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WHAT IS DUALITY?

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Duality gives a satisfying connection between two different areas of strong interaction physics, Regge poles at high energy and resonances at low energy. This interlocking gives powerful bootstrap conditions, and together with the assumption that certain channels do not contain resonances it gives strong restrictions on the hadron spectrum. Since there is some confusion about the term duality, we shall explain what is meant by the various forms of duality (FESR duality, local duality), and what is meant by "building up", and we shall show in what way antidual models (such as the generalized interference model) come into conflict with basic empirical facts.

Duality expresses the relationship between two descriptions of the hadronic scattering amplitude. At low energy (LE) the description by direct-channel resonances is simple and useful:

At low energy the data show prominent peaks as a function of energy, and one may try the approximation of resonance saturation, i.e. of neglecting the non-resonating background. The second description is the exchange of Regge poles, and it is useful at high energies (HE), where typical features are forward peaks, energy dependence s^α , and structure at fixed t .

The two descriptions are very different; resonance formation corresponds to poles in the s-channel, Regge exchange to poles in the t-channel. Duality says that there are direct relationships between these two descriptions, that they are equivalent in a certain sense. In complete contrast, the interference models postulate that one must add the two descriptions. (If lowest order perturbation theory was relevant to strong interactions, one would be led to adding the diagrams.)

Each of the two descriptions (resonances, Regge poles) is by itself an approximation to a complete description:

<u>Approximation</u>	<u>Complete description</u>	<u>Neglected</u>
Resonance saturation	(1) Fixed t dispersion relation (2) Partial wave expansion	Non-resonating background
Regge-pole dominance	Regge expansion	Cuts Background integral.

It is clear that we can choose one or the other complete description, and we can certainly not add them. The controversy arises only when one makes approximations. FESR duality follows from the assumption that it is a reasonable low-energy approximation to neglect the non-resonating background, and a reasonable high-energy approximation to neglect the background integral in the j-plane. FESR duality is nothing more than these two assumptions plus analyticity^{*)}. In contrast, the interference models assume that Regge poles are a good approximation to the non-resonating background, and resonances a good approximation to the background integral in the j-plane.

The question what to do about the two descriptions first arose in the intermediate energy region (IE), where one has direct evidence for both mechanisms. For example, in $K^-p \rightarrow K^-p$ between $p_L = 1.3$ and 2.5 GeV/c one has a strong forward peak, a dip at a fixed t-value, and a secondary peak.

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*) From the point of view of the controversy between interference models and duality, Regge cuts are on the same footing as Regge poles. It is only the Regge background integral which is necessarily neglected in the high-energy approximation.

These features are characteristic of an exchange mechanism. On the other hand, one has in the same energy range strong Y^* resonances which show up as bumps in the total cross-section. Should we add the two mechanisms? The Finite-Energy Sum Rules (FESR) make a good tool for gaining insight into these questions.

FESR^{1,2)} contain nothing more than analyticity and the assumption that the high-energy amplitude can be represented by a Regge expansion. One considers an invariant amplitude $A(s,t)$ for fixed t , e.g. for πN scattering. Our energy variable is $\nu = (s-u)/(4M_N)$; for $t = 0$, this is the laboratory energy of the π . In the ν -plane we have the right-hand cut for $\nu \geq \nu_{\text{threshold}}$ and a left-hand cut symmetric to it. (We shall leave out the bound-state poles just to save time.) We consider the contour C shown in Fig. 1. According to Cauchy's theorem $\oint_C A(\nu t) d\nu = 0$. The part of A which is even in ν leads to a trivial identity. Let us therefore assume that A is odd. The parts of the contour C which lead along the cuts (from $\pm\nu_{\text{th}}$ to $\nu = \pm N$) give (4i) times

$$\int_{\text{th}}^N d\nu \mathcal{D}_\nu A(\nu, t)$$

where $\mathcal{D}_\nu A \equiv (1/2i) [A(\nu + i\epsilon) - A(\nu - i\epsilon)]$ is the discontinuity across the cut. For $t \leq 4\mu_\pi^2$ we have $\mathcal{D}_\nu A = \text{Im } A$.

Along the two semicircles (i.e. at high energy $|\nu| = N$) we replace $A(\nu, t)$ by its Regge expansion:

$$A = \sum_{\text{poles}} \beta \frac{1 - e^{-i\pi\alpha}}{\sin \pi\alpha} \nu^\alpha + \text{cuts} + \int dj \dots$$

After having done this replacement, we deform the semicircles. Different deformations are needed depending on the value of α . For $\alpha > -1$ we contract the upper semicircle to a straight path from $\nu = +N + i\epsilon$ to $\nu = -N + i\epsilon$ (and similarly in the lower half-plane). This gives (4i) times

$$- \sum_{\alpha > -1} \int_0^N d\nu \mathcal{D}_\nu A_{\text{Regge}} = - \sum_{\alpha > -1} \beta \frac{N^{\alpha+1}}{\alpha+1}$$

For $\alpha < -1$ we expand the semicircle and first go along the cut from $\nu = +N$ to $\nu = +\infty$, and then along an infinite semicircle. The contribution of the latter is zero because $\alpha < -1$. The contribution of the four pieces from $\pm N$ to $\pm\infty$ are (4i) times

$$\sum_{\alpha < -1} \int_N^{\infty} d\nu D_{\nu} A_{\text{Regge}} = - \sum_{\alpha < -1} \beta \frac{N^{\alpha+1}}{\alpha+1}.$$

