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MOVING CDD POLES AND COMPLEX REGGE TRAJECTORIES

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A B S T R A C T

A mechanism for generating dynamical fixed cuts in the angular momentum plane, based on moving CDD poles, is proposed. In particular the splitting of Regge poles coupled to the $\pi\pi$ system is studied. Implications on low energy scattering and on high energy phenomenology are discussed.

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INTRODUCTION

It is well known ¹⁾ that the collision of two Regge poles leads to a left-hand cut discontinuity of the trajectory function $\alpha(t)$; this phenomenon has been studied in potential scattering ²⁾. A specific example is the collision of a physical sheet pole with a branch point on the real axis in the angular momentum plane, since this mechanism is equivalent to the collision of a pair of complex conjugate poles near the branch point ^{3),4)}.

In relativistic scattering, where parallel trajectories have been widely accepted, the pole-cut relationship is more important for studying the possibility of complex Regge poles. In this paper we investigate a dynamical model which generates fixed cuts on the real axis, compatible with t channel unitarity. This is achieved by using N/D equations continued into the ℓ plane and our basic assumption is that the D function is dominated by an infinite series of CDD poles, the position of which in the energy plane is a function of the angular momentum ℓ (moving CDD poles). The motivation of introducing this kind of CDD poles is to guarantee the full spectrum of resonances suggested by dual four-point amplitudes. Physically, the assumption that these particles are CDD poles in the one-channel formulation of the unitarity condition, exhibits the fact that the basic mechanism for producing rising trajectories is the coupling to inelastic channels. We remind that models which have tried to explain the rich spectrum without introducing CDD poles (using only Yukawa forces), have failed ⁵⁾. On the other hand, for P wave $\pi\pi$ scattering, it was shown in Ref. 6) that a CDD parametrization deduced from the Lovelace-Veneziano model, leads to a self-consistent description of this channel.

The existence of real CDD poles, moving in the ℓ plane, leads in general to fixed cuts in the ℓ plane through the integral representation of the N function. In particular, for linear CDD trajectories, the corresponding cuts are on the real axis. It is worth mentioning that in potential theory fixed cuts occur on the real axis of the ℓ plane when the potential contains a singular attractive

term at small distances $\overline{V}(r) \sim a/r^2$, $a < 0$. The branch point is just at the place where the most singular part of the potential, including the centrifugal term, changes sign and this physically corresponds to the threshold of falling of particles into the centre ⁷⁾. If we keep this picture also for relativistic scattering, the fixed cuts in our model may be interpreted as due to the strong and short range attraction resulting from the coupling to inelastic channels and represented by the linear CDD trajectories, as mentioned before. Fixed cuts in relativistic scattering are also suggested by field theoretical models ⁸⁾, e.g., ladder diagrams with vector meson exchange give rise to such cuts. Recently fixed cuts have been obtained in a Van Hove model for spinless particles using exchanged fields with a specific daughter structure ⁹⁾.

In Chapter 1, we give the general description of the pole-cut relationship in a model with an infinite series of linear CDD trajectories. We concentrate on $\pi\pi$ scattering and a plausible form of the D function is derived on the basis of the degenerate ρ -f family. In Chapter 2, we study in detail the isotopic spin-one ($I = 1$) channel and we discuss the behaviour of the ρ trajectory. In Chapter 3 the $I = 0$ channel is treated. The D function takes into account extra singularities in the l plane due to Pomeron. The low energy S wave $\pi\pi$ scattering amplitude is extracted and the structure of the leading trajectories is analyzed. The features of diffraction scattering in this model are investigated and several phenomenological implications are discussed.

1. - MODEL FOR FIXED CUTS

In this Chapter we want to illustrate the mechanism for producing fixed cuts in the angular momentum plane (ℓ plane) by introducing real CDD poles, the position of which in the t plane is a linear function of ℓ (linear CDD trajectories). For simplicity we restrict ourselves to the case of neutral spinless and equal mass particles, i.e., pions ($m_\pi^2 = 1$). The case of charged pions is treated in the next Chapters.

