended to two-pion photoproduction on protons and shown to give good agreement with experimental data. It yields vector meson dominance in the sense of a strongly $\rho$ dominated two-pion mass spectrum and moreover explains the observed deviation of the $\rho$ peak from a Breit-Wigner shape as well as the "background". Further experimental tests are proposed.
1. **INTRODUCTION**

High energy diffraction dissociation has generally been described either as effective two-body process (resonance excitation) or by a multi-peripheral approach. In spite of the considerable success of these descriptions, both encounter essential difficulties: in the resonance picture the background remains an open question, while the multi-peripheral picture can account for resonances (by duality arguments \(^1\)) at best in an average sense.

Experimentally, the most successful case for the resonance approach is given by high energy two-pion photoproduction:

\[
\gamma P \rightarrow P \pi^+ \pi^- \tag{1}
\]

Here the two-pion mass spectrum is indeed dominated very strongly by the \(\rho\) peak, and therefore this reaction has been considered in the past mainly under the aspect of \(\rho\) production [vector meson dominance (VMD) \(^2\)]. Nevertheless, the presence of background contributions as well as the asymmetry of the \(\rho\) peak indicate that even here genuine three-body effects cannot be neglected. Thus to interpret the data for reaction (1) in terms of a pure VMD model, these features must first be taken into account, which is presently done in a variety of more or less ad hoc ways: subtraction of "phase space background", modification of the Breit-Wigner resonance form by adding multi-peripheral terms to the \(\rho\) production amplitude \(^3\) or by introducing form factors dependent on the two-pion mass \(^4\), etc.

In this paper we want to present a model which, at high energies, is applicable to the whole of reaction (1), i.e., a genuine three-particle final state description which includes both resonance and multi-peripheral aspects as well as any "background". For hadronic diffraction dissociation in general such a model was recently proposed \(^5\); it describes the diffractive dissociation of a hadron into two or more hadrons in terms of a dual resonance
amplitude 6) and was shown to agree quite well with existing high energy data for the reaction $PP \rightarrow PN \pi^+$. The extension of this model to the photoproduction of two pions will be formulated in Section 2 and compared with experiment in Section 3.

2. THE MODEL

The diffractive photoproduction of two pions is illustrated in Fig. 1, with $\mathbf{P}$ denoting Pomeron exchange. Since $\mathbf{P}$ has the quantum numbers of the vacuum, only the isovector part of the photon can (to first order in $\alpha$) contribute to the process. In order to establish the connection to strong interactions, we shall assume this part of the photon to behave as an isovector vector hadron ($I^G=1^+, J^P=1^-$) of mass zero.*

Analogous to Ref. 5) and bearing in mind that we later on average over nucleon spins, we now write for the matrix element $\mathcal{M}$ describing reaction (1)

$$\mathcal{M} = e^{a/2} t_{PP} \bar{s} V$$

(2)

where $t_{PP} = (p-p')^2$ and $\bar{s} = (p+q)^2$ (cf., Fig. 1); the $PP \mathbf{P}$ form factor $e^{(a/2)^2} t_{PP}$ is determined (by factorization) from elastic proton-proton (or pion-proton) scattering: a $\sim 5 \text{ GeV}^{-2}$. Before we can now introduce a dual resonance form for the amplitude $V$ describing the "reaction" $\mathbf{P} \gamma^* \rightarrow \pi^+ \pi^-$, we have to engage in some kinematics.

In terms of an invariant amplitude $\tilde{\mathcal{A}}(s,t,u)$ we can write

*) This is a considerably weaker assumption than the mass extrapolation hypothesis in the VMD formalism, which in addition postulates the matrix element to be independent of the photon mass.
\[ V = \beta \left\{ \left[ (k^+ q)^- k^- (k^- q)^+ k^+ \right] \epsilon^u_q \right\} \tilde{A}(s, t, u) \]

with
\[ k^\pm = k^+ \pm k^- \]
\[ s = (k^+ + k^-)^2; \quad t = (q - k^-)^2; \quad u = (q - k^+)^2 \]
\[ s + t + u = 2k^2 + t_{pp} \]

