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NEAR FORWARD NEUTRINO REACTIONS ON NUCLEAR TARGETS

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A B S T R A C T

A simple model is considered for near forward neutrino reactions on nuclear targets in the conditions of the CERN experiment. The model accounts for the failure to observe the shadow effect, which is compensated by the elastic interaction and masked by unshadowed terms which grow rapidly away from the forward direction.

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It has been suggested ¹⁾ that, in certain kinematical circumstances, inelastic neutrino reactions on nuclei should show "shadow effects", like those familiar with incident hadrons. In particular the differential cross-section

$$\frac{d^2\sigma}{dq^2 dq_0}$$

(where the variables are those of the final lepton, and final hadrons are summed over) might have a nuclear size dependence more like $A^{2/3}$ than A . Such nuclear optical phenomena have now become quite familiar, theoretically ^{2)*} and experimentally ^{3)*}, for incident photons. They are associated with the coherent generation and propagation of mesons, the vector mesons in the case of the photon, and also the pion and pseudo-vector mesons in the weak interaction case. The lighter mass of the pion means that interesting effects develop at lower energies. However, they are harder to see, because of the relative under-development of neutrino experimentation.

The required kinematical conditions are that the four-momentum transfer q_μ to the hadrons should have a time component q_0 in some sense large enough, and an invariant square q^2 in some sense small enough. The original theory ¹⁾ referred only to the forward process ($q^2 \approx 0$) so, while able to be more specific about q_0 , could say nothing much about the desirable limits on q^2 . This paper presents a step towards extending the theory in the required direction.

For point of departure we have the relation of Adler ⁵⁾ for forward neutrino reactions on nucleons ^{**)} :

$$M^2 \frac{d\sigma}{dq^2} \Big|_{\theta=0} = (3.99 \times 10^{-14}) \int dW^2 \sigma_\pi(W) \frac{M^2}{P_\pi W} \frac{k'_0}{k_0}$$

*) See also, for example : Von Bochmann, Margolis and Tang ⁴⁾, and the many papers there cited.

***) We omit here lepton mass terms ⁵⁾, which, in the muon case, reduce the quoted results by several per cent.

where M is nucleon mass, W is hadron centre-of-mass energy, σ_{π} and p_{π} the corresponding total cross-section and centre-of-mass momentum for pions on nucleons ; k_0 and k'_0 are primary and secondary lepton energies. For a target which is an average of proton and neutron we have made a rough evaluation of the integral representing σ_{π} for $W < 2.13 M$ by the p wave resonance formula (24), and for $W > 2.13 M$ by a constant value of 35 millibarns. The result is given in Fig. 1, and also given separately is the contribution from the low mass region $W < 2.13 M$. For comparison the level at which the quasi-elastic process would contribute, averaged over free neutrons and protons, is indicated.

It would be nice, theoretically, to be able to restrict attention to the highly inelastic region. But for experiments so far performed ^{6),7)}, on the integrated effect, and with a broad spectrum of neutrinos in the few GeV region, it is clear that what happens in the lowly inelastic region is of the first importance. It is this question that we tackle, in a preliminary way, in this paper. In this region it seems reasonable to assume that the formation of the $N^*(1238)$ resonance is the dominant process, and that the propagation of mesons other than the pion is not essential. These are the essential ingredients of the following work. Clearly it is important also to know how the quasi-elastic process is modified by nuclear effects. In this paper we use only a simple Berman ⁸⁾ Fermi gas exclusion factor ; some other considerations will be reported elsewhere.