The t channel reduced partial wave amplitude, $B(\ell, t)$, is defined as usual by ¹⁰⁾

$$A(\ell, t) = (q(t))^{2\ell} B(\ell, t) \quad (1)$$

with $q(t) = (t-4/4)^{\frac{1}{2}}$. The elastic unitarity condition, analytically continued in ℓ , for the amplitude $B(\ell, t)$ is written as follows ($t \geq 4$) :

$$\text{disc}_t B(t+i\epsilon, \ell+i\epsilon) = g_\ell(t) B(t+i\epsilon, \ell+i\epsilon) B(t-i\epsilon, \ell+i\epsilon), \quad (2)$$

and

$$g_\ell(t) = 2 (q(t))^{2\ell+1} / \sqrt{t} .$$

In order to incorporate unitarity in a dynamical scheme it is convenient to use the decomposition

$$B(\ell, t) = \frac{N(\ell, t)}{D(\ell, t)} .$$

The functions N and D satisfy then the following equations

$$\begin{aligned} N(\ell, t) &= \frac{1}{\pi} \int_{-\infty}^0 \frac{\text{disc}_t B(\ell, t')}{t' - t} D(\ell, t') dt' , \\ D(\ell, t) &= \text{CDD-poles} - \frac{1}{\pi} \int_4^\infty \frac{g_\ell(t') N(\ell, t')}{t' - t} dt' . \end{aligned} \quad (3)$$

For the CDD term in Eqs. (3), we consider the following form :

$$\text{CDD-poles} = \sum_{n=0}^{\infty} \frac{g_n(\ell)}{\ell - c_n(t)} \quad (4)$$

where $c_n(t) = c_n(0) + \tilde{c}_n t$ ($\tilde{c}_n > 0$). Correspondingly we obtain the following integral equation for the N function :

$$N(\ell, t) = \frac{1}{\pi} \sum_{n=0}^{\infty} g_n(\ell) \int_{-\infty}^0 \frac{\text{disc}_t B(\ell, t')}{\ell - c_n(t')} \frac{dt'}{t' - t} + \frac{1}{\pi^2} \int_4^{\infty} dt'' K(\ell, t'', t) N(\ell, t'') \quad (5)$$

and

$$K(\ell, t'', t) = g_e(t'') \int_{-\infty}^0 \text{disc}_t B(\ell, t') \frac{dt'}{(t' - t)(t' - t'')} .$$

From the inhomogeneous term of Eq. (5) it can be seen that the CDD poles generate fixed cuts in the N function along the negative real axis in the ℓ plane. The corresponding branch points are at $\ell_{c_n} = c_n(0)$. In order to study the properties of the Regge trajectories one has to look at the zeros of the D function. In the neighbourhood of the real axis in both ℓ and t planes, we obtain :

$$D(\ell, t) = \text{Re } D(\ell, t) - \frac{i}{\pi} \Theta(c_0(0) - \ell) \mathcal{P} \int_4^{\infty} \frac{g_e(t') \text{disc}_\ell N(\ell, t')}{t' - t} dt' - i \Theta(t - 4) \text{Re } N(\ell, t) g_e(t) = 0 \quad (6)$$

The imaginary part of $D(\ell, t)$ for $\ell < c_0(0)$ and $t < 4$ is due to the presence of the branch cuts in the ℓ plane. Having now in mind a situation where a trajectory $\alpha(t)$ collides with the leading branch point at $t = 0$, i.e., $\alpha(0) = c_0(0)$, we can easily see that, due to the second term in Eq. (6), the trajectory function $\alpha(t)$ develops a small imaginary part for negative t [$\alpha(t) = \alpha_R(t) + i\alpha_I(t)$]. For small α_I , an approximate solution is given by :

$$\operatorname{Re} D(\alpha_R, t) = 0$$

$$\alpha_z(t) = \frac{1}{\pi \left(\operatorname{Re} D(\ell, t) \right)'_{\ell=\alpha_R}} \int_4^{\infty} \rho_{\alpha_R}(t') \frac{\operatorname{disc}_\ell N(\ell=\alpha_R, t')}{t' - t} dt' \quad (7)$$

for $\alpha_R < c_0(0)$. The quantity $\operatorname{disc}_\ell N(\ell, t) \equiv \frac{1}{2i} [N(\ell+i\epsilon, t) - N(\ell-i\epsilon, t)]$ satisfies the following integral equation, easily obtained from Eq. (5) :

$$\begin{aligned} \operatorname{disc}_\ell N(\ell, t) = & \sum_{n=0}^{\infty} \frac{g_n(\ell)}{c_n(t) - \ell} \operatorname{disc}_t B(\ell, t = \frac{\ell - c_n(0)}{\tilde{c}_n}) \theta(c_n(0) - \ell) \\ & + \frac{1}{\pi^2} \int_4^{\infty} dt'' K(\ell, t'', t) \operatorname{disc}_\ell N(\ell, t'') \end{aligned} \quad (8)$$

