NUMBERS AND SPECTRUM OF MUONS FROM PROTON-INDUCED EXTRANUCLEAR CASCADES IN SHIELDS

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1. INTRODUCTION

This study is an attempt to produce a mathematical model for the build-up and decay of cascades produced in shields by very high-energy incident particles, by separating (as far as possible) the various quantities of interest, e.g. the energy of the incident beam, the shielding material, etc.

2. SIMPLIFICATIONS IN THE MODEL

There are two basic phenomena which determine the cascade: absorption of the hadrons in the material, and production of secondary particles by such interactions.

2.1 Absorption of the hadrons

As many other authors do, we will assume that hadron absorption in the cascade is due entirely to nuclear interaction, and we will neglect less significant effects of Coulomb absorption, and elastic and inelastic scattering not leading to the production of new particles, meson decay, etc. These items could probably be introduced as a correction at a later stage. The nuclear interaction cross-section $\sigma$ will be taken to be equal to the total inelastic cross-section, which is not very different from the geometrical one, and is a known function of the mass number $A$ of the target material. It is also known to be independent of energy, and even of the kind of incident particle (hadron) above some threshold energy.

With $N_0$ denoting Avogadro's number, the mean free path in g.cm$^{-2}$ is found to be $A/N_0\sigma$, and since $\sigma$ varies roughly as $A^{\frac{2}{3}}$, this mean free path varies as $A^{\frac{1}{3}}$. The relaxation length $\lambda$ is thus this mean free path divided by the density $\rho$:

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\[ \lambda = \frac{A}{N_0 \sigma} \text{ cm}. \]

Figures 1 and 2 are plots of \( \sigma(A) \) at very high energies and of the \( \lambda \sigma \) values derived from the \( \sigma \)'s. It is apparent that protons, neutrons, and pions of either sign have roughly the same total inelastic cross-section.

2.2 **Secondary production**

We define a secondary production function \( F(E_K, E) \) by the following: one incident particle of energy \( E_K \) produces \( F(E_K, E) \) dE secondaries with energies in the range \( E, E+dE \), from one interaction. At first sight it would appear that the form of the production function would vary according to the type of incident particle and also to the type of secondaries produced. However, in the energy range of interest to us, we have already seen that all particles behave in the same way as regards nuclear absorption, and thus it seems reasonable to assume, in a first approximation, that all particles produce a similar spectrum. This is equivalent to the case of a cascade made up entirely of one type of hadron, and we shall see that using this assumption we can derive several very interesting and general formulae.

We assume that there exists some low-energy \( \eta \) below which no secondaries are produced. In practice, this is the energy at which charged-particle stopping becomes so important that the particles no longer have nuclear interactions (remembering that the majority of the hadrons in the cascade are charged ones). We will assume that this cut-off is sharp, and will be taken care of in the following by a proper choice of the lower integration limit.

2.3 **Summary**

Although the simplifications made in our model may appear drastic, recent investigation by O'Brien has shown that results obtained from these assumptions are in good agreement both with experimental data and Monte-Carlo calculations at high and low energies.
3. NUMBERS AND ENERGY SPECTRA OF THE VARIOUS GENERATIONS OF SECONDARIES

3.1 Generations in the cascade

We first introduce the idea of generations of particles. When an incident particle has an interaction in the shield, it will produce secondary particles which have an energy distribution given by the production function discussed in Section 2.2. These secondary particles are said to be in the 'first generation'. Any such particle, with an energy greater than the cut-off energy $\eta$, will produce particles of the 'second generation' when it interacts with a nucleus. In general, the $n^{th}$ generation is produced from interactions of the $(n-1)^{st}$. Thus at any given time, we can distinguish particles by the generation to which they belong. That this is independent of the particle's position in space is clearly seen when we remember that these particles only loose energy when they have a nuclear interaction. Further, the probability that a particle will reach a certain position in the shield depends only on its origin and on the cross-section, and this, as we have already seen, is independent of energy; thus this probability is the same for every particle.

3.2 Numbers and energy spectra

In all the following calculations, we will assume that the cascade is generated by a single incident particle of energy $E_0$.