Finally, for $\alpha = -1$ one evaluates the semicircles directly, and obtains a result which is the limit of the cases $\alpha \gtrsim -1$. [A_{Regge} is purely real and finite for $\alpha = \text{odd}$, therefore $\beta \sim (\alpha + 1)\bar{\beta}$ for $\alpha \approx -1$, with $\bar{\beta} = \text{finite}$ at $\alpha = -1$]. Putting the Regge terms on the right-hand side (RHS) we have:

$$\int_{th}^N d\nu D_{\nu} A(\nu, t) = \sum_{\text{poles}} \beta \frac{N^{\alpha+1}}{\alpha+1} + \text{cuts} + \int dj \dots (1)$$

This is the (zeroth moment) FESR. It gives an explicit connection between the LE average on the LHS and the HE Regge terms on the RHS:

$$\begin{array}{ccc} \text{LE} & \xleftrightarrow{\text{analyt.}} & \text{HE} \\ < 0, N > & & \text{at } N \end{array}.$$

The connection comes from analyticity. It imposes a consistency condition on Regge parameters.

A very straightforward (and non-controversial) application of FESR is to choose N rather high, $t = 0$, to feed in on the LHS the experimental total cross-sections, $\text{Im } A' = p_L \sigma_{\text{tot}}$, and to obtain additional restrictions on the HE Regge parameters. When FESR were first applied to πN charge exchange (CEX) at $t = 0$, i.e. to $\frac{1}{2}[\sigma_{\text{tot}}(\pi^- p) - \sigma_{\text{tot}}(\pi^+ p)]$, by Logunov et al., Igi et al., and Horn and myself, we found that a one- ρ -pole description for $\nu \geq N$, $p_N = 3.5 \text{ GeV}/c$, could also satisfy the additional restriction imposed by FESR's²⁾. It is quite misleading to use the words "testing FESR's"; what one is testing is a particularly simple Regge parametrization. If one has enough accurate data (HE data, FESR's) one can determine subsidiary Regge poles or Regge cuts. [There is good evidence for Regge cuts, but the data are not good enough to pin them down, nor do we have a satisfactory theoretical method of calculating them.]

Although the information used in FESR is a low-energy average, the FESR give restrictions on the same Regge poles which are relevant above N . More precisely: the relative importance of a ρ' is the same in the Regge fit to data above N and in the RHS of the FESR. If a ρ' can be neglected above N , it can equally well be neglected on the RHS of the FESR. Note that the daughter pole will become very important for $\nu \rightarrow 0$, it will blow up if $\alpha < 0$, and the integral will diverge if $\alpha < -1$. But all this is irrelevant because the daughter contribution on the RHS of the FESR will be negligible. In other words, if a FESR with the ρ alone should fail by a factor of 2, it is not possible to blame this on daughters which are unimportant for $\nu \geq N$.

After having convinced oneself that a one-Regge-pole approximation to πN CEX at $t = 0$ is good for $\nu \geq N$, one can rewrite the approximate FESR:

$$\int_0^N \text{Im} (A' - A'_\rho) d\nu \approx 0. \quad (2)$$

This is shown on Fig. 2. We see that on the average the low-energy amplitude is given by the ρ Regge term. We have drawn the low-energy extrapolation of the Regge amplitude. Does it matter what analytic form we choose? Note that it is Eq. (1) which has a precise meaning. It is in Eq. (1) that we discussed the importance of daughters, and correspondingly the difference between ν^α , $P_\alpha(\nu)$, and $Q_{-\alpha-1}(\nu)$. (For convenience of integration we have chosen ν^α .) Figure 2 is merely an illustration, which only becomes possible after having made the approximation of only keeping the leading term in Eq. (1).

Since the full low-energy amplitude is given on the average by the ρ Regge term, the average of the resonances must already be contained in A'_ρ . We call this average duality (or FESR duality).

Let us now show that average duality is incompatible with the old interference model of Barger and Cline³⁾:

Interference models:

$$\int_0^N \text{Im} (\bar{F} - \bar{F}_{\text{Regge}} - \bar{F}_{\text{res}}) d\nu = 0$$

FESR:
$$\int_0^N \text{Im}(\bar{F} - \bar{F}_{\text{Regge}}) dv = 0$$

[Barger and Cline used their model only in the IE region; therefore we should consider higher moment sum rules which suppress the LE region.]
Taking the difference of the two equations, we get

$$\int_0^N \text{Im} \bar{F}_{\text{res}} dv = 0. \quad (3)$$

For the case of bound states or narrow resonances on an unimportant background (in the relevant partial wave) this reduces to

$$\sum_i (m \Gamma_x)_i C_{ts_i} K_i P_{\ell_i}(z_{s_i}) = 0 \quad (4)$$

which is valid for meson-meson scattering. C_{ts} is the isospin crossing matrix, and K_i is an unimportant kinematical factor.

For $t = 0$ and $I_t = 0$ we have a sum of positive numbers, essentially $\sum_i g_i^2$, which cannot possibly be zero. In Fig. 3 we give the example of πN scattering with $I_t = 0$ at $t = 0$. Again, the $P + P'$ Regge terms go through the average of the experimental points. The interference model would add the positive resonance contributions to the Regge terms and would fail dramatically.

The generalized interference model⁴⁾ (discussed by Donnachie at this meeting) still keeps the superconvergence condition $\int_0^N \text{Im} \bar{F}_{\text{res}} dv = 0$. But the authors point out that the contribution of one resonance to the integral may well be negative. An example for a non-positive resonance contribution is the $S_{11}(1570)$, whose true contribution to the integral is about zero because $g^2 \approx \text{imaginary}$. However, the generalized interference model is in contradiction with the following crucial empirical facts:

- i) Resonance dominance is empirically very good for peripheral partial waves up to $p_L \approx 2 \text{ GeV}/c$. This is particularly true for $I_t \neq 0$. [See Harari and Zarmi⁵⁾]. No Regge term can be added to the resonances in these partial waves, because resonances alone approximate these partial waves very well.

