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HIGH ENERGY PION-NUCLEON SCATTERING DATA
AND THE POMERANCHUK THEOREM

R. Wit *)
CERN - Geneva

A B S T R A C T

The high energy data for the $\pi^\pm - N$ forward scattering are analyzed in connection with the possible violation of the Pommeranchuk theorem. No strong evidence for such a violation is found ; the importance of further, precise measurements of $\text{Re } T^\pm$ is emphasized.

*) On leave from the Jagellonian University, Cracow, Poland.

There are two striking features of the total meson-nucleon cross-sections resulting from the recent Serpukhov data ¹⁾ :

- (i) - in the momentum range from 25-65 GeV/c the total cross-sections seem to be essentially constant ; a small increasing tendency is not excluded ;
- (ii) - the differences $\sigma_{\pi^+p} - \sigma_{\pi^+n}$ and $\sigma_{\pi^-p} - \sigma_{\pi^-n}$ appear to be constant in the considered energy range. In any case a strong decreasing trend of them is not observed.

These conclusions have raised again the question of the validity of Pomeranchuk's theorem from both phenomenological ²⁾ and more rigorous points of view ³⁾. According to Barger and Phillips one obtains good fits to the new IHEP-CERN values of the total cross-sections by adding to the well-known Regge poles a vacuum cut which is a slowly decreasing function of energy as compared to the leading pole term. The parametrization they used for the πN scattering was of the following form

$$\frac{T^+(\omega) + T^-(\omega)}{2} = i\gamma_p \omega - \gamma_{p'} \omega^{\alpha_{p'}} e^{-i\frac{\pi}{2}\alpha_{p'}} + i\gamma_c \omega \frac{1}{(\ln \omega - \beta - i\frac{\pi}{2})^\lambda}, \quad (1a)$$

$$\frac{T^-(\omega) - T^+(\omega)}{2} = i\gamma_g \omega^{\alpha_g} e^{-i\frac{\pi}{2}\alpha_g} \quad (1b)$$

where $T^+(\omega)$ and $T^-(\omega)$ describe the π^+p and π^-p forward scattering amplitudes. By ω we denote the meson lab. energy (in GeV) ; the units are $\hbar = c = \text{GeV} = 1$ and the optical theorem is used in the form $\sigma_{\pi}^{\pm}(\omega) = \text{Im } T^{\pm}(\omega)/k$, where $k^2 = \omega^2 - 0.0196$.

Some recently discussed models ^{4),5)} yield additional support for an introduction of such a leading vacuum cut dominating over secondary Regge poles at high energies. Since on the other hand they predict constant total cross-sections at infinity and negligible contribution from the real part to the scattering amplitude in the asymptotic energy region, they do not lead to any difficulty with the Pomeranchuk theorem.

As pointed out several years ago by Martin ⁶⁾ and discussed recently again in several aspects by Eden ³⁾ the proofs of the Pomeranchuk theorem contain some ad hoc assumptions which might be in general not true and therefore it is possible to construct a counter-example to Pomeranchuk's theorem not violating the general properties of the forward scattering amplitudes which follow from axiomatic field theory ⁷⁾. To construct such a counter-example it is sufficient, e.g., to reject only the commonly considered "obvious" additional condition

$$\lim_{\omega \rightarrow \infty} |T^{\pm}(\omega)| / \omega \ln \omega = 0.$$

Therefore it seems to be interesting to analyze the existing experimental data under the assumption

$$\lim_{\omega \rightarrow \infty} (\sigma_T^{-}(\omega) - \sigma_T^{+}(\omega)) = 2\epsilon \quad (2)$$

and to evaluate the quantity ϵ from the best fits using, for example, the χ^2 test. We do not include in our analysis any invariance property like, e.g., isospin invariance ^{*)}; all the results we have obtained are based merely on analyticity, $s \rightleftharpoons u$ crossing, dominance of the scattering amplitude by the imaginary part and, of course, the specific form of our high energy parametrization. We restrict ourselves to the case of πN scattering for the following reasons :

- 1) - The present experimental data seem to indicate that if the Pomeranchuk theorem is violated the "smallest" violation takes place for the πN scattering. Therefore any indication for the non-zero asymptotic limit between the total cross-sections for the pion-nucleon scattering would lead at the same time to more serious problems for kaon-nucleon and nucleon-nucleon scattering.