Consistently with our general assumption of CDD dominance, the potential $\operatorname{disc}_t B(\ell, t)$ ($t < 0$) may be considered as a small perturbation allowing for an iterative treatment of the above equation. We obtain

$$\begin{aligned} \operatorname{disc}_\ell N(\ell, t) = & \sum_{n=0}^{\infty} g_n(\ell) \operatorname{disc}_t B(\ell, t = \frac{\ell - c_n(0)}{\tilde{c}_n}) \theta(c_n(0) - \ell) \\ & \left[\frac{1}{c_n(t) - \ell} + \frac{1}{\pi^2} \int_4^{\infty} dt'' \frac{K(\ell, t'', t)}{c_n(t'') - \ell} + \text{higher orders in } \operatorname{disc}_t B \right] \end{aligned} \quad (9)$$

Keeping only terms of first order with respect to the potential $\operatorname{disc}_t B(\ell, t)$, we end up with the simple expression

$$\operatorname{disc}_\ell N(\ell, t) = \sum_{n=0}^{\infty} \frac{g_n(\ell)}{c_n(t) - \ell} \operatorname{disc}_t B(\ell, t = \frac{\ell - c_n(0)}{\tilde{c}_n}) \theta(c_n(0) - \ell) \quad (10)$$

We would like to mention that the essential structure in $\text{disc}_\ell^N(l, t)$ coming from $\text{disc}_t^B(l, t)$ is independent of our first order approximation. From Eqs. (7) and (10), we obtain :

$$\alpha_I(t) = \frac{1}{\pi (\text{Re } D(l, t))'} \sum_{n=0}^{\infty} g_n(\alpha_n) \text{disc}_t^B(l = \alpha_n, t' = \frac{\alpha_n(t) - c_n(0)}{\tilde{c}_n}) .$$

$$\theta(c_n(0) - \alpha_n(t)) \int_0^{\infty} \frac{\rho_{\alpha_n(t)}(t') dt'}{(t' - t)(c_n(t') - \alpha_n(t))} . \quad (11)$$

This formula gives the imaginary part α_I near the upper lip of the ℓ cut ; therefore the Regge pole $\alpha(t)$ is on the unphysical sheet if $\alpha_I(t) < 0$. Obviously, from the reality of the D function, a complex conjugate pole appears near the other lip of the cut. We observe that for a given potential $\text{disc}_t^B(l, t)$, Eq. (11) provides a model for the splitting of the Regge poles near the branch point and a quantitative study of $\alpha_I(t)$ is possible if the structure of $\text{Re } D(l, t)$ is known.

2. - I = 1 $\pi\pi$ CHANNEL

We apply this model to the scattering of physical pions and we study the most interesting channels with isospin 1 and 0. First we consider the channel with $I = 1$ where no singularities due to the Pomeron exist in the ℓ plane.

A simple ansatz for $\text{Re } D^{I=1}(l, t)$ can be constructed as follows. Imposing an infinite number of zeros at $\ell = \alpha_p(t) - n$ ($n = 0, 1, \dots$) corresponding to the usual daughter spectrum of the leading degenerate ρ -f trajectory¹¹⁾, $\alpha_p(t) = \alpha_p(0) + \alpha_p' t$, and inserting an infinite number of moving CDD poles, we obtain for a power behaved (for large t) $\text{Re } D^{I=1}(l, t)$ the plausible form

$$\text{Re } D^{I=1}(l, t) = \frac{\Gamma(x - \alpha_p(t))}{\Gamma(\ell - \alpha_p(t))} . \quad (12)$$

Furthermore, the parameter x turns out to be $x = 2\ell - \alpha_p(0)$, from the following asymptotic conditions :

- (a) $A(\ell, t) \rightarrow \text{const} + O(t^{\alpha_p(0)-1})$ for $t \rightarrow +\infty$, a behaviour suggested by the Regge model (neglecting logarithmic terms) and corresponding to the exchange of the Pomeron and the degenerate pole $\alpha_p(t)$;
- (b) $\text{disc}_t A(\ell, t) \rightarrow \text{const} (\neq 0)$ for $t \rightarrow -\infty$ ¹²⁾ ;
- (c) the N function has a power behaviour for large t , independent of ℓ .