- ii) Because the peripheral resonances sit on almost no background, unitarity requires g^2 to be approximately real and positive.

In order to see the contradiction between these two empirical facts and the generalized interference model, we evaluate the funny superconvergence relation implied by the interference model, $\int_0^N \text{Im } F_{\text{Res}} ds = 0$ at $t = 0$ and at $t = -0.5 \text{ GeV}^2$ and take the difference. By taking the difference we suppress the low partial waves (e.g. the S-waves drop out). Therefore the superconvergence integral will be dominated by the peripheral partial waves and cannot possibly vanish because of the empirical fact (point ii) explained above.

The generalized interference model was invented because of the loop story, because the authors wanted to interpret Regge loops as being different from resonance loops. (See the discussion of local duality below.) However, it is important to note that the generalized interference model is already in contradiction with average duality (FESR duality), as we explained in the preceding paragraph.

Let us look back at Fig. 3; note in particular the region between the 1238 and the 1688. The full amplitude is below the Regge amplitude. The authors of the generalized interference model conclude that these holes are due to two new hypothetical resonances with negative g^2 , one between the 1238 and the 1520, the other between the 1520 and the 1688.

Jengo⁶⁾ tries to construct interference models by considering "superconvergent resonance contributions", e.g.

$$\frac{\exp \left[- (s_{th} - s)^{1/4} \right]}{E - E_{res} + i\gamma (s - s_{th})^{1/2}}$$

Depending on the choice of E_r and γ , the imaginary part of this function might look as follows:

The resonance proper is accompanied by an equally large negative contribution. What this discussion shows is that analyticity by itself is not sufficient; if one constructs models that are crazy enough, one can get around duality. Somewhere along the line one needs some empirical input.

Let us recall that the classical superconvergence relations⁷⁾ referred to amplitudes which were, at high energy, either "zero" (exotic exchanges) or suppressed (double helicity flip and charge exchange). It is precisely for such amplitudes that the isospin or the spin crossing matrix in Eq. (4) has negative elements, therefore the expression (4) can be zero with all $g^2 \sim (m\Gamma x)$ real and positive. This beautiful interlocking of two apparently very different properties (high energy, low energy) of scattering amplitudes is required by the classical superconvergence relations. It is a pity to see this satisfying concept misused in the generalized interference model.

Up to now we have only used analyticity and Regge-pole dominance at high energy. Resonances are obviously present at low energy, and we have shown that they are already included on the average in the dominant Regge term continued to low energy. Now we shall make the additional assumption that the low-energy amplitude is actually dominated by resonances. With this assumption FESR express a particularly beautiful relationship. In the example of πN CEX one has:

$$\begin{array}{c} \text{LE average} \\ \text{of } N^* \text{ resonances} \end{array} \longleftrightarrow \rho \text{ Regge term} .$$

This is a linear relation (FESR) between the particles in two different channels. Of course it was always clear that there had to be some connection between the poles in the crossed channels, but nobody realized that such a simple, linear connection existed (FESR duality).

Global duality (= average duality = FESR duality) is nothing more than what follows from the three assumptions:

- (1) Analyticity
- (2) HE Regge dominance (for $s \geq s_N$, one pole per quantum number)
- (3) LE resonance dominance (for $s \leq s_N$, prominent, well-known resonances).

Assumptions (2) and (3) are necessarily not exact. For saturation by the known resonances to be a reasonable approximation, we have to choose the limit of integration N rather low, $p_L \approx 1.2-2.0$ GeV/c. [At present the same is true if we use phase shifts.] Can we expect that Regge gives a reasonable description down to these momenta? We already pointed out how well this works for $K^-p \rightarrow K^-p$ down to $p_L = 1.3$ GeV/c, even in the polarization. The same is true for the sum of $\sigma_{\text{tot}}(\pi^-p)$ and $\sigma_{\text{tot}}(\pi^+p)$, (see Fig. 3). On the other hand, in the difference, (see Fig. 2), one has to go higher, to ~ 2.5 GeV/c. If one takes a low N in the difference, one will have large fluctuations, and therefore large errors. In that case the only useful statement will be that $A'_{\text{Regge}}(-)$ is small compared to $A'_{\text{Regge}}(+)$. These large errors for strongly fluctuating amplitudes are inherent in FESR duality, and one should always try to estimate them. It is absolutely misleading to choose one fixed s_N and to consider the FESR with the Regge-pole approximation as an exact equation.

If one applies FESR to πN CEX, and uses LE resonance dominance and HE Regge dominance, one finds nice correlations between the N^* 's and the ρ Regge amplitude (for details see Ref. 1). In the LE energy integral for B "all" resonances happen to come in with the same sign. This generates a large Regge amplitude B_ρ . On the other hand, the LE resonances cancel in the LE integral for A' . This generates a small Regge amplitude A'_ρ , and we expect $\nu B \gg A'$ at high energies. This is indeed observed; the differential cross-section has a dip at $t = 0$, where the large amplitude B cannot contribute because of angular momentum conservation. The explanation of the dip at $t \approx -0.6$ GeV², $\alpha_\rho = 0$, can be given similarly in terms of N^* resonances: the integral $\int d\nu \nu^3 \text{Im } B$ is dominated by the prominent intermediate energy resonances 1520, 1688, 1920. They all vanish simultaneously for $t \approx -0.5$ GeV². [This roughly corresponds to the first zero of $P'_\ell(z)$ in the resonance numerator.] It then follows from FESR that B_ρ vanishes at the same t -value.