^{*)} Since at the present moment there are no data for $\pi^+ p$ scattering for the Serpukhov energy range we assume, as in Ref. 2), $\sigma_T(\pi^- n) = \sigma_T(\pi^+ p)$.

- 2) - In the momentum range from 10-65 GeV/c we have much more experimental data for $\sigma_T(\pi^-p)$, $\sigma_T(\pi^+p)$ and $\sigma_T(\pi^-n)$ than for any other reactions.
- 3) - There exists a considerable amount of experimental values ⁸⁾ of the ratio $\alpha^\pm = \text{Re } T^\pm / \text{Im } T^\pm$ up to 26 GeV/c. This fact seems to be rather important since the parameters describing the high energy behaviour of $\sigma_T^\pm$ describe at the same time the behaviour of $\text{Re } T^\pm$. Thus any fit to the total cross-sections can be compared to the independent measurements of $\text{Re } T^\pm$.

In addition to the asymptotic relation (2) we assume also the dominance of the imaginary part of the scattering amplitude, at least for one of the physical amplitudes $T^\pm(\omega)$; e.g.,

$$\lim_{\omega \rightarrow \infty} \text{Re } T^+(\omega) / \text{Im } T^+(\omega) = 0. \quad (3)$$

Due to the assumption (2) the total cross-sections, if not increasing in the high energy range, are at least constant and in this case we get using only analyticity and $s \rightleftharpoons u$ crossing symmetry ⁹⁾

$$\frac{\text{Re } T^+(\omega) + \text{Re } T^-(\omega)}{\text{Im } T^+(\omega) + \text{Im } T^-(\omega)} \xrightarrow{\omega \rightarrow \infty} 0 \quad (4)$$

unless the Froissart bound is violated. On the other hand we obtain from the dispersion formula

$$\frac{\text{Re } T^-(\omega) - \text{Re } T^+(\omega)}{\omega \ln \omega} \xrightarrow{\omega \rightarrow \infty} -\frac{1}{2} \epsilon$$

and therefore

$$\frac{\operatorname{Re} T^-(\omega) - \operatorname{Re} T^+(\omega)}{\operatorname{Im} T^-(\omega) + \operatorname{Im} T^+(\omega)} = \frac{\operatorname{Re} T^-(\omega) + \operatorname{Re} T^+(\omega)}{\operatorname{Im} T^-(\omega) + \operatorname{Im} T^+(\omega)}$$

$$- \frac{2 \operatorname{Re} T^+(\omega)}{\operatorname{Im} T^-(\omega) + \operatorname{Im} T^+(\omega)} \xrightarrow{\omega \rightarrow \infty} 0$$

but then follows

$$\frac{\ln \omega}{\sigma_{\pi}^{\pm}(\omega)} \xrightarrow{\omega \rightarrow \infty} 0. \quad (5)$$

This result seems to be of some general interest. In the Regge description of the high energy processes the leading term is of the form $s^{\alpha(t)} \times (\ln s)^{\beta(t)} \times (\ln \ln s)^{\gamma(t)} \dots$ and the maximum value of $\alpha(0)$ is equal to 1. For $\alpha(0) = 1$ the value of $\beta(0)$ cannot be greater than one also ¹⁰⁾. Putting $\beta(0) = 1$ and $\alpha(0) = 1$ we get $\gamma(0) < 0$, etc. Therefore relation (5) violates in some respect the simple Regge description of the high energy scattering although we are not in contradiction with the Froissart upper bound $\sigma_{\pi}(\omega) < (\ln \omega)^2$.