We use the optical model equation ^{*)}

$$(\nabla^2 + k^2 - V)\chi = \vec{U} \cdot \vec{\ell} e^{i\vec{q} \cdot \vec{x}} \quad (1)$$

^{*)} Coulomb effects are quite neglected. It could be feared that they would be of relative order $Z\alpha$. But the analysis of Nachtmann ⁹⁾, although strictly adapted to more highly inelastic processes, indicates that this is not so and that the Coulomb effects are probably rather small.

where $k^2 = q_0^2 - m_\pi^2$; V is the optical potential for the propagation of pions, and $\vec{U}$ is the "transition-potential" for the coherent generation of pions by leptons ; $\vec{\ell}$ is the space part of the lepton pair wave function

$$\ell_\nu = \bar{p} i \gamma_\nu (1 + \gamma_5) \nu$$

and we will neglect lepton mass, so that $q_\nu \ell_\nu = 0$ and ℓ_0 can be eliminated in terms of $\vec{\ell}$. A formal construction of the operators $\vec{U}$ and V from fundamental interactions can be given along familiar lines ¹⁰⁾. It tells us that $\vec{U}$ and V are in general energy-dependent non-local operators, and little more. Progress then depends on simple practical guesses and phenomenological analysis. For the scattering of low energy pions this study is well developed ^{*}) but for the resonance region it is only beginning ¹³⁾. It is therefore not now sensible to attempt very elaborate computations in our more complicated case, and we will do only very simple things. In particular we will tackle only the case of a plane wave propagating in an infinite medium, although this may certainly be rather remote from the actual case of rather long wave-length pions in finite nuclei. Moreover for the optical potential we will use only a simple forward scattering amplitude approximation.

Taking $\vec{x} = 0$ in (1), it reduces in the homogeneous plane wave case to

$$(m_\pi^2 + q^2 + V) \chi = - U_3 \ell_3 \quad (2)$$

where ℓ_3 is the component of $\vec{\ell}$ along $\vec{q}$, and the other components cannot now contribute (by rotational symmetry). So a first point is this : the transverse components of $\vec{\ell}$ do not generate a coherent pion wave and so are not shadowed (always ignoring heavier mesons). In the isobar dominated model it will be seen that not only are the amplitudes

^{*}) See for example Ericsons ¹¹⁾ and Koltun ¹²⁾.

for forward pion production, by pions and leptons respectively, in the ratio V/U_3 , but that this ratio holds for all processes. What is important then is the combination

$$U_3 \ell_3 + VX \quad (3)$$

which from (2) has the value

$$\begin{aligned} U_3 \ell_3 - V U_3 \ell_3 (m_\pi^2 + q^2 + V)^{-1} \\ = U_3 \ell_3 \frac{(m_\pi^2 + q^2)}{(m_\pi^2 + q^2 + V)} \end{aligned} \quad (4)$$

So a second point is this : the longitudinal part of the lepton wave is effectively damped by a factor ^{*)}

$$\frac{m_\pi^2 + q^2}{m_\pi^2 + q^2 + V} \quad (5)$$

Some idea of the importance of the effect may be given by Fig. 2. The forward double differential cross-section $d^2\sigma/dq^2 dW^2$, where W is centre-of-mass energy ($W^2 = M^2 + 2q_0 M$, where M is nucleon mass) is calculated in the resonance model to be described. The lower curve is then obtained by multiplying by the square of the modulus of (5), taking a modified forward scattering approximation for V :

*) Note that this "damping" factor may actually exceed unity ¹⁴⁾, at low energies for example, when $\text{Re } V$ is negative and $\text{Im } V$ small (see, for example, Fig. 2). Exploratory calculations suggest that this is not numerically important in the integrated effect.

$$V = - \rho \sigma_{\pi}(w)(\cot \delta + i) \frac{q_0^2}{\sqrt{q_0^2 - m_{\pi}^2}}$$

where ρ is nucleon density ; σ_{π} and δ are given in (24). When m_{π} is neglected this reduces to the formula previously quoted ¹⁾ but it is probably a better off mass shell continuation ⁵⁾ for finite pion mass and p wave scattering.

When the muon continues in the direction of the neutrino it is clear that $\vec{l}$ must be purely longitudinal, and the reduction factor (5) applies to the total process, as before ¹⁾. However, for non-forward reactions there is a transverse part which is not shadowed. Our business then is to calculate the relative importance of these two parts. This we do in a simple isobar model described in the next Section. It is found that in the conditions of experiments performed so far the transverse part is sufficient, along with the quasi-elastic process, to prevent a large shadow effect in the longitudinal contribution from showing up decisively.