In the integral $\int \text{Im } A' \nu^2 d\nu$ the dominating IE resonances all vanish for $t \approx -0.2 \text{ GeV}^2$. Therefore the HE amplitude A' must vanish at the same t -value. This illustrates the power and the limitations of FESR's from a practical point of view. Just by looking at HE data it is very hard to establish this zero in A' , because the helicity flip amplitude dominates. Since $d\sigma/dt$ is a sum of four squares, it is hard to disentangle and to determine the signs of the amplitudes. At LE the disentangling has been done for us by the people who determined the J^P of the resonances or did the complete phase-shift analysis. Therefore FESR allow us to determine each HE amplitude separately. This is the power. The limitation comes from the fact that FESR cannot practically discriminate between Regge poles and Regge cuts. In connection with A' the question arises: Is it possible that the pole term in A' vanishes at $\alpha_\rho = 0$, but that the (absorptive) cut moves this zero from $t = -0.6 \text{ GeV}^2$ to $t = -0.2 \text{ GeV}^2$? FESR tell us what the amplitude is; it does not tell us whether it is a pole or a cut.

We have discussed how the low-energy N^* resonances "generate via analyticity" or "build-up via analyticity" the high-energy ρ Regge pole. This type of "building up" is at the root of the FESR bootstrap (to be discussed below). However, the term "building up" has been used in different ways and has also been misused. Let me make a few comments.

The concept of resonance dominance cannot possibly be defined for the real part of the amplitude, where one has long-range tails $\sim 1/s$. The integral over it is zero if we integrate symmetrically, and it diverges if we integrate to ∞ . All these difficulties are absent if we integrate $\text{Im } F$, and the result hardly depends on how we treat the tails; one always gets approximately the narrow resonance result. In a resonance dominance scheme one must construct $\text{Re } F$ via fixed t -dispersion relations. Take, for example, K^+p scattering. There are approximately no resonances, and indeed $\text{Im } (F - F_p) = 0$. On the other hand, $\text{Re } F \neq 0$. In a dispersion relation sense this real part is generated by the tails of the Y^* resonances which occur for negative s . The term "building up" has a direct meaning only for the imaginary part.

Applying resonance saturation in the imaginary part to reactions like $KN-KN$, $NN-NN$, which do not contain resonances, and observing that the experimental total cross-sections in these channels are constant (from $p_L = 1.5$

to $p_L = 20 \text{ GeV}/c$) leads inescapably to the conclusion that the Pomeron is outside the resonance saturation scheme and must be treated separately. (See Harari's talk at this conference.)

Building up a Regge pole via analyticity from low-energy resonances has teeth. It has predictive power in the case of the A'_ρ amplitude, and in the case of the B_ρ amplitude it is a highly restrictive interlocking of two entirely different quantities (LE resonances, HE Regge) which were both known already. In contrast, the term "building up" has also been used in the sense that HE resonances can build up the HE Regge pole, i.e. in the sense of resonance saturation for $s \rightarrow \infty$. This is trivially possible with hypothetical resonances, since strongly overlapping resonances in each partial wave look like non-resonating background. It is also uneconomical (except in closed form as in the Veneziano model), because these resonances at high energy must be highly correlated. Resonance saturation at high energy (for one amplitude), or in other words the assumption that the high-energy amplitude is "built up" by high-energy resonances, is an empty statement, because it is only through this statement that the high-energy resonances can be determined.

Our discussion up to now was restricted to $t \leq 0$. Let us now go to $t = m_\rho^2$. Is a building up still possible? How can s-channel resonances, which are regular at $t = m_\rho^2$, build up a pole at $t = m_\rho^2$?

In the sense of fixed t-dispersion relations it is certainly true that the t-channel poles are generated by the singularities in the s-plane (s-channel and u-channel singularities):

$$A(s, t) = \frac{1}{\pi} \int ds' \frac{D_s A(s', t)}{s' - s} \quad (5)$$

The right-hand side is determined by the s-plane singularities, namely the discontinuity function $D_s A$. The discontinuity function does not contain the t-channel poles:

$$D_s A = \text{regular at } t = m_{\text{res}}^2$$

$$\begin{aligned}
 D_s A &\sim D_s \frac{\beta P_\alpha(-z_t)}{m^2 - t} \\
 &\sim D_s \beta \frac{(-s)^{\alpha(t)}}{\sin \pi \alpha(t)} \sim \beta s^{\alpha(t)}.
 \end{aligned}$$

The t-channel poles, which are contained in $A(s,t)$, are generated by the divergence of the integral over the s-channel (and u-channel) singularities. The new element in the FESR approach is the fact that we need no longer know $D_s A(s',t)$ for $s' \rightarrow \infty$; we only have to know it for $s' < s_N$, and then the particular Regge ansatz allows us to generate the asymptotic behaviour, which in turn generates the t-channel poles.

It is trivial to say that s-channel (and u-channel) singularities build up the t-channel singularities. This is nothing more than what is expressed by fixed t-dispersion relations. The non-trivial part is that the s-channel spectral function, $D_s A$, can be approximated, for $s \leq s_N$, by poles, and that the asymptotic term in s (i.e. the leading Regge pole) is a reasonable approximation down to $s = s_N$.

Average duality (= FESR duality) is not something you can prove *a priori*; rather it follows from two empirical approximations (pole dominance at LE and at HE) and from analyticity.