With the ansatz (12) for $\text{Re } D^{I=1}$ we can study now the low energy scattering amplitude for $I = 1$ and physical ℓ values ($\ell = 1, 3, \dots$) and also investigate the properties of the Regge poles (trajectories and residues) near the fixed cuts. The P wave low energy $\pi\pi$ scattering amplitude is easily obtained with the use of the N and D functions. We have (for $\ell = 1$, $t \geq 4$) :

$$D_1(t) = \frac{\Gamma(2 - \alpha_p(0) - \alpha_p(t))}{\Gamma(1 - \alpha_p(t))} - i \rho_1(t) N_1(t) \theta(t-4),$$

$$N_1(t) = \frac{1}{\pi} \int_{-\infty}^0 \frac{\text{disc}_t B_1^{I=1}(t')}{t' - t} \frac{\Gamma(2 - \alpha_p(0) - \alpha_p(t'))}{\Gamma(1 - \alpha_p(t'))} dt' . \quad (13)$$

With the potential $\text{disc}_t B_1^{I=1}(t)$ given by the Lovelace-Veneziano ^{11), 13)} model, these equations give a consistent description of the ρ meson and the electromagnetic form factor of the pion, as shown in Ref. 6).

We investigate the detailed properties of the leading ρ trajectory following the prescription given in the last Chapter.

From the expansion of the ansatz (12), we obtain the CDD pole structure

$$\frac{\Gamma(2\ell - \alpha_p(0) - \alpha_p(t))}{\Gamma(\ell - \alpha_p(t))} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\alpha_p(0) - \ell - n)} \cdot \frac{1}{2\ell - \alpha_p(0) - \alpha_p(t) + n} \quad (14)$$

[From this formula one can identify the parameters $g_n(\ell)$ and $c_n(t)$.] The fixed branch points are at $\ell_{c_n} = \alpha_p(0) - n/2$ and the collision of the p trajectory with the leading branch point ($n = 0$) occurs at $t = 0$. The corresponding imaginary part $\alpha_{Ip}(t)$ is given by Eq. (11) which, using formulae (12) and (14), becomes :

$$\alpha_{Ip}(t) = \frac{t \sin \pi \alpha_p'(t)}{2\pi} \text{disc}_t B^{I=1}(\ell = \alpha_p(t), 2t) \int_4^{\infty} \frac{\rho_{\alpha_p(t)}(t') dt'}{(t' - t)(2t - t')} \quad (15)$$

This expression takes into account only the leading branch cut and is therefore valid only for $-1/2 \alpha_p' < t \leq 0$. For smaller values of t ($t < -1/2 \alpha_p'$) similar expressions, due to the lower branch points ($n=1, 2, \dots$), can be easily found. From the expression (15) the behaviour of $\alpha_{Ip}(t)$ near $t = 0$ can be extracted by using the threshold behaviour ¹⁴⁾ of $\text{disc}_t B^{I=1}(\ell, t)$. We find

$$\alpha_{Ip}(t) \longrightarrow \text{const} \left[(a_0^0)^2 - \frac{5}{2} (a_0^2)^2 \right] (-t)^{7/2} \text{ for } t \rightarrow 0^- \quad (16)$$

where a_0^0 and a_0^2 are the S wave scattering lengths with $I = 0$ and $I = 2$ respectively. Because of the smallness of the scattering lengths and the strong suppression due to the threshold factor in (16), the imaginary part α_{Ip} is expected to be very small near the forward direction ($t=0$). In Fig. 1 the behaviour of $\alpha_{Ip}(t)$, obtained from Eq. (16), is shown in the region $-1.2 (\text{GeV}/c)^2 < t < 0$. For the potential $\text{disc}_t B^{I=1}$ we use the following model for the structure of the s and u channels. At low energies (up to the ρ region in the crossed channels), phenomenological phase shifts for S and P waves ¹⁵⁾ are used, whereas at higher energies the Veneziano resonances are inserted ¹³⁾ in the Froissart-Gribov representation for $\text{disc}_t B^{I=1}$.