THE 33 RESONANCE

A simplified theory of neutrino reactions involving the 33 resonance will be outlined, assembling elements from previous papers ^{15)-20) *}, with attention to the separation of longitudinal and transverse amplitudes.

Consider the process

$$\nu(k) + P(p) \rightarrow \mu^-(k') + P(p') + \pi^+(q')$$

^{*}) Many other papers are cited by Adler ²⁰⁾.

of positive pion production on a proton. The four-momenta are indicated in brackets. As usual it will be supposed that the lepton wave functions appear in the amplitude only in the combination

$$\ell_\nu = \bar{\psi} i \gamma_\nu (1 + \gamma_5) \nu$$

The mass of the muon will be neglected, so that

$$q_\nu \ell_\nu = 0 \quad (6)$$

where

$$q_\nu = k_\nu - k'_\nu.$$

The hypothesis of dominance by a state of $J = \frac{3}{2}$ implies that the dependence of the amplitude on the final hadron states is only through a quantity of appropriate transformation character ; it can be taken to be a two-spinor-three-vector $\vec{R}$ satisfying

$$\vec{\sigma} \cdot \vec{R} = 0. \quad (7)$$

We work in the c.m. system of the final hadrons :

$$\vec{p}' + \vec{q}' = \vec{p} + \vec{q} = 0. \quad (8)$$

The general invariant amplitude $\mathcal{M}$ is a combination of eight terms ¹⁸⁾. For simplicity we retain only those axial and vector terms with minimal dependence on the initial momentum $\vec{q}$:

$$m = \vec{c} \cdot (\vec{R}^* p) \quad , \quad \vec{c} = c_A \vec{e} + c_V \vec{e} \times i \vec{q} \quad (9)$$

where C_V and C_A are coefficients to be discussed, P is the proton two-spinor, and $\vec{e}$ and $\vec{q}$ are the space parts in the system (8) of the four-vectors l_μ and q_μ . It will be convenient to consider at the same time the invariant amplitude for the reaction

$$\pi^+(q) + P(p) \rightarrow \pi^+(q') + P(p')$$

which we write

$$i \vec{q} \cdot (\vec{R}^* p) \quad (10)$$

absorbing a multiplicative coefficient into the definition of $\vec{R}$.

For the 33 state the conventional dispersion theory^{16),17),20)} amounts largely to a resonant final-state enhancement of the renormalized one nucleon exchange diagram. With an incident pion the relevant diagram is

and for the neutrino

These diagrams differ only at the top vertex. The covariant forms of these vertices are

$$\sqrt{2} g i \gamma_5, \quad \ell_\nu \frac{G^P}{\sqrt{2}} \left\{ i \gamma_\nu (1 + \lambda \gamma_3) + (1 - \xi) \sigma_{\mu\nu} \frac{q_\mu}{2M} \right\}$$

where $g (\approx 13.5)$ is the pion-nucleon coupling constant, $G^P (\approx 1.00 \times 10^{-5} M^{-2})$ the $\Delta S = 0$ weak coupling, $\lambda (\approx 1.2)$ the axial to vector ratio, and $\xi (\approx 4.7)$ is the weak magnetism constant ($\mu_p - \mu_n$). Making a small $\vec{q}$ approximation in the final hadron centre-of-mass system, these reduce to

$$\frac{g}{\sqrt{2} M} \vec{\sigma} \cdot i \vec{q} \quad (11)$$

for the pion, and for the leptons

$$- \frac{G^P}{\sqrt{2}} \ell_0 + \frac{G^P}{\sqrt{2}} \vec{\ell} \cdot \left(\frac{2\vec{p}' - \vec{q}}{2M} - \frac{\xi}{2M} i \vec{q} \times \vec{\sigma} - \lambda \vec{\sigma} \right) \quad (12)$$

retaining only those axial and vector terms of lowest order in centre-of-mass momenta. In fact only the terms containing $\vec{\sigma}$ contribute to the 33 final state. Then comparing (11) and (12) with (9) and (10)