We shall now show how one can predict the meson masses (and couplings) from the baryon masses and couplings. This example is analogous to the $\pi\pi$ bootstrap described elsewhere⁸⁾. We consider the s-channel $\bar{K}N \rightarrow \bar{K}N$ with $I_s = 0$ and saturate the low-energy region with the Y_0^* 's⁹⁾. The u-channel, $KN \rightarrow KN$, contains no (or only very weak) resonances. In our resonance saturation scheme the left-hand cut (u-channel imaginary part) will be zero, and the t-channel Regge poles will be exchange degenerate. Therefore we combine each odd-signature Regge term (e.g. the ρ) with the corresponding even-signature term (e.g. the A_2) into a single Regge term which will have only a right-hand cut. We further assume that the ρ - A_2 trajectory is degenerate with the ω - f trajectory, and that the ϕ - f' trajectory decouples from $N\bar{N}$. Since we consider only $I_s = 0$, the two t-channel isospins occur in one definite linear combination and can be combined into one term. This leaves

us with one t-channel Regge pole. (The Pomeron must be left out in a resonance saturation scheme; see the talk of Harari at this meeting.) If we now take the ratio of the two lowest moment FESR we obtain

$$\frac{1}{N} \frac{\int_0^N \nu D_\nu A'(\nu, t) d\nu}{\int_0^N D_\nu A'(\nu, t) d\nu} = \frac{\alpha + 1}{\alpha + 2} \quad (6)$$

and similarly for the helicity flip amplitude B. (If we had a Regge pole with definite signature, e.g. odd signature, we should have to take the ratio of two even moments.) We choose the limit of integration N half-way between the $Y_0^*(\frac{5}{2}^+, 1815)$ and the $Y_0^*(\frac{7}{2}^-, 2100)$. For our application at $t \geq m_\rho^2$ the leading Y_0^* sequence, $\frac{1}{2}^+ - \frac{3}{2}^- - \frac{5}{2}^+$, completely dominates over the resonances in lower partial waves¹⁰⁾, which are therefore neglected. Equation (6) is now a simple algebraic equation. The input on the left-hand side requires the masses and couplings of the Y_0^* 's, the output on the right-hand side is the meson trajectory $\alpha(t)$ corresponding to the degenerate ρ , ω , A_2 , f trajectories. The output trajectory is shown on Fig. 4a. The $\alpha(t)$ determined from the flip amplitude B and from the non-flip amplitude A' agree nicely with each other and with the experimental meson masses, ρ , ω , A_2 , f , g . In Fig. 4b we show how the Y_0^* 's manage to generate the meson trajectory $\alpha(t)$. The FESR's with one-Regge-term dominance for $s > s_N$ and resonance dominance below s_N imply that the low-energy extrapolation of the former gives an average description of the latter in the imaginary part. The imaginary part of the resonances (δ functions in the narrow resonance approximation) are shown by the vertical lines, the height representing the strength of the δ -function. The dotted line shows the imaginary part of the Regge amplitude. As we go to larger t-values, the higher s-channel resonances grow more rapidly, because of their higher J-values. Therefore the dotted lines (Regge) must grow more rapidly and are characterized by a larger α . Concluding, we can state that we have successfully computed the meson trajectory (i.e. the meson masses) using as an input only the baryon masses and couplings.

We could have also computed the meson couplings, e.g. $(\Gamma_{\rho K \bar{K}} \cdot \Gamma_{\rho NN}^{(A')})$. However, since these numbers cannot be compared with experiment, we refer to the example of $\pi\pi \rightarrow \pi\pi$, where output widths of the t-channel resonances were consistent within 10-20% with the input widths of the s-channel resonances⁸⁾

One might fear that the resonance saturation becomes bad at $t = m_\rho^2$ because the partial wave series diverges. However, in the narrow width approximation for the t-channel ρ , it is only the partial wave series for $\text{Re } A$ that diverges at $t = m_\rho^2$, while the partial wave series for $\text{Im } A$ converges since $\text{Im } A$ is regular at $t = m_\rho^2$. More important, however, is the following remark: As t becomes positive and large, i.e. $z_s \gg +1$, the partial wave series becomes a useless tool. Rather, one must consider the Regge expansion in the direct channel, as in the original application of Regge (without crossing) in 1958. It says that resonance saturation in $\int D_s A ds$ becomes an excellent approximation as $t \rightarrow +\infty$. We have therefore the following situation:

- i) resonance saturation in the s-channel is empirically fairly good for $|z_s| \leq 1$;
- ii) it is theoretically excellent for $|z_s| \rightarrow \infty$;
- iii) there is no reason, why it should be bad at $t = m_\rho^2$. One can actually make some estimates at $t = m_\rho^2$, and one finds that resonance saturation is indeed better at $t = m_\rho^2$ than at $t = 0$ (see Ref. 8).

Let me remind you that many people published "proofs" that duality was impossible¹¹⁾ until Veneziano published his beautiful model¹²⁾. Since one example is stronger than a thousand "proofs" to the contrary, people had to accept the fact that duality was possible. But then they argued that duality was just one possibility, and that the opposite model, i.e. the interference model, is an equally good possibility. I think that the example of the FESR bootstrap above, where we calculated the meson masses from the baryon masses, shows that FESR duality is not only a possibility, but that it is realized in nature.

The integral $\int_0^N \text{Im } F_{\text{res}} ds$ does all the building up in our bootstrap, while it is supposed to vanish in the generalized interference model, see Eq. (3). It is hard to see how such an integral could vanish at $t = m_\rho^2$, because it is dominated by the peripheral resonances $\Lambda(1115)$, $Y_0^*(1520, \frac{3}{2}^-)$, $Y_0^*(1815, \frac{5}{2}^+)$, which all have a g^2 which is very nearly real and positive.