and with \( \epsilon^u_q \) and \( \beta \) denoting the photon polarization and the
over-all normalization, respectively. The expression in curly
brackets in (3) is the only gauge-invariant form that can be
constructed. The amplitude \( \tilde{A}(s, t, u) \) contains (as easily seen,
e.g., from the OPE terms) a kinematical singularity at \( (k^+ q) = \frac{1}{2}(s - t_{pp}) = 0 \), which we remove explicitly

\[ \tilde{A}(s, t, u) = A(s, t, u) / (k^+ q) \]

For this \( A(s, t, u) \), free of kinematical singularities, we now
introduce a dual resonance form \( 6 \)

\[ A(s, t, u) = B(1 - \alpha_p(s) - \alpha_{A_1}(u)) + B(1 - \alpha_p(t) - \alpha_{A_1}(t)) + \eta B(-\alpha_p(t) - \alpha_{A_1}(u)) \]

with \( B(x, y) = \Gamma(x) \Gamma(y) / \Gamma(x+y) \). Here we have taken the (degenerate)
\( \pi \) and \( A_1 \) trajectories to dominate the \( t \) and \( u \) channels, and
the \( \rho \) trajectory the (two-pion) \( s \) channel. As constructed,
\( A(s, t, u) \) has the correct pole structure and asymptotic limits in
all three channels. Since the emitted two-pion system has isospin
one, \( V \) must be antisymmetric and hence \( A \) symmetric under \( t-u \)
crossing; this fixes the relative weight and sign for the first two
terms in (6). The parameter \( \eta \) describes the relative coupling
strengths of \( \pi \) and \( A_1 \) exchange: \( \eta = \pm 1 \) implies only odd or
even (parent) signature, respectively. We shall for the following
assume equal couplings, setting \( \eta = 0 \).
The differential cross-section for reaction (1) is given in terms of the above amplitude by

\[
\frac{d\sigma}{|M|^2} = \frac{2m^2}{(2\pi)^6 (s-m^2)} \frac{d^3k^+}{2k^+} \frac{d^3k^-}{2k^-} |M|^2 \delta^{(4)}(p', k^+ + k^- - \mu - q')
\]  

(7)

where \( m \) denotes the nucleon mass and \( |M|^2 \) the squared matrix element (2) averaged over photon polarization and nucleon spin:

\[
|M|^2 = \frac{\beta^2}{2} \bar{\epsilon}^2 e^{\alpha t_{pp}} (s-4p^2) \sin^3 \theta \sin^2 \theta |A(s,t,u)|^2
\]  

(8)

Here \( \hat{\theta} \) is the angle of one of the two pions with respect to the incoming photon in the two-pion CMS (Jackson angle), and \( \mu \) the pion mass.

The trajectories \( \alpha_{\pi} \) and \( \alpha_\rho \) are taken as usual

\[
\alpha_{\pi}(x) = -\mu^2 + x
\]

\[
\alpha_\rho(x) = 1 - m_\rho^2 + x + i(x-\mu^2) \frac{m_\rho p^\rho}{(m_\rho^2 - \mu^2)} \Theta(x-\mu^2)
\]  

(9)

where the imaginary part of \( \alpha_\rho \) vanishes below the two-pion threshold and for \( x=m_\rho^2 \) correctly reproduces the \( \rho \) width \( \Gamma_\rho \).

3. COMPARISON WITH EXPERIMENT

As already stressed in the Introduction, our model applies to the whole of the reaction \( P \to P \pi^+ \pi^- \) in the diffractive region \( p^+ \geq 4 \text{ GeV}/c \) for \( |t| = m_{\pi}^2 \leq 1.5 \text{ GeV}^2 \); it therefore should not be compared with any results for \( P \to P \rho \), but rather to the complete data from which these results were extracted.
The total diffractive cross-section for reaction (1) at laboratory energies above 4-4.5 GeV/c is found to be approximately 19 fb^{7-10}, which determines β = 4 GeV^{-5}. All results of our model from here on will hence be absolute predictions.

