CM-P00058189

Ref.TH.1164-CERN

BUILDING MESON BARYON SCATTERING AMPLITUDES FROM DUALITY

P.R. Auvil ^{*) +)}, F. Halzen ^{")} and C. Michael
CERN - Geneva

A B S T R A C T

We extend duality constraints to include the interrelation of the structure of the s channel spectrum with that of u and t channels. Predictions for t channel meson couplings are presented within an $SU(3)$ framework.

-
- ^{*)} Permanent address : Northwestern University, Evanston, Ill., U.S.A.
- ⁺⁾ Research supported in part by the AFOSR, USAF, under AFOSR Grant No 69-1761.
- ^{")} Aspirant NFWO, on leave of absence from the University of Leuven, Belgium.

Ref.TH.1164-CERN

21 May 1970

In meson-baryon scattering the problem of finding SU_3 representations for the s channel exchange degenerate Regge trajectories such that they do not imply exotic contributions in any channel has been solved ¹⁾. The simplest cases which accommodate the physically observed particles are : (i) $\underline{N}$ $1 + 8_1$ of $3/2^-$, $7/2^-$, ... exchange degenerate with 8_1 of $1/2^+$, $5/2^+$, ... and (ii) $\underline{\Delta}$ $8_{-1/3} + 10$ of $3/2^+$, $7/2^+$, ... exchange degenerate with $8_{-1/3}$ of $5/2^-$, $7/2^-$, ... The suffixes represent the F/D values of the octets into meson baryon. Generalizations of these two simple and separate degeneracies which include extra states are easily handled. For example, the F/D value of the $\underline{N}$ solution can be brought into agreement with the experimental F/D value by a small admixture of decuplet. Correspondingly, the simplest t channel spectrum which does not imply s and u channel exotics is $TV : 1+8$ of 2^+ , 4^+ , ... exchange degenerate with 8 of 1^- , 3^- , ... where the F/D values of these mesons into baryon-antibaryon are arbitrary ²⁾.

This spectrum does not reflect all duality restrictions on meson-baryon scattering. Since one can build the amplitude in either the s or the u channels which are formally the same, one has additional constraints. One approach is the eigenvector method where one ignores the dynamical part of the amplitude and considers only the SU_3 eigenvectors. There are three eigenvectors of the SU_3 , (s,u) crossing matrix which contain no exotic states ; V_1 and V_2 are crossing even and V_3 is crossing odd. This approach has been discussed by Lovelace ³⁾ and others ⁴⁾. For the (s,u) crossing even amplitude A they find additional restrictions ; a combination of the $\underline{N}$ and $\underline{\Delta}$ spectra is now a solution, provided these trajectories are degenerate. However, we know that the B amplitude must be odd under crossing. Lovelace noted that because there is only one odd eigenvector, V_3 , no satisfactory solution exists for this case. In fact it can be shown that, in order to get reasonable F/D values, the minimal s channel spectrum contains five unsignatured trajectories. As this does not seem to be physically acceptable, we consider the dynamical part of the amplitude as well.

It is difficult to impose the total input of duality plus crossing on the most general form for the amplitude. We approach the problem in a way suggested by the crossing structure of duality diagrams ⁵⁾ and consider a bivariate structure for the amplitude where (s,t) , (u,t) and (s,u) terms are added separately, where the (s,t) term has absorptive parts only in $\underline{s}$ and $\underline{t}$, etc. For example, such a construction is manifest in the Veneziano model ⁶⁾. In meson-baryon scattering crossing symmetry provides self-consistency equations for the (s,u) term.

With the bivariate approach one now elegantly avoids the unwanted eigenvector constraints. The most general form for the amplitude which has no absorptive parts in exotic channels, is, e.g., for the $(s-u)$ A term,

$$A = K_1^s(s,u) V_1 + K_2^s(s,u) V_2 + K_3^a(s,u) V_3$$

where the superscript indicates symmetry or antisymmetry under $s-u$ interchange. For B, K_1 and K_2 must be antisymmetric and K_3 symmetric. Note that the eigenvector method is equivalent to setting $K_3 = 0$ in A and $K_1 = K_2 = 0$ in B. Since we have only three independent functions but five independent SU_3 amplitudes, there exist two relations among these amplitudes for all $\underline{s}$ and $\underline{u}$. They can be compared to the particle spectrum either by passing to the Regge limit in which case we can relate the Regge residues among exchange degenerate trajectories or by performing a local average in $\underline{s}$ to relate resonance couplings. The restrictions on the spectrum are, of course, the same in both cases being identical to the usual duality results ¹⁾. Hence for the (s,u) term the crossing constraints are just a restriction on the symmetry of $K_{1,2,3}$. Thus, any choice of the functions with the right symmetry produces an amplitude satisfying the duality constraints on the spectrum ; of course, this must be made to match the physical spectrum.