Let me point out that the generalized interference model is an anti-resonance-dominance model in the sense that it postulates that resonances alone give a vanishing contribution to the low-energy integral, even in those cases where the low-energy integral of the full amplitude is large.

We now come to the question of exchange degeneracy. We shall have to go a little bit beyond global duality. Consider $\bar{K}N \rightarrow \bar{K}N$ in the s-channel, $KN \rightarrow KN$ in the u-channel. The FESR always involve an integral from $-N$ to $+N$ over $D_\nu A$, where $\nu = (s-u)/(4M)$. As t becomes more positive, the gap where $D_\nu A = 0$, i.e. the gap between the s-cut and the u-cut, becomes larger, and the two regions of integration become more and more separated. Considering even and odd moment FESR simultaneously, it follows that the two regions will approximately satisfy corresponding FESR's separately.

$$\frac{1}{N} \int_{-N}^0 D_\nu A d\nu \approx \beta_{\text{odd}} \frac{N^{\alpha_{\text{odd}}}}{\alpha_{\text{odd}}+1} - \beta_{\text{even}} \frac{N^{\alpha_{\text{even}}}}{\alpha_{\text{even}}+1}.$$

If we use resonance saturation on the LE side and if we use the empirical fact that $KN \rightarrow KN$ show no (or only very weak) resonances, the left-hand side of this equation will be zero. From this, together with the corresponding equation for the higher moments, it follows that

$$\alpha_{\text{odd}} \approx \alpha_{\text{even}} \quad \beta_{\text{odd}} \approx \beta_{\text{even}}.$$

These equations express exchange degeneracy. Note that we have not invoked local duality. Another way to argue is to state that the whole FESR scheme is a linear scheme. Therefore each amplitude, in our case $\bar{K}N \rightarrow \bar{K}N$, can be considered separately. With resonance saturation there is no u-channel cut in this amplitude, and one gets exchange degeneracy.

This conclusion could not be drawn in other dynamical schemes such as the N/D bootstrap scheme, because N/D is based on unitarity, which is a non-linear relation and requires the inclusion of coupled channels. In our example, $\bar{K}N \rightarrow \Lambda\pi$ and $\bar{K}N \rightarrow \Sigma\pi$ are important. But in these channels there are important baryon exchange forces which might destroy exchange degeneracy in $\bar{K}N \rightarrow \bar{K}N$.

A clean experimental test of the predicted exchange degeneracy is shown in Fig. 5¹⁰⁾. We see in Fig. 5a that the Λ_α trajectory $1/2^+ - 5/2^+ - \dots$ is indeed quite accurately degenerate with the Λ_γ trajectory $3/2^- - 7/2^- - \dots$. But it is not only the trajectory functions that are degenerate, but also the residue functions. If we draw in Fig. 5b a smooth curve through the couplings of $1/2^+ - 5/2^+ - 9/2^+$, and another smooth curve with the same shape

but different normalization through $\frac{3}{2}^- - \frac{7}{2}^-$, we see that the normalization is about the same in both cases. In Fig. 5b the strength of the resonances is compared at $t = m_\rho^2$. This enhances the peripheral resonances over those in central partial waves. In Fig. 5a we have only plotted those Y_0^* resonances which are important at $t = m_\rho^2$.

The confrontation of exchange degeneracy with SU_3 gives many beautiful results about coupling ratios (singlet to octet, or F/D) and mixing angles¹³⁾. Most of these results were previously obtained in the quark model. We have here a very exciting and simple connection of two different parts of strong interaction physics which are usually considered separate¹⁴⁾.

We now come to a more detailed connection between Regge exchanges and direct-channel resonances. We shall see that the duality holds not only on the average from $\nu = 0$ to $\nu = N$, but even approximately locally for each intermediate energy (local duality)¹⁵⁾.

We consider the helicity flip amplitude for πN charge exchange, $B^{(-)}$. We shall partial-wave analyse $B^{(-)}$ in the direct channel and thereby obtain circles on the Argand diagrams corresponding to the prominent experimental N^* resonances.

The usual high-energy parametrization is

$$B_{CEX} = \beta \left(\frac{E}{E_0} \right)^{\alpha-1} \frac{1 - e^{-i\pi\alpha}}{\sin(\pi\alpha) \Gamma(\alpha)} \quad (7)$$

The factor $1/\Gamma(\alpha)$ kills the poles which would otherwise appear at $\alpha = -1, -3, \dots$ (ghost-killing factor). It also produces zeros in $|B_{CEX}|$ at $\alpha = 0, -2, \dots$, i.e. at the unphysical wrong signature points. These zeros are observed as dips in $d\sigma/dt$ at $t \approx -0.6$ and $t \approx -2.5$ (GeV)². E is the laboratory energy of the π . β is the residue function and depends on t . However, an exponential t -dependence of β can always be absorbed by a redefinition of E_0 . The fits to the high-energy data for $0 \geq t \geq -3.0$ (GeV)² show that $\beta \approx$ constant for $E_0 = 0.7$ GeV in the parametrization (7). We now take this high-energy fit and extrapolate it down to $p_L = 1.7$ GeV/c; it fits the data roughly, even at these energies. In particular, the dip at $t = -0.6$ (GeV)² persists down to this energy, and the magnitudes of the near forward peak and of the secondary peak are correctly given by the extrapolation.