When this particle interacts in the shield it produces $F(E_0, E) \, dE$ particles in the energy range $E, E+dE$. If we now integrate this function over all permissible energies, we will obtain the number of particles in the first generation at, as we shall call it, the first population $\Pi_1$. Thus

$$\Pi_1 = \int_{E=\eta}^{E_0} F(E_0, E) \, dE.$$  \hspace{1cm} (1)

Note that as we have defined it, $\Pi_1$ is the number of particles in the first generation which are capable of having cascade interactions, since all have an energy greater than $\eta$. We also assume here that no
secondary particle is emitted with an energy greater than that of the
incident particle, this being a consequence of kinematics, as any energy
which could be supplied by the nucleus is clearly negligible at the scale
considered. We also define the multiplicity of particles produced from
a nuclear interaction by the ratio of the successive populations;
obviously:

\[ M_1 = \Pi_1 \]  \hspace{1cm} (2)

where \( M_1 \) is the first multiplicity. The others are defined from the
formula:

\[ \Pi_{n+1} = M_{n+1} \cdot \Pi_n = M_{n+1} \cdot M_n \cdot \Pi_{n-1} = M_{n+1} \cdot M_n \ldots M_2 \cdot M_1 \]  \hspace{1cm} (3)

Note that only \( M_1 \) is a true multiplicity, since it is the number of par-
ticles produced from a single incident particle; the others are averaged
over each population.

In order to calculate these quantities, we must first know the
energy spectrum of each population; thus we introduce, corresponding
to the \( n \)th generation, an energy distribution function \( f_n(E_0, E) \) which
will naturally depend on the initial energy of the incident particle
and the secondary production function \( F(E_K, E) \), \( E_K \) being the energy of
the incident particle in the previous generation. This is defined in
a way that is similar to the secondary production function, i.e.
\( f_n(E_0, E) \, dE \) is the number of particles of the \( n \)th generation in the
energy range \( E, E + dE \) produced by the single incident particle of energy
\( E_0 \). It is therefore obvious that

\[ f_1(E_0, E) = F(E_0, E) \]  \hspace{1cm} (4)

In order to calculate the second spectrum, we use the fact that each
particle of the first generation of energy \( E_K \) is a source of \( F(E_K, E) \, dE \)
particles in the energy range \( E, E + dE \) of the second generation. By inte-
grating over all energies \( E_K \), we obtain the function \( f_1 \), the energy
spectrum of the second generation, i.e.

\[ f_2(E_0, E) = \int_{E_K=E}^{E_0} f_1(E_0, E) \cdot F(E_K, E) \, dE_K \]
The lower limit is \( E \), as no contribution can come from incident particles of energy lower than \( E \). From this it is easy to see that the spectrum of the \( n^{\text{th}} \) generation is given by:

\[
\phi_n(E_0,E) = \int_{E_K=E}^{E_0} \phi_{n-1}(E_0,E_K) \cdot F(E_K,E) \, dE_K .
\]  

(5)

The successive populations are then easily obtained from their corresponding spectra by a method that is similar to the one already used in calculating \( \Pi_1 \) thus:

\[
\Pi_n = \int_{E=\eta}^{E_0} \phi_n(E_0,E) \, dE .
\]  

(6)

4. DEVELOPMENT OF THE HADRON SHOWER ALONG AN AXIS

We have not introduced any dependence on the emission angle into our production formula, we strive to find the properties of the cascade development which are independent of the angular distribution.

As we have already stated, the absorption of the shower particles in matter is considered to be governed solely by the nuclear interaction cross-section -- this gives a relaxation length \( \lambda \) for reduction of the particle numbers by a factor \( 1/e \) (neglecting decay in flight, etc.). Using this, we now attempt to find out how the different populations are created and die out in space as the cascade propagates.