According to the above discussion we suggest the investigation of the following parametrization

$$\frac{T^+(\omega) + T^-(\omega)}{2} = i\gamma_p k - \gamma_{p'} k^{\alpha_{p'}} e^{-\frac{i\pi}{2}\alpha_{p'}} + i\gamma_G k (\ln \omega - b - i\pi/2)^{\lambda}, \quad (6a)$$

$$\frac{T^-(\omega) - T^+(\omega)}{2} = -\frac{2}{\pi} \epsilon \omega (\ln \omega - i\pi/2) + \omega \delta + i\gamma_s \omega k^{\alpha_s - 1} e^{-\frac{i\pi}{2}\alpha_s} \quad (6b)$$

which on the other hand can be very easily related to that introduced by Barger and Phillips.

To minimize the number of parameters we put $\alpha_g = \alpha_{P'} = 0.4$ and $\lambda = 1.1$. Because of the particular choice of the logarithmic term in the charge-exchange amplitude the parameter δ , analogous to b , will not be determined by any fit to the total cross-sections only and consequently it will not influence in this case the best choice for the remaining parameters. Therefore we put it simply equal to zero and start with the following six free parameters :

- $\alpha_g, \alpha_{P'}, \alpha_P$ - they correspond to the conventional Regge residues ;
- 2ϵ - the asymptotic difference between the total cross-sections ;
- γ_C - the strength of the cut ; it must be positive and we expect to observe a very strong correlation between ϵ and γ_C ;
- b - it is related to the position of the branch point ω_0 in the complex ω plane by the formula $b = \ln \omega_0$.

Of course, there is no limit for further complication of the asymptotic description of the scattering amplitudes. What we would like to investigate here is, however, evidence for or against a term violating the Pomeranchuk theorem by fitting the data with a minimal number of parameters.

Before starting the numerical analysis we would like to make some remarks concerning the general behaviour of the scattering amplitude which follow from our assumptions. The first one is almost obvious : if the Pomeranchuk theorem is violated the main high energy contribution to $\text{Re } T^\pm$ comes from the charge-exchange amplitude which is not the case for the total cross-sections. Therefore one might expect to obtain rather good fits to $\sigma_T^\pm$ using (6a), (6b). It is, however, difficult to make any prediction how the fits would work for $\alpha^\pm$. Note also that the parametrizations (1a), (1b) and (6a), (6b) lead to different predictions for the energy behaviour of $\alpha^\pm$ and it would be interesting to get an

estimation of the energy at which the differences between these two predictions would be essential.

Another question concerns the balance between cuts and Regge poles. Usually one expects ¹¹⁾ that for the forward scattering the Regge poles dominate while for negative t the cut contributions become more important. However in our case the cuts obviously dominate also in the forward direction for both elastic and charge-exchange scattering.

The most unpleasant situation may, however, arise in connection with the charge-exchange data. Assuming isospin invariance we would not be able to reproduce the almost asymptotic values of the charge-exchange cross-sections (if the falling trend were conserved) for positive, significantly different from zero ϵ and positive γ_3 . Returning to the formula (6b) we see that the first term of the right-hand side may be written in the form $\epsilon \omega e^{-i\pi\mu/2} (\ln\omega - i\pi/2)^\nu / k^\mu$ with the values of μ and ν fixed in advance and equal to 0 and 1, respectively. On the other hand it is almost obvious that in a finite energy range one can obtain two compatible, rather good fits to the experimental values of $\sigma_T^\pm$ and $\alpha^\pm$ for small ϵ, μ but $\nu \sim 1$ and for relatively large values of ϵ and μ . Therefore if our numerical analysis yielded ϵ even different from zero but much smaller than the presently observed values of $\sigma_T^- - \sigma_T^+$ we would interpret this result in favour of a cut in the charge-exchange amplitude in general rather than a violation of the Pomeranchuk theorem itself.