$$C_A = - \frac{\lambda G^P M}{g}, \quad C_V = - \frac{\xi G^P}{2g} \quad (13)$$

We adopt these values for small q^2 . A serious discussion for large q^2 cannot be made with our small $\vec{q}$ approximation. We will rather arbitrarily adopt the q^2 dependence of the nucleon charge form factor :

$$C_A = - \frac{\lambda G^P M}{g} \left(1 + \frac{q^2}{8M^2} \right)^{-2}$$

$$C_V = - \frac{\xi G^P}{2g} \left(1 + \frac{q^2}{8M^2} \right)^{-2} \quad (14)$$

Now there is nothing sacred about the conventional dispersion theory, and the small recoil approximation here is very very crude. It is comforting therefore that for small q^2 there are other supports for the couplings adopted. For the axial part the relation (6) happens to be just the appropriate specialization of the generalized Golberger-Treiman relation. So we will automatically reproduce Adler's relation between forward neutrino and pion reactions ⁵⁾. For the vector part, the usual CVC theory requires a relation with electroproduction and photoproduction amplitudes. Thus the weak amplitude

$$- \frac{\sqrt{2} G^A}{2g} \vec{E} \times i \vec{q} \cdot (\vec{R}^* p)$$

implies an amplitude

$$- \frac{\sqrt{2} e}{2g} \sqrt{\frac{2}{3}} \vec{E} \times i \vec{q} \cdot (\vec{R}^* p)$$

for the formation of the resonant state by photons incident on protons. Then denoting the total resonant photoproduction cross-section by σ_Y

$$\frac{\sigma_Y(W)}{\sigma_\pi(W)} = \frac{\sqrt{2}^2 e^2}{6g^2} \frac{p_Y}{p_\pi}$$

where

$$\begin{aligned} 4 p_Y^2 W^2 &= (W^2 - M^2)^2 \\ 4 p_\pi^2 W^2 &= (W^2 - M^2 - m_\pi^2)^2 - 4 M^2 m_\pi^2. \end{aligned} \quad (15)$$

With the appropriate CG coefficients then

$$\frac{\sigma(\gamma + p \rightarrow \pi^0 + p)}{\sigma(\pi^0 + p \rightarrow \pi^0 + p)} = \frac{F^2 e^2}{4 q^2} \frac{p_\pi}{p_\gamma}$$

This is just the relation of Chew and Low²¹⁾. We have compared it with experiment in Fig. 3. Over the Δ resonance region it works well enough for our purpose. The corresponding relation for charged pions is not so good, because of substantial S wave and $I = \frac{1}{2}$ states completely omitted here.

We require the differential cross-section for producing muons of lab energy k'_0 at angles θ . With the invariant variables

$$q^2 = q_\mu q_\mu = 4 k_0 k'_0 \sin^2 \theta/2 \quad (16)$$

$$W^2 = -(p_\mu + q_\mu)^2 = M^2 - q^2 + 2 M q_0 \quad (17)$$

where M is proton mass,

$$\frac{d^2 \sigma}{dq^2 dW^2} = \frac{1}{32 \pi^2} \frac{1}{4 M^2 k_0^2} \sum \bar{\sum} |m|^2$$

where the average is over proton spin states and the sum over energy conserving final hadron states, and k_0 is neutrino laboratory energy. It is convenient to introduce a function P of W defined by

$$\sum \bar{\sum} (R^*_i p)(p^* R_j) = 16 \delta_{ij} P(W) W^2$$

This quantity occurs also in the pion scattering cross-section σ_π from the amplitude (11), and so can be regarded as an observed quantity :

$$P(W) = \frac{\sigma_\pi(W)}{4\pi W} \quad (18)$$

Then

$$\frac{d^2\sigma}{dq^2 dW^2} = \frac{4W^2}{32\pi^2} \frac{P(W)}{M^2 k_0^2} |\vec{C}|^2 \quad (19)$$