We partial-wave analyse $B(E, z)$ in the direct channel

$$B_\ell(E) \equiv \frac{1}{2} \int_{-1}^{+1} dz P_\ell(z) B(E, z). \quad (8)$$

We integrate out to 180° , but in a first approximation we ignore the small backward peak coming from N exchange. The result of this partial wave analysis is shown in Figs. 6a, 6b, and 6c for fixed ℓ vs. p_L . Surprisingly the B_ℓ describe circles on the Argand diagram. Such circles are usually associated with direct-channel resonances. In Fig. 7 we plot the "resonance" momenta¹⁶⁾ vs. ℓ and compare them with the experimental resonances (1920, 2190, 2420). The correspondence is rather striking, particularly if we remember that there were no free parameters. In Fig. 6d we show B_ℓ for fixed p_L as a function of ℓ . Such a circle corresponds to an N^* Regge pole and indicates that the N^* resonances plotted in Fig. 7 actually belong to an N^* Regge trajectory. (For a quantitative comparison of widths and partial widths with the experimental values, see Ref. (15).

$\text{Im } B_\ell(s)$ has peaks in s , although $\text{Im } B(s, t)$ for fixed t goes like $s^{\alpha(t)}$ and has no peaks. This is easily understood if we recall that the observed resonances form a sequence of peaks with increasing ℓ , with their masses arranged in such a way that they have a very similar t -dependence (although their angular distribution looks very different if plotted on a $\cos \theta$ axis); e.g. their first zero in t is at approximately the same place for all prominent resonances in the energy region under consideration. Since these resonances overlap strongly, they add up to a function which is fairly smooth in s and has only small wiggles. It is important to see that in using the Regge formula (7) we only neglect these remaining wiggles, but we include the main bulk of the resonances. If we go to fixed ℓ we pick out one resonance from this sequence, and it will show up as a peak in $\text{Im } B_\ell$. Note that the wiggles are too small to be observed experimentally for $p_L > 2 \text{ GeV/c}$ and t fixed in the near forward or in the secondary peak regions. Local duality can only work because the N^* resonances in B_{CEX} happen to be highly correlated and strongly overlapping.

Do the circles in Fig. 6 really correspond to genuine resonance circles? The problem is that Eqs. (7) and (8) are regular at $s = m_{\text{res}}^2 - i(m\Gamma)_{\text{res}}$ they do not contain second-sheet poles corresponding to resonances. Let us understand

this problem by discussing a mathematical model, the Veneziano model¹²⁾. We take the example of the s-channel process $\pi^+\pi^- \rightarrow \pi^+\pi^-$. This makes life simple because the u-channel ($\pi^+\pi^+ \rightarrow \pi^+\pi^+$) is exotic and contains no resonances, while the t-channel is identical to the s-channel. The Veneziano model is in this case:

$$A = -C \frac{\Gamma(1-\alpha_s) \Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)} \quad (9)$$

This amplitude has the following properties:

i) $\Gamma(1-\alpha_s)$ has poles at $\alpha_s = 1, 2, 3, \dots$, where $\alpha_s = 0.5 + s$. The residuum of the pole, e.g. at $\alpha_s = 2$, is $\Gamma(1-\alpha_t)/\Gamma(-1-\alpha_t) = (-\alpha_t)(-1-\alpha_t)$. This is a quadratic function in t , therefore also quadratic in z_s , and it corresponds to a superposition of spin 2, 1, 0.

ii) We can write a dispersion relation at fixed t . Because the spectral function consists of poles only, the dispersion integral simplifies and becomes a sum of pole terms:

$$A(s, t) = \frac{1}{\pi} \int_0^\infty ds' \frac{D_s A(s', t)}{s' - s} = \sum_i \frac{\gamma_i^2 P_i(z)}{m_i^2 - s} \quad (10)$$

iii) Where are the t-channel poles in this representation of the Veneziano amplitude? When the dispersion integral in s diverges, one needs subtraction constants, for $t \uparrow m_\rho^2$ one needs two of them, $a_0 + a_1 s$. However, these constants (in s) are functions of t , and they will contain the ρ -pole: $a_0 + a_1 s = (m_\rho^2 - t)^{-1} \cdot \gamma^2 P_1(z_t)$.

iv) The asymptotic behaviour for $s = (1 + i\epsilon)\lambda$, $\lambda \rightarrow \infty$, fixed t , is given by the Stirling formula:

$$\begin{aligned} \frac{\Gamma(1-\alpha_s)}{\Gamma(1-\alpha_s-\alpha_t)} &= \frac{\Gamma(\alpha_s + \alpha_t)}{\Gamma(\alpha_s)} \cdot \frac{\sin \pi(\alpha_s + \alpha_t)}{\sin \pi \alpha_s} \\ &\rightarrow s^{\alpha_t} \cdot e^{-i\pi \alpha_t} \end{aligned} \quad (11)$$

The factor $e^{-i\pi\alpha_t}$ is the Regge phase appropriate for an amplitude with only a right-hand cut in the s -plane.

v) The important property from the point of view of local duality is the fact that the asymptotic representation A_{asy} (Stirling formula, one-term Regge approximation) is a good approximation of A for $|s| \rightarrow \infty$ as long as we are outside a wedge, $|\arg s| > \varepsilon$, around the positive real s axis, where the particle poles are located in this model. A_{asy} does not contain these poles, the approximation breaks down if we penetrate the wedge. Similarly the Regge asymptotic form B_{asy} is a good approximation to the full amplitude B for real s . Both amplitudes contain the resonance circles. But if we go below the physical axis, the approximation breaks down, and B_{asy} does not contain the resonance poles.