Crossing symmetry does not directly impose additional restrictions on the (s,t) term. We simply have to rule out $\underline{s}$ and $\underline{t}$ channel exotics. We can express the most general amplitude as a sum of three SU_3 vectors times dynamical functions of $\underline{s}$ and $\underline{t}$ with, however, no particular symmetry under s - t exchange. Again by considering either the Regge limit or the local average, we recover the standard duality results ^{1),7a,b)}. However, we now have the possibility of relating the $\underline{s}$ and $\underline{t}$ channel spectrum by crossing (to the $\underline{t}$ channel) the (s,t) term, which has been constructed to match the signature and spectrum requirements from a suitable (s,u) term.

To gain some insight into the implications of these constraints on the physical amplitudes, we consider two simple examples which contain the $\underline{N}$ and the $\underline{\Delta}$ trajectories in the (s,u) term. Combining these with suitable (s,t) terms, we look at the predictions of $(f/d)_{\underline{T}}$, $(f/d)_{\underline{V}}$; and the singlet/octet ratio, $S_{\underline{V},\underline{T}}$, for the leading $\underline{t}$ channel trajectories. The first nice point is that the $\underline{N}$ and $\underline{\Delta}$ solutions in any relative combinations predict $(f/d)_{\underline{T}} = (f/d)_{\underline{V}}$. Adding BB factorization to the duality constraints would lead to the same equality. Rosner ^{7a)} has also given some discussion of the implications for the s channel spectrum when $(f/d)_{\underline{T}} = (f/d)_{\underline{V}}$. We shall review direct experimental evidence later.

First we consider the $\underline{N}$ and $\underline{\Delta}$ solutions from an $\underline{s}$ channel point of view where we obtain coupling relations of the resonances to meson baryon. Using the $3/2^{\pm}$, $5/2^{\pm}$, and $7/2^{\pm}$ states in πN as examples, we calculate from the widths the ratio of the coupling of the $\underline{N}$ and $\underline{\Delta}$ states at each J . This ratio is relatively constant for the A amplitude. For fixed $\underline{s}$ this involves the leading power of $\underline{t}$ in the (s,t) term. If we assume that these ratios do not change appreciably as we move to $t = 0$, we determine the $\underline{t}$ channel (f/d) ratio to lie between $+1/2$ and $-1/9$ over this range of $\underline{s}$.

Second we look for a specific model of the $\underline{N} - \underline{\Delta}$ bootstrap with Veneziano functions. In the spirit of the bootstrap approach we require a leading-leading type amplitude where direct poles are dual to leading trajectories, not daughters. For the $(s,u)A$ amplitude, a simple, essentially unique, solution exists which implies $(f/d) = +1/3$, $S_T = S_V \approx 0$. Note that $S_V = 0$ automatically since it does not couple in the problem. Also, since we satisfy the usual duality constraints, the ratio of the coupling of the vector and tensor octets is fixed and S_T is a function of the F/D values. We might add that the degenerate $\underline{N} - \underline{\Delta}$ solution mentioned earlier also requires these values. No leading-leading solution for B is possible; it is ruled out by the required symmetry of $K_{1,2,3}$. In B the lowest states are at spin $\frac{1}{2}$ for $\underline{N}$ and $\frac{3}{2}$ for $\underline{\Delta}$ which would also preclude an elegant leading-leading construction unlike A where both $\underline{N}$ and $\underline{\Delta}$ commence with spin $\frac{3}{2}$ states. Of course, a non leading-leading duality form for B exists but it is arbitrary. The suppression of parity doublets is accomplished by relations between the A and B amplitudes. These relations are not naturally accommodated in a pure pole dual model and we prefer to abandon such constraints concentrating solely on the A amplitude.