It is convenient at this point to express the centre-of-mass components $\vec{l}$ and $\vec{q}$ in (9) in terms of the corresponding laboratory components. This requires the substitutions

$$\begin{aligned} \vec{l}_t &\rightarrow \vec{l}_t \\ \vec{l}_3 &\rightarrow \vec{l}_3 \frac{Mq_0 - q^2}{Wq_0} \\ \vec{q} &\rightarrow \vec{q} \frac{M}{W} \end{aligned} \quad (20)$$

where (6) has been used to eliminate the l_0 arising in the Lorentz transformation, and $\vec{l}_3$ and $\vec{l}_t$ are parts of $\vec{l}$ along and perpendicular to $\vec{q}$:

$$\vec{C} = C_A \vec{l}_3 \frac{Mq_0 - q^2}{Wq_0} + C_A \vec{l}_t + C_V \frac{M}{W} \vec{l} \times i\vec{q} \quad (21)$$

Then, evaluating the products of lepton factors from

$$\begin{aligned} \ell_\mu^* \ell_\nu &= \text{Trace } i\gamma_\mu (1 + \gamma_5) i\gamma_\nu k' i\gamma_\nu (1 + \gamma_5) i\gamma_\mu k \\ &= 8(k'_\mu k_\nu + k_\mu k'_\nu - \delta_{\mu\nu} k \cdot k' + \epsilon_{\mu\nu\rho\sigma} k'_\rho k_\sigma) \end{aligned}$$

we have finally

$$\begin{aligned} \frac{d^2\sigma}{dq^2 dW^2} = & \frac{4W^2}{8\pi^2} \frac{P(W)}{M^2 k_0^2} \times \\ & \left\{ |C_A|^2 \left(\frac{Mq_0 - q^2}{Wq_3} \right)^2 \left[(k_0 + k'_0)^2 - q_3^2 \right] \right. \\ & + |C_A|^2 \frac{q^2}{q_3^2} \left[(k_0 + k'_0)^2 + q_3^2 \right] \\ & + \operatorname{Re} C_A^* C_V 4q^2 \frac{M}{W} (k_0 + k'_0) \\ & \left. + |C_V|^2 q^2 \frac{M^2}{W^2} \left[(k_0 + k'_0)^2 + q_3^2 \right] \right\} \end{aligned} \quad (22)$$

with

$$\begin{aligned} q_3^2 &= q^2 + q_0^2 \\ 2Mq_0 &= W^2 - M^2 + q^2. \end{aligned}$$

The result (22) differs from that of Bell and Berman¹⁵⁾ only by the factors arising in the Lorentz transformation (13), the small recoil approximation having been made formerly in the laboratory system.

This calculation differs from that of Berman and Veltman¹⁸⁾ mainly in that those authors used rather large coupling strengths, based on a misinterpretation of the paper by Gourdin and Salin²²⁾ on photo-production. Berman and Veltman had failed to divine in that paper a hidden convention with respect to Clebsch-Gordan coefficients. The point was cleared up by Bijtebier²³⁾.

As compared with the isobar model of Salin, the important difference is in the axial coupling. That of Salin, as observed by Adler^{24),23)}, did not respect Goldberger-Treiman.

As compared with the detailed dispersion theory, worked out especially by Adler ¹⁸⁾, the present model contains only what are hoped to be the most important terms. We have made a comparison with Adler's total cross-section for the same form factor parameter (the lower set of curves in his Fig. 24). For this purpose we have integrated with the resonance formula (24) up to $W = 1.39$ GeV, the limit chosen by Adler. The resulting curve, Fig. 4, lies a little higher than the highest of the three versions given by Adler, levelling off at about $3.7 \times 10^{-39} \text{ cm}^2$ rather than $3.5 \times 10^{-39} \text{ cm}^2$. Note that this is for the proton-neutron average. With our pure $T = \frac{3}{2}$ model protons are three times as effective as neutrons. Adler, retaining some $T = \frac{1}{2}$ terms, does not have such a large difference.

It should be noted that experimentally ^{25),23)} N^* production seems to be substantially more copious than expected on the Adler theory or, therefore, on our theory.