In the physical case, for B , everything is rotated a little bit such that the poles lie on a ray $s = (1 - i\varepsilon)\lambda$, while the upper boundary of the wedge is rotated into the real s axis and represents the physical region, $\text{Im } s = 0$. The Regge pole formula represents, for $\text{Im } s \geq 0$ and $|s| \rightarrow \infty$, the effect of the resonances. The fact that the Regge pole formula does not contain the second-sheet poles is irrelevant, because it was never supposed to represent the full amplitude on the second sheet, i.e. for $\text{Im } s < 0$.

A more detailed discussion of the question of interpretation of Argand diagram loops generated by Regge exchange has been given elsewhere¹⁷⁾.

When talking about practical aspects of local duality, we must remember that it can only apply at intermediate energies. It obviously fails badly for the narrow, isolated resonances at low energies, such as the 1238 or the 1520. The region of the N^* 's 1920, 2190, 2420, i.e. $p_L = 1.2$ to 3.0 GeV/c, seems best. But even in that region it is a rough approximation. While the position (M^2) and the strength ($g^2 \sim \Gamma x$) are reasonably well approximated in the Regge loops, the detailed shape is not, the width Γ comes out too large by a factor of ~ 1.5 , and the speed on the Argand diagram shows the wrong behaviour.

The greatest obstacle to a detailed investigation of Regge loops is our lack of knowledge of a reliable Regge input for the region

$0 \leq |t| \leq 2.5 \text{ GeV}^2$ for A' and B of P, P', ρ (in the case of $\pi N \rightarrow \pi N$). Reliable means that we should be sure where the various zeros of the amplitudes occur. Note that, for example, the second zero of A'_ρ (its first zero is at $t \approx -0.2 \text{ GeV}^2$) is still very poorly known.

My original motivation, in 1967, to investigate the partial wave projection of Regge amplitudes was to gain a better understanding of FESR duality. The results were more striking than I had expected. But even today an important role of local duality is to give us confidence that average duality is a really meaningful concept, i.e. that resonances are really in some way already contained in a Regge-pole term. Whether local duality can be used for practical purposes, such as obtaining the quantum numbers and good values for the masses of the resonances from a Regge input, remains to be demonstrated. This programme will involve a lot of work, and I have no doubt that on the way one will learn a lot more about strong interaction physics.

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- 16) In order to read off resonance energies, we use the criterion applied in recent phase-shift analyses that the elasticity parameter η should be minimal. For peripheral partial waves, $\ell \approx kR$, there is little background besides the resonances, therefore $\eta = \min$ means that the imaginary part of the elastic amplitude has a maximum. For the charge exchange helicity flip amplitude B_ℓ , a resonance can give a contribution of either sign, depending on J^P and I . It turns out experimentally that all prominent N^* resonances enter with the same positive sign in B_ℓ^{CEX} . Therefore we read off the momentum p_L at which B_ℓ reaches the top of the circle.

See also the talk of R.J.N. Phillips at this meeting.

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Figure captions

- Fig. 1 : Integration contour considered in the derivation of the FESR.
(The Born term poles are not shown.)
- Fig. 2 : Comparison of $2 \operatorname{Im} A'^{(-)} = p_{\text{lab}} [\sigma_{\text{tot}}(\pi^- p) - \sigma_{\text{tot}}(\pi^+ p)]$ with the Regge fit to the high-energy data (8-22 GeV/c, $\alpha_\rho(0) = 0.58$) extrapolated to low energies. On the average, the Regge term is equal to the low-energy amplitude.
- Fig. 3 : Low-energy description on the average of $\operatorname{Im} A'^{(+)}$ = $p_L \frac{1}{2} [\sigma_{\text{tot}}(\pi^- p) + \sigma_{\text{tot}}(\pi^+ p)]$ by the Regge poles P and P' as determined in the high-energy fits of a) Rarita et al.¹⁸⁾ and b) Lindenbaum et al.¹⁹⁾.
- Fig. 4a : Determination of the meson trajectory, $\alpha = \alpha_\rho = \alpha_{A_2} = \alpha_\omega = \alpha_f$, from the Y_0^* resonances ($\frac{1}{2}^+$, $\frac{3}{2}^-$, $\frac{5}{2}^+$) through FESR.
- Fig. 4b : FESR's of several moments force the Regge amplitude (dashed line) to be, on the average, equal to the resonances, whose strength is represented by a vertical line.
- Fig. 5 : a) Shows the exchange-degenerate sequence $\Lambda_\alpha - \Lambda_\gamma$.
b) Shows that those resonances which do not fit into the exchange-degeneracy pattern give very small contributions where it counts dynamically, namely at $t = m_\rho^2$; b) also shows that the resonances included in a) have not only exchange-degenerate masses, but also exchange-degenerate couplings.
- Fig. 6 : The direct-channel partial-wave projection of the ρ Regge exchange amplitude. a)-c) The complex amplitude $B_\ell(p_L)$ for fixed ℓ vs. the laboratory momentum p_L . The numbers along the circles give the values of p_L . d) $B_\ell(p_L)$ for fixed p_L vs. ℓ . The numbers along the circle give the values of ℓ . For normalization we have put $\beta = 1$.

Fig. 7 : Chew-Frautschi plot: a) The established πN resonances,
b) the N^* resonances from our partial-wave analysis of the
 ρ Regge exchange. E_L = lab. energy of $\pi \approx p_L$. E_L is linear
in $(M_{\text{res}})^2$.

FIG.1

FIG.2

FIG.3

FIG. 4a

FIG. 4b

FIG. 5a

FIG. 5b

FIG. 6

FIG.7