Spectrum of Chiral Operators in Strongly Coupled Gauge Theories

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We analyze the large $N$ spectrum of chiral primary operators of three dimensional fixed points of the renormalization group. Using the spacetime picture of the fixed points and the correspondence between anti-de Sitter compactifications and conformal field theories we are able to extract the dimensions of operators in short superconformal multiplets. We write down some of these operators in terms of short distance theories flowing to these non-trivial fixed points in the infrared.

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1. Introduction

Recently, new ways of analyzing conformal field theories have been proposed [1,2,3,4]. Certain conformal field theories are conjectured to be dual to certain space-time compactifications of string theory or M-theory containing anti-de Sitter geometry in the background [1]. Evidence for the conjecture follows from identifying the space-time symmetries with symmetries of the conformal field theory [1,5,2,6,7]. Moreover, in [3,4] a precise recipe was given for computing arbitrary correlation functions in the conformal field theory in terms of the dual space-time theory. The correlation function of operators in the conformal field theory can be computed by analyzing the supergravity action dependence on the boundary of the anti-de Sitter geometry. The correspondence between space-time states and operators in the conformal field theory can be used to test or predict results in conformal field theories. D-brane technology and string duality have recently been used to construct conformal field theories and their conjectured space-time duals [8 – 20,7]. Related work which has appeared recently is [21 – 51].

In [16,7], M-theory duals to three dimensional fixed points [52,53,54] of the renormalization group were given. These theories can be realized as world-volume theories of M2-branes sitting at an ADE singularity. These strongly coupled gauge theories are difficult to analyze and one might hope to learn more about them by using the AdS-CFT correspondence.

In this paper, we identify part of the spectrum of primary chiral operators of three dimensional superconformal field theories. This is done by using the M-theory description of the theory at the fixed points [16,7] and identifying the space-time fields in short supersymmetry multiplets. These fields are in one to one correspondence with chiral operators of the superconformal algebra.

In section two we briefly review the Kaluza Klein spectrum of eleven dimensional supergravity on $AdS_4 \times S^7$ [55,56]. In section three the space-time duals [16,7] of the non-trivial fixed points of the renormalization group are introduced. In this section we analyze the states that survive the orbifold projection and identify the subset of states with the right quantum numbers to be in reduced supersymmetry multiplets. In section four dual short distance theories, related by mirror symmetry [52,53,54], which flow to these fixed points in the infrared are introduced. We give candidate chiral operators in the superconformal field theory using fields in the short distance theories. Section five contains conclusions.
2. Kaluza-Klein Spectrum of Supergravity on \( AdS_4 \times S^7 \)

Kaluza-Klein harmonics in supergravity play a pivotal role in establishing the AdS-CFT correspondence [4]. They belong to small representations of the supersymmetry algebra in the maximally supersymmetric case [57,58]. As such, many of their properties are protected from quantum and stringy corrections. In particular their masses are given by the eigenvalues of the relevant Laplace operator. Therefore, operators in the superconformal field theory which couple to them will be chiral since their dimensions will be protected from quantum corrections [4]. Chiral operators are in small representations of the superconformal algebra (see [59,60] for details). Their dimensions saturate an inequality of the form

\[
D(\mathcal{O}) \geq f(\{L_i\}, \{R_i\}),
\]

where \(\{L_i\}, \{R_i\}\) are the Lorentz and R-symmetry quantum numbers. The function \(f\) is determined by requiring the representation of the superconformal algebra carried by these operators to be unitary.

One can exploit the AdS-CFT correspondence to verify or predict the spectrum of chiral operators of a given superconformal field theory [4,12,13,14,15,17]. In this section we will review the Kaluza-Klein spectrum of supergravity in \( AdS_4 \times S^7 \) [55,56] which we will use in the following sections to determine the spectrum of chiral operators of three dimensional strongly coupled gauge theories. In [12,15] this spectrum was used to determine the chiral operators of an \( \mathcal{N} = 8 \) three dimensional superconformal field theory.