A. Mass spectra

In Fig. 2 we show the comparison of the model with the SLAC 4.7 GeV/c laser beam experiment, which provides at present the highest statistics data at fixed photon energy (≈900 events). All presently available two-pion mass distributions measured for reaction (1) with $P_{lab} > 4.5$ GeV/c $^{7-10}$ are combined to give Fig. 3 (≈2500 events). The agreement between prediction and data in both cases is seen to be very good; in particular, the model correctly reproduces the asymmetry of the $\rho$ peak as well as the magnitude of the cross-section in the high mass region (a comparison is, of course, meaningful only up to $M_{\pi\pi} \approx 1.5$-1.7 GeV, since for the given photon energies reactions leading to higher two-pion masses are very close to threshold and hence hardly diffractive). The cross-section due to the $\rho$ pole alone is found to be 13.3 fb.

The predicted small bumps at 1250 MeV ($\rho'$) and at 1550 MeV ($\rho''$) are due, respectively, to the first daughter ($J^P = 1^-$) of the first recurrence and to both parent ($J^P = 3^-$) and second daughter ($J^P = 1^-$) of the second recurrence of the $\rho$ trajectory. The statistics of the quoted experiments are not yet sufficient to confirm or exclude the existence of such resonances. We shall return to this question later on in the context of counter experiments.

B. Momentum transfer and angular distributions

In Figs. 4 and 5 we compare our $t_{pp}$ distribution at $P_{lab} = 4$ GeV/c (both over-all and for various $M_{\pi\pi}$ intervals) with the DESY data for $2.5 < P_{lab} < 5.8$, which are presently the only available data of proton-proton momentum transfer distributions for reaction (1). Apart from the general agreement, it should be noted that the $t_{pp}$ slope $b(M_{\pi\pi})$
\[
\frac{d\sigma}{dt_{pp}} \sim e^{b(M_{EE})t_{pp}}
\]

(10)
decreases both in prediction and data with increasing two-pion mass \(M_{\pi\pi}\) - a trend which is already discussed in Ref. 5) for the dual resonance description of \(PP\) diffraction dissociation.

In particular, \(b\) decreases significantly even within the \(p\) region; we find \(b = 5.9 \text{ GeV}^{-2}\) at 730 MeV, 5.6 GeV\(^{-2}\) at 760 MeV and 5.2 GeV\(^{-2}\) at 790 MeV. In this model, the \(t_{pp}\) dependence of the \(p\) pole alone would give \(b = 5 \text{ GeV}^{-2}a\), i.e., just the elastic form factor introduced in (2), since the residue of the Veneziano amplitude \(A(s,t,u)\) at the \(p\) pole is independent of \(t_{pp}\). The larger slope values quoted above are due to the additional non-pole contributions and their interference with the \(p\) pole; the \(t_{pp}\) dependence of these contributions is, largely as a consequence of \(t\) channel pion exchange, much stronger than that of the pole term alone *).

We now turn to the pion angular distributions. Shown in Fig. 6 is the pion-photon momentum transfer (t) distribution (integrated over all \(M_{\pi\pi}\) ) at \(P_{lab} = 4 \text{ GeV}/c\), compared with the experimental results for \(3.5 < P_{lab} < 5.8 \text{ GeV}/c\) 11). In Fig. 7 we show, for various \(M_{\pi\pi}\) intervals, both the \(t\) and the \(\cos \theta\) (Jackson angle) distributions together with the data of Ref. 11). Since diffraction dissociation allows only odd partial waves for the outgoing two-pion system, the \(\cos \theta\) distribution is forward-backward symmetric.

*) In most experimental determinations of the \(p\) slope, a non-interfering phase space type "background" is subtracted at fixed \(t_{pp}\); this leads to somewhat larger slopes.
The predicted over-all \( t \) distribution is seen to agree quite well with the data; also in their dependence on \( M_{\pi\pi} \), experimental and theoretical distributions display similar behaviours. Quantitatively the situation is least satisfactory for the \( \cos \theta \) distribution in the interval \( 0.72 \leq M_{\pi\pi} \leq 0.82 \text{ GeV} \), where our model predicts essentially a \( \sin^2 \theta \) behaviour. It would be of great interest to see if this apparent discrepancy persists at higher energies, where non-diffractive effects can with greater certainty be neglected.