We started our numerical analysis by fitting only the total cross-sections with the data from 7.38 GeV/c up, published in Refs, 1), 12), 13). The errors in Ref. 12) were taken equal to 0.1 mb. First we repeated the analysis of Barger and Phillips and the results are given in the second column of Table I. Next we obtained the best fit for the parameters appearing in formulae (6a), (6b) and it is presented in the third column of Table I. What we observe is a rather remarkable change of χ^2 . Also the values of b differ by an order of magnitude corresponding to branch points in the ω plane at 0.34 GeV and $2.2 \cdot 10^4$ GeV respectively. This means that the logarithmic term in the formula (6a) is purely imaginary and relatively large for the present accelerator energy range. One may therefore expect a considerable change of b when the fits include in addition the data for $\text{Re } T^\pm(\omega)$.

The values of parameters given in columns 2 and 3 of Table I lead to different predictions for $\alpha^\pm$. For simplicity we restrict ourselves to the presently most interesting momentum range from 10 GeV/c - 70 GeV/c. The predicted and experimental values of $\alpha^\pm$ are shown in Table II and we observe the following :

- 1) - Barger and Phillips' set of parameters leads to a good agreement with the experimental values of $\alpha_{\text{exp}}^\pm$. Both α^+ and α^- are negative ($\alpha^+ < \alpha^-$) and they change their signs of 128 GeV/c and 86 GeV/c respectively.
- 2) - The present test fit to only the total cross-sections predicts values for $\alpha^\pm$ at least twice as large in magnitude as the experimental values. The α 's are still negative but from 14 GeV/c up $\alpha^+ > \alpha^-$.

Because of these troubles with the real part we performed another fit which included also the experimental values of $\alpha_{\text{exp}}^\pm$ with the possible systematic error equal to ± 0.02 . The new values of the parameters for the lowest χ^2 are given in the fourth column of Table I. Now, as we expected, the most striking change is that of b . The agreement between $\alpha^\pm$ and experiment has been improved very much (cf. columns 6 and 7, Table II) ; as before we have again $\alpha^- < \alpha^+$ for $k \gg 16$ GeV/c ^{*)}. At the same time, however, the value of χ^2 due to $\sigma_T^\pm$ only has increased by about 30% and the value of ϵ has decreased by one third. Putting more emphasis on the measured values of $\alpha_{\text{exp}}^\pm$ themselves and performing the same fit without the above-mentioned systematic error we end up with

$$\sigma_T^-(\omega) - \sigma_T^+(\omega) \xrightarrow{\omega \rightarrow \infty} 0.46 \text{ mb} \quad (7)$$

*) Because the experimental errors are large we cannot say too much about the relative behaviour of α_{exp}^+ and α_{exp}^- . What we observe is that below 20 GeV, $\alpha_{\text{exp}}^+ < \alpha_{\text{exp}}^-$ but near 22 GeV one might expect an intersection of them ⁸⁾.

which is almost equal to one third of the difference $\sigma_T^-(\omega) - \sigma_T^+(\omega)$ observed at the Serpukhov energy range.

The situation cannot change very much even if we introduce δ as an additional free parameter because the term $\delta\omega$ appears in α^+ and α^- with opposite sign. Indeed, the lowest value of χ^2 obtained by fitting the same data with seven free parameters is equal to 153.01. Obviously, any further fit, including also the charge exchange data cannot improve this picture although δ may balance in a large energy range the contributions due to the logarithmic term in the charge exchange amplitude.

Finally we have to discuss the question of the experimental errors for the difference of the total cross-sections. As pointed in Ref. 2) although for the momenta 7-30 GeV/c the values of the errors for the total cross-sections are 0.1 mb, for the difference $\sigma_T^- - \sigma_T^+$ some of the systematic errors cancel each other and therefore for these energies the values of $\sigma_T^- - \sigma_T^+$ are given with a total error of 0.1 mb only. The situation is quite different for the new Serpukhov data. Although the point to point errors are not very large (being equal in almost all cases to 0.15 mb for π^-p and 0.30 mb for π^-n scattering) the additional scale errors involve more serious uncertainty into the data and we have for the difference of the total cross-sections at the highest accelerator energies

$$\sigma_T(\pi^-p) - \sigma_T(\pi^+p) = 1.3 \pm 0.7 \text{ mb}.$$

If in addition one takes into account the rather large values of χ^2 given in columns 4 and 5 of Table I and lists the main unpleasant features ^{*)} of the scattering amplitude which, according to our conside-