We first introduce a longitudinal coordinate \( z \), being the distance of the particle under consideration from the edge of the shield. Let us denote by \( \nu_n(z) \) the number of particles of the \( n^{\text{th}} \) generation at depth \( z \), and let \( \nu(z) \) be the total number of particles at this depth. Then clearly

\[
\nu(z) = \sum_{n=0}^{\infty} \nu_n(z) , \quad z > 0 ,
\]  

(7)

remembering that the cascade is assumed to be initiated by a single incident particle of energy \( E_0 \).
Consider the \((n-1)^{st}\) generation at depth \(z\). On its way through a path differential \(dz\), the component \(\nu_{n-1}(z)\) creates particles of the \(n^{th}\) generation at a rate \(\lambda^{-1} M_n \nu_{n-1}(z) dz\) from cascade interactions. At the same time, the \(n^{th}\) generation experiences a loss equal to \(\lambda^{-1} \nu_n(z) dz\). Thus the change in the numbers of the \(n^{th}\) generation at depth \(z\) is given by:

\[
d\nu_n(z) = \lambda^{-1} M_n \nu_{n-1}(z) dz - \lambda^{-1} \nu_n(z) dz.
\]  

(8)

For convenience in calculation, we introduce here a dimensionless coordinate \(\zeta\) defined by:

\[
\zeta = \lambda^{-1} z.
\]

Substituting this into Eq.(8) we obtain the differential equation:

\[
\frac{d\nu_n(\zeta)}{d\zeta} + \nu_n(\zeta) = M_n \nu_{n-1}(\zeta).
\]

(9)

Using the fact that the initial particle has an interaction probability which is exponential in \(\zeta\), i.e.

\[
\nu_0(\zeta) = e^{-\zeta},
\]

(10)

this differential equation can easily be solved. Using the relation between \(\Pi_n\) and \(M_n\) [Eq. (3)], we find that:

\[
\nu_n(\zeta) = \Pi_n \frac{\zeta^n}{n!} e^{-\zeta}.
\]

(11)

If we now define a new function \(G_n\) by:

\[
G_n(\zeta) = e^{-\zeta} \frac{\zeta^n}{n!},
\]

(12)

we then have, as a solution of Eq.(9),

\[
\nu_n(\zeta) = \Pi_n G_n(\zeta).
\]
The functions $G_n(\zeta)$ are the same as the distribution functions $P_n(\zeta)$ for the Poisson probability distribution (characterized by the fact that the occurrence or non-occurrence of an event is independent of the history) with parameter $\lambda^{-1}$.

Figure 3 gives a plot of the functions $G_n(\zeta)$ which describe the build-up and decay of each generation.

4.1 The decoupling theorem

In the previous section we assumed that the secondaries were produced along an axis. However, nowhere in the calculation did we use the fact that the axis should be straight; in fact, with minor modifications (e.g. $z$ becomes the total distance traversed from the edge of the shield) it is easily seen that all the above holds, no matter what line the particles are propagated along. We can thus see that this decoupling of the numbers in any generation into a product of an energy-dependent part and a distance-dependent part is more general than would at first appear, and so we summarize the result in the following theorem:

When the nuclear interaction cross-section is independent of energy, the dependence of the flux on the coordinate along the direction of flight in successive generations can be totally decoupled from the energy spectrum of each generation.

A consequence of this theorem is that the energy spectrum of a population is the same throughout the shield — only the numbers in each population change with depth.

We thus have the total number of particles at a depth $\zeta$ (assuming an axial development of the cascade), given by:

$$V(\zeta) = \sum_{n=0}^{\infty} \Pi_n \cdot G_n(\zeta), \quad \zeta = \lambda^{-1}z > 0, \quad \Pi_0 = 1.$$ 

It is possible to determine experimentally the populations $\Pi_n$ of the generations of the hadron cascade in a shield. Let us measure the numbers $N(z_K)$ of particles traversing a plane perpendicular to the direction of propagation at $i$ different depths $z_K (K = 1, 2, \ldots, i)$. Substituting in
the above equation, one obtains a system of i linear equations for the
first i populations \( \Pi_i \) to \( \Pi_i \), i being the number of depths at which
measurements were made; i must of course be larger than the number of
significant generations in the cascade.

5. MUONS

Having given expressions for the numbers and energy spectra of
particles in the cascade, it is tempting to derive the same data for the
muons coming from the decay in flight of the pions in the cascade. Let
us assume for the moment that all the particles in the cascade are pions,
which is usually the case when the incident high-energy particle is a
pion and not a proton. At a later stage, corrections can always be
brought in to account for the proton component in a cascade generated
by a proton, and also to take care of the muon-producing kaon component.