We neglect contributions coming from the third double spectral function and we point out that the fixed cuts in our model are present even with vanishing third double spectral function.

We notice that for $-1.2 < t < -0.2 \text{ (GeV/c)}^2$ the ρ pole lies on the unphysical sheet whereas for $-0.2 < t < 0 \text{ (GeV/c)}^2$ we have a pair of complex conjugate poles on the physical sheet which behaves practically like one real pole (very small $\alpha_{I\rho}$, Fig. 1). The small contribution due to the next branch point at $l = \alpha_\rho(0) - \frac{1}{2}$ is shown separately in Fig. 1. The smallness of $\alpha_{I\rho}$ seen from Fig. 1 guarantees the consistency of the approximation involved in Eqs. (6) and (7).

The reduced residue of the ρ Regge pole can be derived using the $N^{I=1}(l, t)$ function [Eqs. (3)] and for small $\alpha_{I\rho}$ and negative t we obtain :

$$\begin{aligned} \text{Res } B_\rho(t) = & \frac{1}{\pi \Gamma(\alpha'_\rho t)} \mathcal{P} \int_{-\infty}^0 dt' \frac{\text{disc}_t B^{I=1}(\alpha_\rho(t), t')}{t' - t} \frac{\Gamma(2\alpha'_\rho t - \alpha'_\rho t')}{\Gamma(\alpha'_\rho t - \alpha'_\rho t')} \\ & + \frac{i}{\pi} \sin(\pi \alpha'_\rho t) \text{disc}_t B^{I=1}(\alpha_\rho(t), 2t) . \end{aligned} \quad (17)$$

We observe that in the forward direction ($t = 0$) of the s channel scattering, the ρ pole decouples and therefore the dominant $I = 1$ term for $s \rightarrow \infty$ in the Sommerfeld-Watson representation comes from the fixed branch point $l_c = \alpha_\rho(0)$, and the corresponding contribution to the forward amplitude behaves like $s^{\alpha_\rho(0)} (\ln s)^{-5/2}$ for $s \rightarrow \infty$. The modulus and the phase of the residue (17) are shown in Fig. 2.

For comparison, we show in Fig. 1 the imaginary part of the ρ trajectory used in Ref. 16) in order to fit the pion-nucleon charge exchange data. The different sign is not essential in this comparison since the phenomenological application refers to a pair of complex conjugate poles, whereas the size of $\alpha_{I\rho}$ is consistent with our prediction in the range of t , $-1 \lesssim t \lesssim -0.5 \text{ (GeV/c)}^2$. On the contrary, near the forward direction, $|t| \lesssim 0.2$, we found a very small imaginary part,

$\alpha_{I\rho} \lesssim 0.004$, and we feel that we are not able to explain the polarization data in this region with ρ exchange alone. However, consistently with the assumed daughter structure in the present model, the contribution of the ρ' may be important at present energies. Finally, we observe in Fig. 2 that the phase of the residue of the pole changes rapidly in the low t region and this may have implications in a detailed phenomenological application, especially for cross-over phenomena ¹⁷⁾.

For completeness we give the imaginary part of the ρ trajectory, due to unitarity, in the physical region of the t channel ($t \geq 4$)

$$\alpha_{I\rho}(t) = \frac{2}{\pi T(\alpha_{\rho'}(t)) \sqrt{t}} (q(t))^{2\alpha_{\rho}(t)+1} \int_{-\infty}^0 dt' \frac{\text{disc}_t B^{I=1}(\alpha_{\rho}(t), t')}{t' - t} \cdot \frac{T(2\alpha_{\rho}(t) - \alpha_{\rho}(0) - \alpha_{\rho}(t'))}{T(\alpha_{\rho}(t) - \alpha_{\rho}(t'))} \quad (18)$$

It has the correct threshold behaviour, as expected, and using $\text{disc}_t B^{I=1}$ as already discussed, we find a small value for $\alpha_{I\rho}(t)$ and especially at $t = m_{\rho}^2$ the corresponding value of the ρ width lies in the region $T_{\rho} = 90 - 112$ MeV for $0.5 < \alpha_{\rho}(0) < 0.6$ and for input value for $T_{\rho} = 110$ MeV. Self-consistency is achieved for $\alpha_{\rho}(0) = 0.57$ ⁶⁾.