The spectrum consists of three families of scalar excitations, two families of pseudoscalar excitations, two families of vector excitations, one family of axial vectors and one family of spin 2 excitations. A given Kaluza Klein excitation transforms in a particular representation of the \( SO(8) \) isometry group of \( S^7 \) and carries the Lorentz quantum numbers of the family to which it belongs. The mass\(^1\) of a given space-time field determines the renormalization group flow properties of the corresponding operator in the conformal field theory [4]. The dimension of the operator that couples to a space-time \( p \)-form is given by \((\Delta + p)(\Delta + p - d) = m^2\), where \(d\) is the number of dimensions in which the conformal field theory lives. Thus tachyonic, massless and massive states couple to relevant, marginal and irrelevant operators respectively [4]. In what follows, we shall only consider those families which contain tachyonic and massless modes corresponding to relevant and marginal operators. These are listed below:

\(^1\) The mass squared of a field is the eigenvalue of the relevant differential operator.
- **Scalar:**
  \[ m^2 = \frac{1}{4} k(k - 6), \quad k \geq 2. \]
  These states transform in the \( k \)-th symmetric traceless representation of \( SO(8) \); their Dynkin labels are \((k, 0, 0, 0)\). The dimensions of the corresponding chiral operators in the CFT are \( \Delta = \frac{k}{2} \).

- **Pseudo-scalar:**
  \[ m^2 = \frac{1}{4} ((k - 1)(k + 1) - 8), \quad k \geq 1. \]
  These states transform in the product representation of the \( 35_c \) with the \( k - 1 \)-th symmetric traceless representation of \( SO(8) \); their Dynkin labels are \((k - 1, 0, 2, 0)\). The dimensions of the corresponding chiral operators in the CFT are \( \Delta = \frac{k + 3}{2} \).

- **Vector**
  \[ m^2 = \frac{1}{4} (k^2 - 1), \quad k \geq 1. \]
  These states transform in the product representation of the \( 28 \) with the \( k - 1 \)-th symmetric traceless representation of \( SO(8) \); their Dynkin labels are \((k - 1, 1, 0, 0)\). The dimensions of the corresponding chiral operators in the CFT are \( \Delta = \frac{k + 3}{2} \).

- **Graviton**
  \[ m^2 = \frac{1}{4} k(k + 6), \quad k \geq 0. \]
  These states transform in the \( k \)-th symmetric traceless representation of \( SO(8) \); their Dynkin labels are \((k, 0, 0, 0)\). The dimensions of the corresponding chiral operators in the CFT are \( \Delta = \frac{k + 6}{2} \).

States in different Kaluza Klein towers can be combined in supermultiplets by the action of the supersymmetry generators. The operators in the first scalar family are the only \( \mathcal{N} = 8 \) superconformal primaries [61].

### 3. States in reduced supersymmetry multiplets

In [16,7] supergravity duals of \( \mathcal{N} = 4 \) three dimensional fixed point theories with ADE global symmetries were given. These theories have dual short distance descriptions related by three dimensional mirror symmetry [52,53,54]. They can be realized as world volume theories of brane configurations in space-time. Taking the limit as in [1] of the supergravity description yields the space-time theory dual to the fixed point. The space-time compactification can be read from the near horizon geometry of the brane configuration.

The brane configuration corresponding to the fixed point is the theory on \( N \) M2-branes at a \( C^2/\Gamma \) singularity\(^2\). The near horizon geometry of this brane configuration is

\(^2\) The discrete group \( \Gamma \in SU(2) \) is one of the ADE discrete groups.
$AdS_4 \times S^3 \times f D_4/\Gamma$, where the $S^3$ is fibered over the $\Gamma$ quotiented four-disk. Therefore, the theory dual to the fixed point is M-theory on $AdS_4 \times S^3 \times f D_4/\Gamma$ \cite{7}. We would like to emphasize the need to include all the degrees of freedom of M-theory and not just the eleven dimensional supergravity ones in order to regulate the singular geometry at the origin of the disk. In \cite{7} all the symmetries of the fixed point were identified with symmetries in space-time, including the ADE global symmetries.