The mean free path in the shield for pion decay in flight is:

\[
\lambda = \frac{\tau \cdot c \cdot E}{m c^2} = \tau \cdot c \cdot \gamma ,
\]

where \( \tau \) is the lifetime of the pion in its own reference system
\((2.6 \times 10^{-3} \text{ sec})\), \( E \) the pion energy in flight, \( m c^2 \) the pion rest energy
\((139.58 \text{ MeV})\), and \( c \) the velocity of light. We find:

\[
\tau \cdot c = 7.8 \times 10^2 \text{ cm} .
\]

This is, of course, very large in comparison with the mean free path for
nuclear interaction \( \lambda \) which is of the order of 15 cm for iron, as an
example. We find thus that the fraction of the pions decaying in flight,
which is \( \lambda / \lambda(E) \), is very small with respect to 1 and depends on pion
energy.

We have now to examine what is the energy spectrum \( f(E_\mu) \) of the muons
produced by the decay of pions of a given energy \( E \). One finds (Rossi) that
at very high energies (neglecting the rest energies) there is an equal pro-
bability of finding the muons between the two energies \( E \) and \( \beta E \), where:

\[
\beta = 1 - \frac{2 \pi c^*}{m_\pi c^2} = 0.57 ,
\]
\( p^* \) being the momentum carried away by the muon in the frame of reference where the decaying pion is at rest (\( p^* = 30 \text{ MeV/c} \)). At the same time, neutrinos are produced with the complementary momentum, i.e. with an uniform spectrum between 0 and \((1-\beta)E\). So if we find a means of calculating the muon spectrum, the neutrino spectrum will be found as well.

Thus for one pion at an energy \( E \), one will have a probability:

\[
f(\mu) \frac{dE}{\mu} = \frac{dE}{(1-\beta)E}, \quad \beta E \leq E \mu \leq E
\]

of finding a muon with one energy between \( E \mu \) and \( E \mu + dE \mu \) within the limits \( \beta E \) and \( E \); outside these limits there are no muons and the integration gives 1. We now go back to our \( \pi \)-meson spectra of the various generations and try to calculate the muon spectrum \( f_n(\mu) \) given by the decay of the pions of a given generation, say the \( n \)th generation. In doing this, we will neglect the absorption of muons in matter. Remembering that \( \ell \) is proportional to \( E \), and that \( \lambda/\ell(E) \) is the fraction of pions which decay in flight, we obtain

\[
f_n(\mu) = \frac{\lambda mc^2}{\tau c(1-\beta)} \int_{E=E_\mu}^{E=\beta^{-1}E_\mu} f_n(E_\theta E) \frac{dE}{E^2},
\]

where \( f_n(E_\theta E) \) is the pion energy spectrum of the \( n \)th generation in a cascade generated by an incident particle of energy \( E_\theta \).

For muons arising from the decay of kaons, the value of \( \beta \) is 0.95, and instead of \( f_n(E_\theta E) \) the corresponding kaon energy spectrum in the \( n \)th generation has to be introduced.
BIBLIOGRAPHY


APPENDIX

A PARTICULAR EXAMPLE

We now take a particular example of a production spectrum and illustrate some of the results which were derived in the first part of the report.

The production function used in this section is the one which was first used by Passow, but more recently generalized by O'Brien; it has the form:

$$F(E_K, E) \, dE = \alpha \frac{E^\lambda}{m} \, dE \quad E_K > \eta$$

$$= 0 \quad \quad E_K < \eta$$

where $\alpha$, $\lambda$, $m$, and $\eta$ are constants. Following the above authors, we will assume that

$$m = \lambda + 1$$

and thus the production function becomes:

$$F(E_K, E) \, dE = \alpha \frac{E^\lambda}{E^{\lambda+1}} \, dE \quad E_K > \eta$$

which has only three constants which need to be determined. Note that this value of $\lambda$ makes $\alpha$ a dimensionless constant.