As the two-pion mass increases beyond the \( p \) region one observes, both experimentally and in our results, an increasing peak near \( \hat{\theta} = 0 \), which eventually dominates completely the ("resonant") \( \sin^2 \hat{\theta} \) behaviour.

Finally, our model predicts as direct consequence of the form (2) a constant distribution for the Treiman-Yang angle \( \varphi \), defined by

\[
\cos \varphi = \left[ \frac{\mathbf{q}' \cdot \mathbf{k}^+ \cdot \mathbf{q} \times \mathbf{k}^+}{\left| \mathbf{p}' \right| \left| \mathbf{p} \right| \left| \mathbf{q} \right| \left| \mathbf{k}^+ \right|} \right]_{\pi \pi \text{ CMS}}
\]  

(11)

It is seen from Fig. 8 that the experimental results from Ref. 11) are quite compatible with this prediction.

C. Some further implications

So far we have found that our model describes rather well the experimentally observed features of two-pion photoproduction on hydrogen; in the following, we want to point out some further experimental tests.

As already mentioned, the slope \( b(M_{\pi\pi}) \) describing the proton-proton momentum transfer distribution \([\text{cf.}, \text{ Eq. (10)}]\) shows an appreciable variation with the invariant two-pion mass \( M_{\pi\pi} \). In Fig. 9 we display the detailed behaviour of \( b(M_{\pi\pi}) \), calculated in the interval \( t_{\min} \leq |t_{pp}| \leq 0.215 \) for \( P_{\text{lab}} = 4.7 \text{ GeV/c} \).
Asymptotically in $M_{\pi\pi}$ we find \[ b = a = 5 \text{ GeV}^{-2} \]. As evident from Fig. 5, the data for $M_{\pi\pi} \approx 1 \text{ GeV}$ agree quite well with the predicted $t_{FP}$ distributions and hence with our predicted slope behaviour. It would be very interesting to see if our results for higher masses can also be experimentally established, i.e., if the experimental $t_{FP}$ distribution indeed becomes steeper again for $M_{\pi\pi} \approx 1.2 \text{ GeV}$ and asymptotically approaches $b = 5 \text{ GeV}^{-2}$.

A further question of great current interest concerns the existence of higher mass vector and tensor mesons as predicted \(^{12}\) by the Veneziano form; these in fact appear as rather small broad bumps in our $M_{\pi\pi}$ distribution (cf. Figs. 2 and 3). A recent counter experiment on the photoproduction of forward pion pairs on carbon \(^{13}\) has not revealed the existence of such resonances. However, for the evaluation of this experiment a $\sin^2 \hat{\xi}$ distribution relative to the incident photon beam was assumed for the two pions in their rest system. As seen from Fig. 7, this assumption can hardly be justified for $M_{\pi\pi} \gtrsim 1 \text{ GeV}$. Furthermore, it is not clear to us to what extent nuclear physics effects (e.g., mass dependent absorption) may affect a comparison between the reactions on carbon and on hydrogen. We therefore find it difficult to consider this experiment as conclusive for the absence of higher two-pion resonances in the reaction $\gamma P \to P \pi^+ \pi^-$. Thus we would consider of great interest a high energy counter experiment on hydrogen, measuring symmetric ($\hat{\xi} = \pi/2$) pion pairs in the forward direction ($t_{FP} = t_{FP}^{\min}$). As pointed out in Ref. 13), this kinematic region is most suitable for the detection of vector mesons; it is also evident from Fig. 5, where the strong pion exchange contributions at higher $M_{\pi\pi}$ are seen to appear most dominant for small $\hat{\xi}$. In Fig. 10 we show for such an experiment at $P_{\text{lab}} = 5 \text{ GeV/c}$ our predicted $(\sigma / dt_{FP} \cos \hat{\xi}) (\hat{\xi} = \pi/2, t_{FP} = t_{FP}^{\min})$ as function of $M_{\pi\pi}$. The expected signal of the $P'$ at 1250 MeV is predicted to be a factor two above the "background". The further bump at 1550 MeV (cf. Figs. 2 and 3) has largely disappeared; this indicates that it is essentially due to the parent $(J^P = 3^-)$ contribution, which in contrast to $J^P = 1^-$ does not lead to a maximum for $\hat{\xi} = \pi/2$. 
It should be pointed out here that these conclusions concerning higher vector mesons hold only for the simple Veneziano form (6) without satellites. It is possible of course to introduce satellite terms which bring about no change for \( M \lesssim 1 \text{ GeV} \) and yet substantially alter the high mass behaviour.