The procedure for finding the chiral spectrum is analogous to that of \cite{17}. We know that the space-time realization of the fixed point is the theory of $N$ M2-branes sitting at an ADE singularity. The presence of the ADE singularity breaks the transverse $SO(8)$ global symmetry to $SO(4)_{ADE} \times SO(4)_E$. Furthermore, the discrete group $\Gamma$ acts in $SU(2)_\Gamma \subset SO(4)_{ADE}$, which breaks the $SO(8)$ isometry group of $S^7$ to $SU(2)_{ADE} \times SU(2)_L \times SU(2)_R$. The unbroken supersymmetry charges of the theory on the M2-branes transform in a $(2,1,2)$ representation of $SU(2)_{ADE} \times SU(2)_L \times SU(2)_R$. Therefore, we identify $SU(2)_{ADE} \times SU(2)_R$ as the R-symmetry of the three dimensional fixed point theory. The $SU(2)_L$ symmetry acts as an additional global symmetry group and will be kept for most of the analysis.

The geometry of the configuration above is singular, since the action of $\Gamma$ leaves an invariant $S^3$ at the origin. However, we assume that the Kaluza-Klein modes invariant under this action will correspond to chiral operators in the field theory. Therefore, one must decompose the Kaluza Klein $SO(8)$ representations of $AdS_4 \times S^7$ under $SU(2)_\Gamma \times SU(2)_{ADE} \times SU(2)_L \times SU(2)_R$, and keep only those states in $SU(2)_\Gamma$ representations which have neutral components under the action of $\Gamma$. As will be elaborated later on, only a subset of the states surviving the projection will have the right quantum numbers to be chiral operators of the superconformal field theory.

The branching rules, $SO(8) \to SU(2)_\Gamma \times SU(2)_{ADE} \times SU(2)_L \times SU(2)_R$, for the $SO(8)$ representations corresponding to relevant and marginal operators are given in the appendix\textsuperscript{3}. Let us consider the case $\Gamma = A_{k-1} = Z_k$ which acts in the 2 of $SU(2)_\Gamma$ as

$$Z^1 \to \alpha Z^1,$$

$$Z^2 \to \alpha^{-1} Z^2,$$  \hspace{1cm} (3.1)

where $\alpha = e^{\frac{2\pi i}{k}}$. The $SU(2)_\Gamma$ representations which have an invariant component under the $Z_k$ action\textsuperscript{4} are those which are a product of an even number of 2's. Therefore we will

\textsuperscript{3} See [62].

\textsuperscript{4} Performing the same analysis for D and E discrete groups will just further restrict the allowed representations.
keep only the odd dimensional representations of $SU(2)_R$ that appear in the branching rules.

The first family of scalar excitations in the previous section has five Kaluza-Klein modes which are either tachyonic or massless. The states that survive the projection are\footnote{The triplet $(a, b, c)$ corresponds to an $SU(2)_{ADE} \times SU(2)_L \times SU(2)_R$ representation.}

- $k = 2 \quad m^2 = -2$
  $(3, 1, 1) \oplus (1, 3, 3) \oplus (1, 1, 1)$
  These states couple to a $\Delta = 2$ operator of the fixed point theory.
- $k = 3 \quad m^2 = -\frac{9}{4}$
  $(3, 2, 2) \oplus (1, 4, 4) \oplus (1, 2, 2)$
  These states couple to a $\Delta = 3$ operator of the fixed point theory.
- $k = 4 \quad m^2 = -2$
  $(5, 1, 1) \oplus (3, 3, 3) \oplus (1, 5, 5) \oplus (3, 1, 1) \oplus (1, 3, 3) \oplus (1, 1, 1)$
  These states couple to a $\Delta = 2$ operator of the fixed point theory.
- $k = 5 \quad m^2 = -\frac{5}{4}$
  $(5, 2, 2) \oplus (3, 4, 4) \oplus (1, 6, 6) \oplus (3, 2, 2) \oplus (1, 4, 4) \oplus (1, 2, 2)$
  These states couple to a $\Delta = \frac{5}{2}$ operator of the fixed point theory.
- $k = 6 \quad m^2 = 0$
  $(7, 1, 1) \oplus (5, 3, 3) \oplus (3, 5, 5) \oplus (1, 7, 7) \oplus (5, 1, 1) \oplus (3, 3, 3) \oplus (1, 5, 5) \oplus (3, 1, 1) \oplus (1, 3, 3) \oplus (1, 1, 1)$
  These states couple to a $\Delta = 3$ operator of the fixed point theory.