3. - I = 0 $\pi\pi$ CHANNEL

For the $I = 0$ channel, assuming exchange degeneracy, we impose the same structure of $\text{Re}D(\ell, t)$ as before, for the description of the resonance pattern. Without destroying this degeneracy and the asymptotic behaviour with respect to t (see Chapter 2) we may introduce Pomeron effects if we multiply the ansatz (12) by a simple meromorphic term as follows.

$$\text{Re}D^{I=0}(\ell, t) = \frac{\Gamma(2\ell - \alpha_f(0) - \alpha_f(t))}{\Gamma(\ell - \alpha_f(t))} \frac{\ell - \alpha_p(t)}{\ell - 1 - \gamma t} \quad (19)$$

with $\alpha_p(t) = 1 + \alpha'_p t$ and $\alpha_f(t) = \alpha_f(t)$. This modification amounts to the introduction of a Pomeron-like moving pole α_p and a fixed cut at $\ell = 1$.

In this channel, the form (19) leads to two tachyons with $\ell = 0$, due to the moving poles α_f and α_p at $t = -\alpha_f(0)/\alpha'_f$ and $t = -1/\alpha'_p$ respectively. If one chooses properly the parameters α'_p and γ these ghost states can be eliminated. In fact, for

$$\alpha'_p = \frac{\alpha'_f}{2\alpha_f(0)} \quad , \quad \gamma = \frac{\alpha'_f}{\alpha_f(0)} \quad (20)$$

the coupling of these tachyons vanishes. We see that the Pomeron-like pole $\alpha_p(t)$ has approximately the same slope as the exchange-degenerate η - f trajectory ($\alpha_f(0) \simeq \frac{1}{2}$). The linearity of $\alpha_p(t)$ may be a good approximation for negative t whereas for positive t this form can only be accepted in the region $\alpha_p(t) < 2$ in order to avoid particle states on this trajectory. Using Eq. (19) we first investigate low energy ($t \lesssim 1(\text{GeV})^2$) $\pi\pi$ S wave scattering for $I = 0$. The N/D system for $\ell = 0$ takes the form :

$$N_0(t) = \frac{1}{2\pi} \int_{-\infty}^0 \frac{\text{disc}_t B_0^{I=0}(t')}{t' - t} \frac{\Gamma(1 - \alpha_f(0) - \alpha_f(t'))}{\Gamma(1 - \alpha_f(t'))} dt',$$

$$D_0(t) = \frac{1}{2} \frac{\Gamma(1 - \alpha_f(0) - \alpha_f(t))}{\Gamma(1 - \alpha_f(t))} - i \left(\frac{t-4}{t} \right)^{1/2} N_0(t) \theta(t-4). \quad (21)$$

The lowest CDD pole gives a subthreshold zero at $t = (1 - 2\alpha_f(0))/(\alpha_f') \simeq 0$ in the amplitude $A_0(t)$. This feature is in agreement with the results of various theoretical models ¹⁸⁾ (current algebra, dispersion relations) and in general it leads to a small scattering length a_0^0 . We perform a numerical evaluation of the phase shift $\delta_0^0(t)$ using as an input for the calculation of $\text{disc}_t B_0^{I=0}(t)$, $t < 0$ the following model :

- i) for the nearby singularities the phenomenological S and P wave phase shifts ¹⁵⁾ (with the down-up form of δ_0^0) are used ;
- ii) for the short range part the exchange of Veneziano resonances is used, and,
- iii) for large negative t ($t \rightarrow -\infty$) a constant value of $\text{disc}_t B_0^{I=0}$ is used consistently with our assumptions on the asymptotic behaviour (see Chapter 3).

The resulting phase shift is shown in Fig. 3, and it is compared with the input data ¹⁵⁾. The scattering length a_0^0 is found in the range $a_0^0 = 0.2 \div 0.4$ (m_π^{-1}) depending on the point where $\text{disc}_t B_0^{I=0}$ is assumed to take its asymptotic value. The sensitivity of a_0^0 reflects the well-known fact that the high mass Veneziano states give a large contribution to $\text{disc}_t B(t)$ for $I = 0$ ¹⁹⁾.

Next we investigate the complex structure of the leading poles f^0 and P for negative t , following the same procedure as in Chapter 2 and taking into account the additional branch point at $\ell = 1$.