In order to obtain a value for $\lambda$, O'Brien requests that the inelasticity should become a given constant at high energy, and he obtains the value $\lambda = 0.216$. This is the value which we will use throughout this example. Once $\lambda$ is chosen, the values of $\alpha$ and $\eta$ are found in a unique manner by fitting the multiplicity curve $\Pi_1(E_0)$ to the experimental data on secondary particle production in cosmic rays (Fig. 4). The fit shows that with $\lambda = 0.216$, only one couple of values

$$\alpha \approx 0.64 \quad \quad \eta \approx 300 \text{ MeV}$$

is possible if one takes the multiplicity of secondaries produced by cosmic rays as a reference.
The multiplicity curves given be formula (1) are:

\[ M_1 \equiv \Pi_1 = \alpha \int_{E=\eta}^{E_0} \frac{dE}{E^{\lambda+1}} = \frac{\alpha}{\lambda} \left[ \left( \frac{E_0}{\eta} \right)^\lambda - 1 \right]. \]

In order to keep our graphs as general as possible, and also for ease of calculation, we define new variables:

\[ x_0 = \frac{E_0}{\eta} \quad \text{and} \quad x = \frac{E}{\eta}, \]

and thus obtain

\[ M_1 = \frac{\alpha}{\lambda} \left[ x_0^\lambda - 1 \right]. \]

Figure 5 is a plot of this multiplicity (for \( \lambda = 0.216 \)) against \( x_0 \) for various values of \( \alpha \).

We shall take the value of \( \lambda \) to be the same for all shield materials although it is quite possible that it has slight material dependence. The only material-dependent quantity in the formula is, of course, the constant \( \alpha \).

Recently, Grote, Hagedorn and Ranft have presented formulae for the production of secondary particles from proton-proton and proton-nucleus collisions at high energies which have been tested by experiments up to 80 GeV. Figure 6 shows the energy spectra of protons, antiprotons, positive and negative pions, and kaons integrated over all angles from a proton-proton collision at 30 GeV, for example, plotted as a function of secondary particle momenta. The sum of these spectra and the hadron spectrum with the above-mentioned values of \( \lambda, \alpha, \eta \) are also drawn, and are seen to be in extremely good agreement. Thus our values are confirmed.

We now derive expressions for the various quantities of interest in the cascade, starting with the energy spectra. Clearly:

\[ f_1(E_0, E) = F(E_0, E) = \frac{\alpha E_0^{\lambda}}{E^{\lambda+1}}. \]
The remaining spectra are calculated from the formula given in Section 3; this yields:

\[ f_2(E_0, E) = \alpha^2 \frac{E_0^\ell}{E^{\ell+1}} \log \frac{E_0}{E} \]

\[ f_3(E_0, E) = \frac{\alpha^3}{2} \frac{E_0^\ell}{E^{\ell+1}} \log^2 \frac{E_0}{E} , \]

and in general

\[ f_n(E_0, E) = \frac{\alpha^n}{(n-1)!} \frac{E_0^\ell}{E^{\ell+1}} \log^{n-1} \frac{E_0}{E} , \]

or equivalently

\[ f_n(E_0, E) = \frac{\alpha^n}{(n-1)!} \frac{1}{E_0^{\ell+1}} \left( \frac{E_0}{E} \right)^{\ell+1} \log^2 \left( \frac{E_0}{E} \right) \left( \frac{E_0}{E} \right)^{\ell+1} \log \left( \frac{E_0}{E} \right)^{\ell+1} \left( \frac{E_0}{E} \right)^{\ell+1} \]

Energy spectra are normally drawn as functions of energy. In order to keep things completely general, however, we rearrange the above formula to obtain:

\[ \gamma_n = \frac{(\ell+1)^{n-1}}{\alpha^n} \cdot E_0 \cdot f_n(E_0, E) = \frac{1}{(n-1)!} \left( \frac{E_0}{E} \right)^{\ell+1} \left( \frac{E_0}{E} \right)^{\ell+1} \left( \frac{E_0}{E} \right)^{\ell+1} \left( \frac{E_0}{E} \right)^{\ell+1} \]

and plot this as a function of \((E/E_0)^{\ell+1}\). The result, which is thus independent of the choice of \(\alpha, \ell, \) and \(\eta\), is shown in Fig. 7.