The empirical evidence for the vector and tensor meson exchange couplings to baryons is rather weak, but we summarize some remarks here.

- (i) - The electric, anomalous magnetic, and magnetic couplings of the baryons relate approximately to the A' , A and B amplitude respectively. This leads to $|f/d|_V \gg 1$ for A' and (in the $SU(6)_W$ model) to $(f/d)_V \sim 2/3$ for B ⁸⁾. The isoscalar anomalous magnetic moment of the nucleon is very small. This together with the known suppression of φ_{NN} leads, for A , to $\omega_{NN} = \varphi_{NN} \simeq 0$. Irrespective of any mixing between ω and φ , this implies that $(f/d)_V = 1/3$ and $S_V = 0$ in A .
- (ii) - Regge fits to total cross-sections⁹⁾ yield $(f/d)_T \simeq (f/d)_V \simeq -2$ in A' .

(iii) - The FESR (finite energy sum rules) results for πN ¹⁰⁾ agree with measurements of R and A parameters ¹¹⁾ and lead to the conclusion that $\gamma B/A' \simeq +1$ for P and f_0 Regge contributions. This means that A ($\simeq A' - \gamma B$ near $t = 0$) is suppressed and $f_{0NN} \simeq 0$. As we expect $f'_{NN} \simeq 0$, $(f/d)_T = 1/3$ and $S_T = 0$ are needed to arrange the decoupling from NN.

In conclusion it seems that the additional duality constraints imposed by crossing are in agreement with experiment at least within the context of these simple models. For the A amplitude we obtained $(f/d)_V = (f/d)_T \simeq 1/3$ and $S_V = S_T \simeq 0$ either from the baryon couplings, or from a bootstrap Veneziano formalism, or from N- Δ trajectory degeneracy. This is consistent with the results of Regge analysis supplemented by FESR.

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge fruitful discussions with Drs. C. Lovelace, C. Schmid and J. Weyers. We are grateful to Professors J. Prentki and W. Thirring for the hospitality extended to us in the Theoretical Study Division.

REFERENCES

- 1) R.H. Capps - Phys.Rev.Letters 22, 215 (1969) ;
J. Mandula, C. Rebbi, R. Slansky, J. Weyers and G. Zweig - Phys.
Rev.Letters 22, 1147 (1969) ;
V. Barger and C. Michael - Phys.Rev. 186, 1592 (1969).
- 2) J. Rosner - Phys.Rev.Letters 21, 950 (1968) ;
J. Rosner, C. Rebbi and R. Slansky - Phys.Rev. 188, 2367 (1970).
- 3) C. Lovelace - CERN Preprint TH 1123 (1970).
- 4) M. Rimpault and Ph. Salin - Bordeaux Preprint (1970) ;
R. Capps - "Bootstrap Conditions from the Veneziano Representation",
Purdue Preprint (1970).
- 5) H. Harari - Phys.Rev.Letters 22, 562 (1969) ;
J. Rosner - Phys.Rev.Letters 22, 689 (1969).
- 6) G. Veneziano - Nuovo Cimento 57A, 190 (1968).
- 7a) J. Rosner - Phys.Rev.Letters 24, 173 (1970).
- 7b) R.C. Arnold - Phys.Rev.Letters 14, 657 (1965) ;
C.B. Chiu and J. Finkelstein - Phys.Letters 27B, 510 (1968) ;
J. Rosner - Phys.Rev.Letters 21, 950 (1968) ;
H.J. Lipkin - Nuclear Phys. B9, 349 (1969) ;
M. Kugler - Phys.Rev. 180, 1538 (1969).
- 8) B. Sakita and K.C. Wali - Phys.Rev.Letters 14, 404 (1965).
- 9) See, e.g., : V. Barger and M. Olsson - Phys.Rev. 146, 1080 (1966).
- 10) V. Barger and R.J.N. Phillips - Phys.Rev. 187, 2210 (1969).
- 11) B. Amblard, G. Cozzila, Y. Ducros, M. Hannail, A. de Lesquen,
J.P. Merlo, J. Morchet and L. Van Rossum - Paper presented
at the Lund International Conference on Elementary Particles,
Lund (1969).