The second family of pseudoscalar excitations has three Kaluza-Klein modes which are tachyonic or massless. The states that survive the projection are

- $k = 1 \quad m^2 = -2$
  $(3, 3, 1) \oplus (1, 1, 3) \oplus (1, 1, 1)$
  These states couple to a $\Delta = 2$ operator of the fixed point theory.
- $k = 2 \quad m^2 = -\frac{5}{4}$
  $(3, 4, 2) \oplus (3, 2, 2) \oplus (1, 2, 4) \oplus (3, 2, 2) \oplus (1, 2, 2) \oplus (1, 2, 2)$
  These states couple to a $\Delta = \frac{5}{2}$ operator of the fixed point theory.
- $k = 3 \quad m^2 = 0$
  $(5, 3, 1) \oplus (3, 5, 3) \oplus (3, 1, 3) \oplus (3, 3, 3) \oplus (1, 3, 4) \oplus (3, 1, 3) \oplus (3, 3, 3) \oplus (3, 1, 3) \oplus (3, 3, 1) \oplus (1, 3, 3) \oplus (1, 3, 3) \oplus (1, 1, 1)$
  These states couple to a $\Delta = 3$ operator of the fixed point theory.
The massless vector excitation is

\[ k = 1 \quad m^2 = 0 \]

\[ (3,1,1) \oplus (1,1,3) \oplus (1,3,1) \oplus (1,1,1) \]

The gauge boson couples to a \( \Delta = 2 \) global symmetry current operator of the fixed point theory in the adjoint of \( SO(4) \).

The massless graviton excitation is

\[ k = 0 \quad m^2 = 0 \]

\[ (1,1,1) \]

The graviton couples to the \( \Delta = 3 \) energy-momentum tensor of the fixed point theory.

As mentioned earlier the dimensions of superconformal chiral operators are uniquely determined by their Lorentz and R-symmetry quantum numbers. Therefore, only a subset of the allowed projected states will have the correct quantum numbers to be in reduced supersymmetry multiplets. The relation between the dimension of chiral operators and quantum numbers can be obtained by analyzing the allowed unitary representations of the superconformal algebra. The theories we are considering are three dimensional superconformal field theories with \( SO(4) \) R-symmetry. The scaling dimensions of primary chiral fields are determined in terms of the Lorentz \( Spin(3) \simeq SU(2) \) quantum number \( j \), and the \( SO(4) \simeq SU(2)_\text{ALE} \times SU(2)_R \) highest weights \(^6 (h_1,h_2) \) as \([59,60] \)

\[ D(O) = h_1 + h_2 \quad (j = 0) \]

\[ D(O) = 1 + j + h_1 + h_2 \quad (j \neq 0). \]

Using (3.2), the superconformal primaries from the scalar family are given by

\[ k = 2 \quad \Delta = 1 \quad (3,1,1) \oplus (1,3,3) \]

\[ k = 3 \quad \Delta = \frac{3}{2} \quad (3,2,2) \oplus (1,4,4) \]

\[ k = 4 \quad \Delta = 2 \quad (3,3,3) \]

\[ k = 5 \quad \Delta = \frac{5}{2} \quad (5,2,2) \oplus (3,4,4) \]

\[ k = 6 \quad \Delta = 3 \quad (5,3,3) \oplus (3,5,5). \]

None of the surviving states from the pseudoscalar tower have the right quantum numbers to be \( \mathcal{N} = 4 \) superconformal primaries. The massless vector and the graviton correspond to conserved currents in the superconformal field theory. They couple to the global symmetry current in the adjoint of the R-symmetry and to the energy momentum respectively. Since they are conserved currents, their dimensions are protected from quantum corrections.

\(^6 \) Our conventions for \( h_i \ i = 1,2 \) are that the dimension \( d \) of an \( SU(2) \) representation is \( d = 2h_i + 1 \).
4. Superconformal primary operators

In the previous section we have obtained the spectrum of states in reduced supersymmetry multiplets. In this section we write some of the corresponding chiral operators in terms of fields in short distance theories. Each fixed point has two short distance descriptions, related by three dimensional mirror symmetry [52,53,54], which flow to the fixed point in the infrared. As we shall see, some of the operators can be realized in both short distance theories, some in only one of the mirror pairs and some of the operators cannot be written in terms of short distance degrees of freedom.