PION-LEPTON EQUIVALENCE AND OPTICAL POTENTIALS

The amplitudes for the formation of the resonant state, by incident pions and lepton-pairs respectively, are given by (9) and (10). In terms of laboratory co-ordinates [see (20)] they are

$$\frac{M}{W} i \vec{q} \cdot (\vec{R}^* p)$$

$$(C_A \vec{e}_3 \frac{M q_0 - q^2}{W q_0} + C_A \vec{e}_t + C_V \frac{M}{W} \vec{e} \times i \vec{q}) \cdot (\vec{R}^* p) \quad (23)$$

There is a manifest equivalence, up to a kinematical factor, between pions and longitudinal lepton pairs. This equivalence is independent of the final state, and applies in particular for the forward production of pions. In the simple conventional approximation the optical potentials V and U are just proportional to the amplitudes for production of single pions by pions and lepton pairs respectively,

incident on a single nucleon, averaged over spin and isospin at the target nucleon (which we take to be at rest, neglecting Fermi motion). The observations are the basis for the assertion that coherent pions and lepton pairs are effective in the combination (3).

It must be admitted that this simplified account of the many-body aspects of the problem would be most apt for a dilute system of slowly moving uncorrelated weak scattering centres. The nucleus is more probably a concentrated system of fast moving correlated scattering centres, which, particularly in the resonant region, scatter strongly. So our calculations are indeed of a somewhat exploratory character. It may be possible to find the courage to do something more elaborate when the study of pion-nucleus interactions in this energy region is more developed.

CALCULATIONS

We have made some calculations of the cross-section (22), with particular attention to the relative importance of the first term with respect to the others. The first term is that arising from the longitudinal component l_3 of the lepton-pair wave function, and so is subject to shadowing, while the other components are not - in this simplified treatment.

For the pion-nucleon cross-section in the resonant region, averaged over proton and neutron, we use the p wave resonance formula ^{21),26)}

$$\sigma_{\pi}(W) = \frac{2}{3} \frac{8\pi}{P_{\pi}^2} \frac{1}{1 + (\cot \delta)^2} \quad (24)$$

$$\cot \delta = \frac{0.085 \left(\left(\frac{W}{M} \right)^2 - 1 \right) (1.742 - \left(\frac{W}{M} \right)^2)}{0.742 \left(\frac{P_{\pi}}{M} \right)^3}$$

with p_π given by (15). From (18) then

$$\begin{aligned} P(W) &= \frac{4\pi}{3Wp_\pi^3} \frac{1}{1 + (\cot \delta)^2} \\ &= 1.85 \frac{M^2}{Wp_\pi^3} \frac{10^{-28}}{1 + (\cot \delta)^2} \text{ cm}^2 \end{aligned} \quad (25)$$

From (14) and (22), with $\lambda = 1.2$, $\xi = 4.7$, $g = 13.5$,
 $G^p = 1.00 \times 10^{-5} / M^2$,

$$\begin{aligned} M^4 \frac{d^2 \sigma}{dq^2 dW^2} &= (2.78 \times 10^{-14}) \frac{M^2 P(W)}{k_0^2} \left(1 + \frac{q^2}{.81 M^2}\right)^{-4} \\ &\left\{ 1.44 \left(\frac{q_0 M - q^2}{M q_3} \right)^2 \left[(k_0 + k'_0)^2 - q_3^2 \right] \right. \\ &\quad + 1.44 \frac{W^2 q^2}{M^2 q_3^2} \left[(k_0 + k'_0)^2 + q_3^2 \right] \\ &\quad + 11.3 \frac{W q^2}{M^2} [k_0 + k'_0] \\ &\quad \left. + 5.52 \frac{q^2}{M^2} \left[(k_0 + k'_0)^2 + q_3^2 \right] \right\} \end{aligned} \quad (26)$$

for the proton-neutron average.