4. CONCLUSIONS

The general agreement found between experiment and the model presented here seems to indicate that a dual resonance approach to diffraction dissociation can in fact provide a suitable framework for the understanding of two-pion photoproduction: it not only leads to vector meson dominance in the sense of a strongly \( P \) dominated two-pion mass spectrum, but also accounts for the observed deviation from a Breit-Wigner form and for the background, features which in a VMD approach call for additional explanations.

In the context of diffraction dissociation in general, high energy two-meson photoproduction is moreover of considerable interest, since it is the only experimentally accessible process with a three-body final state where a dual resonance approach is not faced with the well-known problem of parity doubling \(^{14}\) (for fermions on linear trajectories). Therefore we consider the present analysis to provide significant further support for the picture proposed in 5) and to encourage further attempts to understand high energy hadronic interactions in terms of a dual resonance framework.

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FIGURE CAPTIONS

Figure 1: Diffraction dissociation diagram.

Figure 2: Experimental [Ref. 7] and theoretical two-pion mass spectrum.

Figure 3: Experimental [compilation of Refs. 7)-10] and theoretical two-pion mass spectrum.

Figure 4: Experimental [Ref. 9] and theoretical over-all proton-proton momentum transfer distributions.

Figure 5: Experimental [Ref. 9] and theoretical proton-proton momentum transfer distributions in various $M_{NN}$ intervals.

Figure 6: Experimental [Ref. 11] and theoretical over-all $\not{p} p$ momentum transfer distributions.

Figure 7: Experimental [Ref. 11] and theoretical $\not{p} p$ momentum transfer and Jackson angle distributions for various $M_{NN}$ intervals.

Figure 8: Experimental [Ref. 11] and theoretical over-all Treiman-Yang angle distribution [cf., Eq. (11)].

Figure 9: Slope parameters $b$ [cf., Eq. (10)] as function of $M_{NN}$.
Figure 10:

Predicted differential cross-section

\[ X = \left[ \frac{d\sigma}{dt_{pp}} \frac{d\cos \theta}{d\cos \theta} \right] \theta = \theta_{1/2}, \ t_{pp} = t_{pp}^{min} \]

as function of \( M_{\tilde{q} \tilde{q}} \), in arbitrary units; the inset shows the high mass region enlarged by a factor ten.
FIG. 1
$\gamma P \rightarrow P \pi^+ \pi^-$
$P_Y^{LAB} = 4.7 \text{ GeV/c}$
\[ \gamma P \rightarrow P \pi^+ \pi^- \]
\[ 2.5 \leq p_{\text{LAB}}^y \leq 5.8 \]

\[ \frac{d\sigma}{dt_{pp}} (\mu b/\text{GeV}^2) \]

\[ -t_{pp} (\text{GeV}^2) \]

FIG. 4
\( \gamma P \rightarrow P \pi^+ \pi^- \)

\( 3.5 \leq p_{\text{LAB}}^\gamma \leq 5.8 \)
\[ \gamma P \rightarrow P \pi^+ \pi^- \quad \text{if} \quad 3.5 \leq P_{\text{LAB}}^\gamma \leq 5.8 \]