The short distance theories have a moduli space of vacua with a Higgs and a Coulomb branch which intersect at the origin. There the theories are at a non-trivial fixed point of the renormalization group with ADE global symmetry. In the $Z_k$ case we are considering, the fixed point theory has a global $SU(k)$ symmetry. The two mirror theories are:

- The theory of $N$ D2-branes at a $C^2/Z_k$ orbifold singularity. These are the quiver gauge theories considered in [63,64,65]. The gauge group is $G = \left( \prod_{i=1}^{k} U(N) \right) / U(1)$ with $k$ bifundamental hypermultiplets. In $\mathcal{N} = 1$ language, there are $2k$ chiral superfields $Q_i, \tilde{Q}^\dagger_i$ both transforming in the $N_i \times \bar{N}_{i+1}$ of $U(N)_i \times U(N)_{i+1}$.  

- The theory of $N$ D2-branes probing $k$ D6-branes in Type I’ orientifold $T^3/Z_2\Omega$ [52]. This is a $U(N)$ gauge theory with $k$ hypermultiplets in the fundamental representation and one hypermultiplet in the adjoint. In $\mathcal{N} = 1$ language, there are $2k$ chiral superfields $q_i, \tilde{q}^\dagger_i$ transforming in the $N$ of $U(N)$.

These three dimensional $\mathcal{N} = 4$ gauge theories have an $SO(4) \simeq SU(2)_1 \times SU(2)_2$ global R-symmetry. The massless multiplets transform in some representation of the R-symmetry which can be inferred by dimensional reduction of $d = 6, \mathcal{N} = 1$ theories. The scalars in the vector multiplet transform in the $(3,1)$. Upon dualizing the photon along the Coulomb branch one gets an additional scalar transforming in the $(1,1)$. Therefore along the Coulomb branch the field content is that of a hypermultiplet with scalars transforming in the $(3 \oplus 1, 1)$. Both scalars in the usual hypermultiplet transform as a $(1,2)$.

Mirror symmetry acts by exchanging the Coulomb (Higgs) branch of one theory with the Higgs (Coulomb) branch of the other. Hence, under this symmetry the scalars in the vector (hyper) multiplet of one theory will transform in the $SU(2)$ factor of the R-symmetry in which the scalars in the hyper (vector) multiplets of the other theory transform. Once

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7 The index $i$ is cyclic.
the R-symmetry transformation properties of multiplets in one theory are determined, those of the mirror theory multiplets are also known.

In section three we identified $SU(2)_{ADE} \times SU(2)_R$ as the R-symmetry of the theory at the fixed point. In the quiver construction [63,64,65] the hypermultiplets are naturally charged under the unbroken $SU(2)_{ADE}$. Therefore, the fields in the quiver theory transform as

$$\Phi_i^A \oplus \Phi_i \to (1, 3 \oplus 1), \quad A = 1, 2, 3$$
$$Q_i, \tilde{Q}_i \to (2, 1),$$

where $\Phi_i^A$ are the vector multiplet scalars and $\Phi_i$ is the dualized photon. As explained above this implies that the mirror theory fields transform as

$$\phi^A \oplus \phi \to (3 \oplus 1, 1), \quad A = 1, 2, 3$$
$$q_i, \tilde{q}_i \to (1, 2),$$

where $\phi^A, \phi$ are the vector multiplet scalars and the dual photon.

We want to write down gauge invariant operators corresponding to the supergravity states in small supersymmetry multiplets (3.3). A careful analysis shows that not all of them can be written in terms of the short distance fields. Concretely, while one can always write combination with the right dimension and R-symmetry charges in at least one of the mirror theories, it is not always possible to do it in a gauge invariant way.

As an example, consider the $\Delta = \frac{3}{2}$ operator appearing in (3.3), with R-symmetry numbers $(1, 4)^8$. To write this operator in the quiver theory, one has to fully symmetrize three hypermultiplet scalar fields and this cannot be made in a gauge invariant way. Obviously, this is impossible in the probe theory as well. The same conclusion holds for all operators in the second and fourth lines of (3.3).