To obtain some orientation on the relative importance of the various terms, suppose

$$\begin{aligned} q^2 &\ll M^2, \quad \theta (\approx q^2 / 4k_0 k'_0) \ll 1 \\ q_0 (= k_0 - k'_0) &\ll k_0 + k'_0 \end{aligned}$$

Then

$$q_3^2 (= q^2 + q_0^2) \ll (k_0 + k_0')^2$$

so that in the square brackets we can ignore q_3^2 , and also the difference between k_0' and k_0 . If in addition we replace W , inside the curly bracket, by the resonant value $1.32 M$, then

$$M^2 \frac{d\sigma}{dq^2} \approx 16.0 \times 10^{-14} \int dW^2 \frac{P(W)}{M^2} \times \quad (27)$$

$$\left\{ \frac{(q_0 M - q^2)^2}{M^2 q_3^2} + 1.74 \frac{q^2}{q_3^2} + 5.2 \frac{q^2}{M k_0} + 3.84 \frac{q^2}{M^2} \right\}$$

The first term here is the longitudinal axial term which is vulnerable to shadowing. The remaining terms may be labelled respectively the transverse (axial, interference, and vector) terms. Now

$$M q_0 = \frac{1}{2} (W^2 - M^2 + q^2)$$

so that for small enough q^2

$$q_3^2 = q_0^2 = M^2 (.37)^2 = .14 M^2 \quad (28)$$

on resonance. So that for small q^2 and $k_0 > M$ it is the transverse axial which is the largest competitor with the longitudinal term. Substituting the values (28) in (27), for small q^2

$$\frac{d\sigma}{dq^2} \propto 1 + 16.3 \frac{q^2}{M^2} + 5.2 \frac{q^2}{M k_0} \quad (29)$$

where the first term is the longitudinal. The transverse contributions become comparable, when k_0 is a few GeV or more, when

$$q^2 \approx 0.05 M^2 \quad (30)$$

This corresponds to an angle

$$\theta \approx \frac{\sqrt{q^2}}{k_0} \text{ radians} \approx 12.7 \left(\frac{M}{k_0} \right) \text{ degrees.} \quad (31)$$

which is only a few degrees when k_0 is a few GeV.

The experimental information so far available ^{6),7)} has been obtained with a broad spectrum of neutrinos in the few GeV range, and integrating in general up to rather large muon angles. Because of the remarks just made, it seems worth while to analyze in more detail only the small angle ($< 5^\circ$) comparison of carbon and lead by Borer et al. ⁷⁾. In making this analysis we are well aware of the unreliability of conclusions based on a plane wave in a homogeneous medium for processes actually involving small systems and long wave-lengths.

In these spark chamber experiments only the angle, and to some extent the energy, of the secondary lepton are observed. So we have to integrate our expressions accordingly, including an average over the incident neutrino spectrum. We have used the version of the CERN neutrino spectrum (kindly supplied to us in numerical form by W. Venus) illustrated in Fig. 5. It is said to be fairly reliable in the region between 2 and 5 GeV, but otherwise somewhat impressionistic. As in the experiment of Borer et al., we have required a minimum muon energy of 1.2 GeV. In integrating over the final hadron mass W we have used the resonance formula (24) for $(W/M)^2 < 2.13$. Above this somewhat arbitrarily defined resonance region we have no right to use our formulae away from the forward direction (where they reduce to Adler's PCAC relation ⁵⁾). So the contributions that we obtain from the high

mass region, quoted separately in the tables, are to be treated with reserve - especially as regards the transverse parts, which go to zero in the Adler forward limit. They have been obtained by giving σ_{π} (averaged over protons and neutrons) the constant value 35 millibarns.

Figure 6 gives the resulting longitudinal, transverse, and total, differential cross-sections ($d\sigma/d \cos \theta$), for a single nucleon, averaged over proton and neutron. The transverse-longitudinal cross-over point is at about 3.1° . Thus the mean neutrino energy for this purpose is, from (31), about 4 GeV. This is well above the maximum in the neutrino spectrum (at about 1 GeV) because of the cut-off on lepton energy (required to exceed 1.2 GeV) and because $d\sigma/d \cos \theta$ (in contrast with $d\sigma/dq^2$) increases roughly linearly with neutrino energy.