The imaginary part of P, $\alpha_{IP}^{(1)}(t)$, for $t < 0$ due to the branch point at $\ell = 1$ is :

$$\alpha_{IP}^{(1)}(t) = R(t) \theta(1 - \alpha_P(t)) \text{disc}_t B^{I=0}(\alpha_P(t), \frac{t}{2}) .$$

$$\frac{t \alpha_P'(1 - \alpha_f(0)) \Gamma(2 - 2\alpha_f(0) + \alpha_P' t (2 - \alpha_f(0)))}{\Gamma(\alpha_P(t)(1 - \alpha_f(0)))} \int_4^\infty \frac{\rho_{\alpha_P(t)}(t') dt'}{(t' - t)(\alpha_P' t - \gamma t')} \quad (22)$$

and due to the branch point at $\ell = \alpha_f(0)$:

$$\alpha_{IP}^{(2)}(t) = R(t) \theta(\alpha_f(0) - \alpha_P(t)) \text{disc}_t B^{I=0}(\alpha_P(t), 2t) .$$

$$\frac{\alpha_P(t) - 1 - \frac{2\alpha_P'}{\alpha_f'}(\alpha_P(t) - \alpha_f(0))}{\Gamma(\alpha_f(0) - \alpha_P(t))(\alpha_P(t) - 1 - \frac{2\gamma}{\alpha_f'}(\alpha_P(t) - \alpha_f(0)))} \int_4^\infty \frac{\rho_{\alpha_P(t)}(t') dt'}{(t' - t)(2\alpha_P(t) - \alpha_f(0) - \alpha_f(t))} , \quad (23)$$

where

$$R(t) = - \frac{1}{\pi} \frac{(\alpha_P(t) - 1 - \gamma t) \Gamma(\alpha_P(t) - \alpha_f(t))}{\Gamma(2\alpha_P(t) - \alpha_f(0) - \alpha_f(t))} .$$

Similar contributions to α_{IP} come from the non-leading branch points for $\ell < \alpha_f(0) - \frac{1}{2}$. Analogous expressions are easily found for the f trajectory. We only remark that the f trajectory develops an imaginary part even for positive t because it collides with the leading cut at $t = m_\rho^2$. In Fig. 4, the imaginary parts of f and P are drawn for negative t. Their magnitude is smaller than the

magnitude of $\alpha_{I\rho}$ (ρ trajectory). The f^0 remains on the physical sheet for $-1.2 \leq t \leq 0$ (GeV/c)², and the Pomeron is practically a real pole on the physical sheet (very small and positive imaginary part) for $-0.6 < t < 0$ (GeV/c)². In Fig. 4 we observe that the imaginary part of the f trajectory vanishes at $t = 0$ and it is easy to show that both P and f decouple at $t = 0$ and therefore they do not contribute to the total cross-section. Hence, the moving pole $\alpha_p(t)$ appearing in this model cannot represent the basic mechanism for diffraction scattering, so that the cut at $\ell = 1$ dominates the high energy forward scattering and it leads to a decreasing cross-section like $(\ln s)^{-5/2}$. Nevertheless, the moving pole $\alpha_p(t)$ contributes outside the forward peak.

A detailed investigation of the diffractive component needs the isolation of the leading part of the discontinuity $\text{disc}_\ell B(\ell, t)$ of the amplitude $B(\ell, t)$ along the fixed cut, for $\ell \simeq 1$ and $t \simeq 0$. We find

$$\text{disc}_\ell B(\ell, t) \simeq \text{const } |\mathbb{F}(t)|^2 (1-\ell)^{3/2} \left[\mathbb{F}^{-1}(t) + \frac{1}{4\pi\gamma} \text{Re } N^{I=0}(\ell=1, t) \right] \quad (24)$$

where

$$\mathbb{F}(t) = \frac{\Gamma(2 - \alpha_f(0) - \alpha_f(t))}{\Gamma(1 - \alpha_f(t))} \quad (25)$$

After the Sommerfeld-Watson transformation, the asymptotic form of the diffraction amplitude $A_d(s, t)$, due to the singularity (24) is obtained :

$$A_d(s, t) \simeq \text{const } |\mathbb{F}(t)|^2 \frac{i s}{(\ln s)^{5/2}} \left[\mathbb{F}^{-1}(t) + \frac{1}{4\pi\gamma} \text{Re } N^{I=0}(\ell=1, t) \right] \quad (26)$$

for $s \rightarrow \infty$ and t small.