The operators which can be written in one or both mirror pairs are given by:

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8 Recall that the first and the third entries in each triplet are the R-symmetry quantum numbers.
• Quiver theory

\[ \Delta = 1 \quad \left(3, 1\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^A \left( Q_i^\alpha \tilde{Q}_i^\beta \right) \]

\[ \Delta = 2 \quad \left(3, 3\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^A \left( \tilde{Q}_i^\alpha \Phi_{i+1}^B Q_i^\beta - \tilde{Q}_{i+1}^\alpha \Phi_{i+1}^B Q_{i+1}^\beta \right) \]

\[ \Delta = 3 \quad \left(3, 5\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^A \left( \tilde{Q}_i^\alpha \Phi_{i+1}^B (\Phi^C_{i+1} Q_i^\beta - \tilde{Q}_{i+1}^\alpha \Phi_{i+1}^B \Phi^C_{i+1} Q_{i+1}^\beta) \right) \]

\[ \Delta = 3 \quad \left(5, 3\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^{(A \sigma^C)} \left( \tilde{Q}_i^\alpha \Phi_{i+1}^B \Phi^C_{i+1} Q_i^\beta - \tilde{Q}_{i+1}^\alpha \Phi_{i+1}^B \Phi^C_{i+1} Q_{i+1}^\beta \right) \]

• Probe theory

\[ \Delta = 1 \quad \left(1, 3\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^A \left( q_i^\alpha \tilde{q}_i^\beta \right) \]

\[ \Delta = 2 \quad \left(3, 3\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^A \left( \tilde{q}_i^\alpha \phi^B q_i^\beta \right) \]

\[ \Delta = 3 \quad \left(3, 5\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^A \left( \tilde{q}_i^\alpha \phi^B (\phi^C q_i^\beta) \right) \]

\[ \Delta = 3 \quad \left(5, 3\right) \quad \sum_{i=1}^{k} \sigma_{\alpha \beta}^{(A \sigma^C)} \left( \tilde{q}_i^\alpha \phi^B \tilde{q}_i^\beta \tilde{q}_i^\gamma \right) \]

In the above expressions we used the Pauli matrices \( \sigma^A \) for both \( SU(2)_{ADE} \) and \( SU(2)_R \). The gauge and \( SU(2)_L \) indices are suppressed.

5. Conclusions

Maldacena’s [1] conjecture of the AdS-CFT correspondence opens a new avenue of understanding for non-trivial conformal field theories. By performing rather simple calculations in supergravity one can infer information about field theories that might not be easily obtainable otherwise. In this paper we find part of the spectrum of chiral operators of three dimensional superconformal field theories. This is done by using the space-time picture of the conformal field theory. Analyzing which supergravity states are in short supersymmetry multiplets yields the spectrum of chiral operators of the three dimensional theories. Furthermore, we are able to realize some of these operators in terms of short distance theories which flow to these superconformal fixed points in the infrared.
Acknowledgments

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Appendix A. SO(8) branching rules