*) They are :

- (i) - violation of isospin invariance ;
- (ii) - breakdown of the Regge-type description of the leading term ;
- (iii) - dominating cuts in the forward direction ;
- (iv) - mainly charge-exchange contribution to the real part of the scattering amplitudes.

rations would follow from a violation of the Pomeranchuk theorem, we see that at present there are no good reasons to believe that $\sigma_{\mathbb{T}}^{-}(\omega) - \sigma_{\mathbb{T}}^{+}(\omega) \neq 0$. Since on the other hand, according to Barger and Phillips, an inclusion of a small ρ type cut improves the fit to the difference $\sigma_{\mathbb{T}}(\pi^{-}p) - \sigma_{\mathbb{T}}(\pi^{+}p)$ we would like to suggest a rather simple explanation of the presently observed troubles with the Pomeranchuk theorem : in some respects the ρ cut in the charge-exchange amplitude can simulate the Pomeranchuk theorem violating cut. It is, however, clear from the presented analysis that any more definite statement cannot be made until the experimental data (especially for $\alpha_{\text{exp}}^{\pm}$) are more precise.

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Parameters	Barger and Phillips' fit to $\sigma_{\pi}^{\pm}$	Present test fit to $\sigma_{\pi}^{\pm}$	Present test fit to $\sigma_{\pi}^{\pm}$ and $\alpha^{\pm}$ including systematic errors	Present test fit to $\sigma_{\pi}^{\pm}$ and $\alpha^{\pm}$ without systematic errors
χ_P	85.79	82.62	46.85	51.39
$\chi_{P'}$	184.20	130.37	94.48	75.38
χ_S	13.26	5.55	7.50	10.0
χ_c	-224.90	-	-	-
b	- 1.08	10.53	- 0.214	- 0.724
χ_c	-	3.46	1.94	1.034
e	-	1.19	0.83	0.587
χ^2	104.62	60.53	116.80 *)	163.93
Number of experimental points	58	58	79	79
*) The partial values of χ^2 due to $\sigma_{\pi}^{\pm}$ and $\alpha^{\pm}$ are 86.97 and 29.83, respectively.				

- TABLE I -

Best fits to the total cross-sections and the real parts of the scattering amplitudes for different parametrizations

Momentum (GeV/c)	Barger and Phillips' fit		Present test fit to $\sigma_{\pi^{\pm}}$		Present test fit to $\sigma_{\pi^{\pm}}$ and $\alpha^{\pm}$ including systematic errors		Present test fit to $\sigma_{\pi^{\pm}}$ and $\alpha^{\pm}$ without systematic errors		Experimental values of	
	α^{-}	α^{+}	α^{-}	α^{+}	α^{-}	α^{+}	α^{-}	α^{+}	α^{-}	α^{+}
10	-0.164	-0.239	-0.454	-0.463	-0.226	-0.245	-0.185	-0.220	-0.167 $\pm$ 0.02	-0.215 $\pm$ 0.010
20	-0.093	-0.140	-0.340	-0.308	-0.149	-0.133	-0.126	-0.129	-0.117 $\pm$ 0.025	-0.144 $\pm$ 0.031
30	-0.058	-0.095	-0.285	-0.233	-0.144	-0.082	-0.100	-0.089		
40	-0.039	-0.068	-0.251	-0.186	-0.093	-0.052	-0.084	-0.064		
50	-0.025	-0.050	-0.228	-0.153	-0.080	-0.031	-0.074	-0.048		
60	-0.015	-0.038	-0.210	-0.128	-0.070	-0.016	-0.067	-0.035		
70	-0.008	-0.028	-0.196	-0.109	-0.063	-0.004	-0.061	-0.026		
Change of sign at the	86	128	7300	218	7300	72	7300	114		

- TABLE II -
Predicted and experimental values of $\alpha^{\pm}$

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