We observe that $A_d(s, t)$ is proportional to $|F(t)|^2$ where $F(t) \sim F_{\pi\pi}(t)$, the pion electromagnetic form factor suggested by Veneziano-like models²⁰⁾. The connection between diffraction and electromagnetic structure of the hadrons has been widely advertised recently, and in this model it is a result of exchange degeneracy combined with the existence of a fixed singularity at $\ell = 1$ in the vacuum channel^{21), 22)}. The over-all constant in the expression (26) is proportional to the combination of S wave $\pi\pi$ scattering lengths, $\text{const} \simeq (a_0^0)^2 + 5(a_0^2)^2$, and we can easily find that the total cross-section for a definite isospin in the s channel and for a typical energy $s = 40 \text{ (GeV)}^2$ is $\sigma_{\text{tot}} = 1 - 4 \text{ mb}$ for $0.2 < a_0^0 < 0.4 \text{ m}^{-1}$.

The shape of the diffraction peak resulting from the expression (26) with the same $\text{disc}_t B^{I=0}(\ell, t)$ as before, is given in Fig. 5. The main features of $d\sigma/dt$ come from the form factor $F(t)$, but breaks of the slope can be observed (a usual phenomenon in high energy elastic scattering) mainly due to the oscillatory behaviour of $\text{Re} N^{I=0}(\ell=1, t)$. It is a general feature of our model that a structure of the scattering amplitude for negative t is expected (breaks, dips, bumps) when ℓ plane singularities of the kind discussed above are dominant. The reason is that the parameters of these singularities (trajectories, residues, discontinuities) depend on the potential function $\text{disc}_t B^I(\ell, t)$ which in general oscillates in the low t region.

4. - CONCLUSION

We have shown that moving CDD poles provide a mechanism for generating fixed cuts in the angular momentum plane which are compatible with t channel unitarity and which are of square root type. As a result, the trajectories communicating with this channel become complex after colliding with these branch points. The imaginary part of the trajectory functions depends on the potential functions of the N/D equations continued in the ℓ plane.

We considered a particular model for the real part of trajectories in $\pi\pi$ scattering compatible with the daughter structure of dual models with linearly rising trajectories. In the $I = 0$ channel a Pomeron-like singularity has been inserted.

We studied the splitting of the leading trajectories using reasonable $\pi\pi$ potentials and it was found that the leading trajectories decouple at $t = 0$. Consequently, the forward scattering is dominated at high energies by the leading fixed branch points. Especially the diffraction amplitude is dominated by the branch point at $\ell = 1$ and it is found that the t dependence of $d\sigma/dt$ is mainly described by the electromagnetic form factor of the interacting particles (pions).

In addition, the low energy $\pi\pi$ scattering is investigated within this model in a consistent way.

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FIGURE CAPTIONS

Figure 1 Imaginary part of the ρ trajectory as a function of t for $\alpha_\rho(0) = 0.5$ and $\alpha'_\rho = 1/2m_\rho^2$ (curve a). Curve b gives the contribution due to the second branch point. Dashed curves show the imaginary parts as used in Ref. 16).

Figure 2 Modulus $|\text{Res}_\rho B|$ and phase φ_B of the residue of the amplitude $B^{I=1}(\ell, t)$ at the ρ Regge pole ($\alpha_\rho(0) = 0.5$, $\alpha'_\rho = 1/2m_\rho^2$).

Figure 3 $I = 0$ $\pi\pi$ S wave phase shift δ_0^0 . Curve a corresponds to cutting off the Veneziano resonances in $\text{disc}_t B^{I=0}$ at $t \simeq -4.2 (\text{GeV}/c)^2$, and curve b at $t \simeq -1.8 (\text{GeV}/c)^2$, respectively ($\alpha_\rho(0) = 0.5$, $\alpha'_\rho = 1/2m_\rho^2$). Experimental points correspond to the down-up solution from Ref. 15).

Figure 4 Imaginary parts of the Pomeron trajectory (full line) and of the f trajectory (dashed line). ($\alpha_\rho(0) = 0.5$, $\alpha'_\rho = 1/2m_\rho^2$).

Figure 5 $d\delta/dt$ versus t for high energies.

FIG.1

FIG.2

FIG. 3

FIG.4

FIG.5