<table>
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<th>$SO(8)$</th>
<th>$SU(2)^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>scalar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m^2 = -2$</td>
<td>$(2,0,0,0)$</td>
<td>$(3,3,1,1) \oplus (2,2,2,2) \oplus (1,1,3,3) \oplus (1,1,1,1)$</td>
</tr>
<tr>
<td></td>
<td>$35_v$</td>
<td>$(4,4,1,1) \oplus (3,3,2,2) \oplus (2,2,3,3) \oplus (1,1,4,4)\oplus$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2,2,1,1) \oplus (1,1,2,2)$</td>
</tr>
<tr>
<td>$m^2 = -\frac{9}{4}$</td>
<td>$(3,0,0,0)$</td>
<td>$(5,5,1,1) \oplus (4,4,2,2) \oplus (3,3,3,3) \oplus (2,2,4,4)\oplus$</td>
</tr>
<tr>
<td></td>
<td>$112_v$</td>
<td>$(1,1,5,5) \oplus (3,3,1,1) \oplus (2,2,2,2) \oplus (1,1,3,3)\oplus$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(1,1,1,1)$</td>
</tr>
<tr>
<td>$m^2 = -2$</td>
<td>$(4,0,0,0)$</td>
<td>$(6,6,1,1) \oplus (5,5,2,2) \oplus (4,4,3,3) \oplus (3,3,4,4)\oplus$</td>
</tr>
<tr>
<td></td>
<td>$294_v$</td>
<td>$(2,2,5,5) \oplus (1,1,6,6) \oplus (4,4,1,1) \oplus (3,3,2,2)\oplus$</td>
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<tr>
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<td></td>
<td>$(2,2,3,3) \oplus (1,1,4,4) \oplus (2,2,1,1) \oplus (1,1,2,2)$</td>
</tr>
<tr>
<td>$m^2 = -\frac{5}{4}$</td>
<td>$(5,0,0,0)$</td>
<td>$(7,7,1,1) \oplus (6,6,2,2) \oplus (5,5,3,3) \oplus (4,4,4,4)\oplus$</td>
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<td>$672_v'$</td>
<td>$(3,3,5,5) \oplus (2,2,6,6) \oplus (1,1,7,7) \oplus (5,5,1,1)\oplus$</td>
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<td>$(4,4,2,2) \oplus (3,3,3,3) \oplus (2,2,4,4) \oplus (1,1,5,5)\oplus$</td>
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<tr>
<td></td>
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<td>$(3,3,1,1) \oplus (2,2,2,2) \oplus (1,1,3,3) \oplus (1,1,1,1)$</td>
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<tr>
<td>pseudoscalar</td>
<td></td>
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</tr>
<tr>
<td>$m^2 = -2$</td>
<td>$(0,0,2,0)$</td>
<td>$(1,3,3,1) \oplus (2,2,2,2) \oplus (3,1,1,3) \oplus (1,1,1,1)$</td>
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<tr>
<td></td>
<td>$35_c$</td>
<td>$(2,4,3,1) \oplus (1,3,4,2) \oplus (3,3,2,2) \oplus (4,2,1,3)\oplus$</td>
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<td></td>
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<td>$(2,2,3,1) \oplus (3,1,2,2) \oplus (2,2,1,1) \oplus (1,1,2,2)$</td>
</tr>
<tr>
<td>$m^2 = -\frac{5}{4}$</td>
<td>$(1,0,2,0)$</td>
<td>$(3,5,3,1) \oplus (2,4,4,2) \oplus (4,4,2,2) \oplus (1,3,5,3)\oplus$</td>
</tr>
<tr>
<td></td>
<td>$224_{cv}$</td>
<td>$(5,3,1,3) \oplus (3,3,3,3) \oplus (4,2,2,4) \oplus (2,2,4,4)\oplus$</td>
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<td></td>
<td>$(3,1,3,5) \oplus (2,4,2,2) \oplus (3,3,1,3) \oplus (1,3,3,3)\oplus$</td>
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<tr>
<td></td>
<td></td>
<td>$(3,3,3,1) \oplus (2,2,2,4) \oplus (4,2,2,2) \oplus (2,2,4,2)\oplus$</td>
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<td></td>
<td></td>
<td>$(3,1,3,3) \oplus (1,3,1,3) \oplus (3,3,1,1) \oplus (1,3,3,1)\oplus$</td>
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<td>$(2,2,2,2) \oplus (2,2,2,2) \oplus (3,1,3,1) \oplus (3,1,1,3)\oplus$</td>
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<tr>
<td></td>
<td></td>
<td>$(1,1,3,3) \oplus (1,1,1,1)$</td>
</tr>
<tr>
<td>vector</td>
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<td></td>
</tr>
<tr>
<td>$m^2 = 0$</td>
<td>$(2,0,2,0)$</td>
<td>$(2,2,2,2) \oplus (1,3,1,1) \oplus (1,1,1,3) \oplus (1,1,3,1)\oplus$</td>
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<td>$840_s$</td>
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<tr>
<td>$m^2 = 0$</td>
<td>$(0,1,0,0)$</td>
<td>$(2,2,2,2) \oplus (1,3,1,1) \oplus (1,1,1,3) \oplus (1,1,3,1)\oplus$</td>
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<td></td>
<td>$28$</td>
<td>$(3,1,1,1)$</td>
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11
References

R. Kallosh, J. Kumar and A. Rajaraman, “Special Conformal Symmetry of World Volume Actions”, hep-th/9712073,