Simple integration of the above spectra yields the successive populations \(\Pi_n\), but in this case it is useful to express these as functions of:

\[ x_0 = \frac{E_0}{\eta} \quad \text{and} \quad x = \frac{E}{\eta} . \]

Thus we have, as before,

\[ \Pi_1 = \frac{\alpha}{\ell} \left( x_0 - 1 \right) \]
\[ \Pi_2 = \left( \frac{\alpha}{\beta} \right)^2 \left[ \frac{\varphi}{x_0} \log x_0 - (x_0 - 1) \right] \]

\[ \Pi_3 = \left( \frac{\alpha}{\beta} \right)^3 \left[ \frac{x_0^2 \log^2 x_0}{2} - x_0^2 \log x_0 + (x_0 - 1) \right] . \]

This leads to the general formula (which is easily proved by induction):

\[ \Pi_n = \left( \frac{\alpha}{\beta} \right)^n \left[ \left( -1 \right)^n + x_0 \sum_{r=0}^{n-1} \left( -1 \right)^{n-r-1} \frac{\log x_0^r}{r!} \right] . \]

In this case we plot \( (\beta/\alpha)^n \Pi_n \) as a function of \( x_0^\varphi \), and the result is shown in Fig. 8.

Assuming the whole cascade consists in pions (no proton component), one derives the muon spectrum

\[ f_n(E_0, E_\mu) = \frac{\lambda mc^2}{\tau c(1-\beta)} \frac{\alpha^n}{(\varphi+1)^{n-1}} \frac{1}{(n-1)!} \frac{1}{E_0} \times \]

\[ \int_{E = E_\mu}^{\beta^{-1}E_\mu \text{ or } E_0} \left( \frac{E_0}{E} \right)^{\varphi+1} \log^{n-1} \left( \frac{E_0}{E} \right) \frac{dE}{E^2} . \]

Taking into account that

\[ \log^{n-1} \left( \frac{E_0}{E} \right)^{\varphi+1} = (\varphi+1)^{n-1} \log^{n-1} \frac{E_0}{E} ; \quad \frac{dE}{E^2} = -\alpha \left( \frac{1}{E} \right) \]

one finds:

\[ f_n(E_0, E) = \frac{\lambda mc^2}{\tau c(1-\beta)E_0^\varphi} \frac{\alpha^n}{(n-1)!} \int_{E = E_\mu}^{E_\mu} \left( \frac{E_0}{E} \right)^{\varphi+1} \log^{n-1} \left( \frac{E_0}{E} \right) \alpha \frac{E_0}{E} . \]

or \( E_0 \) if \( \beta^{-1}E_\mu > E_0 \)
Let us introduce the functions

\[ P_n(x) = \frac{1}{(n-1)!} \int_1^x x^{n-1} \log^{n-1} x \, dx . \]

Then

\[ f_n(E_0, \mu) = \frac{\lambda mc^2}{\tau c(1-\beta)E_0} \alpha^n \left[ P_n\left( \frac{E_0}{E_\mu} \right) - P_n\left( \frac{\beta E_0}{E_\mu} \right) \right], \]

if

\[ E_\mu < E_0 \beta \]

and

\[ f_n(E_0, \mu) = \frac{\lambda mc^2}{\tau c(1-\beta)E_0} \alpha^n P_n\left( \frac{E_0}{E_\mu} \right) \]

if

\[ \beta E_0 < E_\mu < E_0 \]

as \( P_n(1) = 0 \). Plots of \( P_n(x) \) for \( \beta = 0.216 \) are found in Fig. 9.
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DISCUSSION

Paper: **Numbers and spectrum of muons from proton-induced extranuclear cascades in shields**

AWSCHALOM: Did you include the probability of decay versus probability interaction for a pion?

BARBIER: Yes, via the mean free path for pion decay in flight, which is a function of pion energy $E$

$$L = \tau \frac{E}{mc^2}$$

($\tau$ pion life-time, $mc^2$ pion rest energy), the fraction of pions decaying in flight is then $L/\lambda$.

MAEDA:

i) Your assumption $\sigma_{\text{inelastic}} = \text{const.}$ is valid up to what energy? What is the energy range that has been considered in your calculation?

ii) Your calculation is one-dimensional. Is your result realistic?

BARBIER:

i) The inelastic cross-section was assumed constant from 300 MeV to infinity.

ii) Lateral spread calculations have been made assuming an angular distribution of the secondary particles according to the law $e^{-p^0/\text{const.}}$. This law ties up energy spectrum and angular spread. Results include angular spreads of secondaries from any generation, assuming this generation is distributed along the axis. Details will be published in a forthcoming CERN report.