In Table 1 are listed the various contributions integrated up to a muon angle of 5° , which defines the small angle sample of Borer et al. The first column lists the results with the formulae given above, and in addition the cross-section for the quasi-elastic process. The latter is calculated in a conventional way, with the axial form factor set equal to the weak magnetic, neglecting Fermi motion, and incorporating a Berman²⁴⁾ symmetric Fermi gas exclusion factor. This is supposed to simulate the carbon nucleus.

In the second column, the Pb nucleus is naively simulated by a collection of free neutrons and protons. The inelastic cross-sections are reduced by some 10% because for our pure $I = \frac{3}{2}$ model the neutrons are only one third as effective as the protons. Of course the actual process is not pure $I = \frac{3}{2}$, and so we have exaggerated the effect of the neutron-proton asymmetry. The very important increase in the quasi-elastic cross-section for nucleon, now computed with the appropriate asymmetric Berman exclusion factor, is to be noted. This arises from the reaction going on neutrons only, and because for $N = Z$ the process is entirely suppressed in the forward direction by the exclusion principle (in the Berman model). In the third column of Table 1, a very crude allowance for shadowing has been made by not allowing σ_{π} in the formula to exceed the geometrical value

$$\frac{2\pi R^2}{A}$$

with $R = 1.2 A^{1/3}$ Fermi, and $A = 208$. Again this is very likely an exaggeration of the difference between lead and carbon ; we would expect some shadowing already in carbon (see the experimental results quoted by, for example, Ericson and Locher ²⁷⁾) and to be incomplete in Pb (see, for example, the optical model calculations of Lovseth and Froyland ¹⁴⁾). Despite these exaggerations of the reduction in the inelastic cross-sections, the increase of the elastic pretty well cancels it. For the ratio of cross-sections per nucleon in Pb and C we have

$$\begin{aligned} \frac{\sigma_v(Pb)}{\sigma_v(C)} &= \frac{2.413}{2.104} = 1.15 && \text{without shadowing} \\ &= \frac{2.024}{2.104} = .96 && \text{with shadowing} \end{aligned}$$

to be compared with the experimental

$$.92 \pm .15$$

As observed by the experimenters, the experimental error is too large to conclude that departure from simple additivity, in nuclei, of nucleon inelastic cross-sections, has been observed. However, if the error were to be reduced it is clear that the relative efficiencies of different nuclei for quasi-elastic processes would be a question of capital importance. In this connection it is notable that the most relevant experiment ²⁸⁾ (actually done in iron) was quite comparable with the total absence of an exclusion effect. This is very curious, both on theoretical grounds and because of the essential (if quantitatively problematic) exclusion effect in the inverse reaction, muon capture. The question certainly requires further investigation.

Note that in Table 1 the shadowing reduction allowed in the longitudinal inelastic is even greater than would correspond to a simple $A^{2/3}$ law (itself probably an exaggeration of the effect ¹⁴). Nevertheless the shadowing gives only a 20% reduction in the total effect, $1.15 \rightarrow 0.96$. The situation improves of course with angular definition. Table 2 is the same as Table 1, except that the angular bound is 2° instead of 5° . The over-all shadowing effect is then a 40% reduction :

$$\frac{\sigma_v(P_L)}{\sigma_v(C)} = \frac{.391}{.319} = 1.23 \quad \text{no shadow}$$

$$= \frac{.281}{.319} = .88 \quad \text{with shadow}$$

Of course the reliability of the simple Fermi gas treatment of the quasi-elastic process is still more critical.

CONCLUSION

The theory developed here is naïve in conception and crude in execution. It could be put forward only into a theoretical vacuum. Two main conclusions seem to be worth noting. One is the importance, even for rather energetic neutrinos, of lowly inelastic processes - N^* production and quasi-elastic scattering. The latter, at least in the simple Berman account, works against the shadow effect in the comparison between light and heavy nuclei ; more experimental information on the nuclear effects for this process would be very desirable. The second main conclusion is the existence of a relatively unshadowed background, in the inelastic process, which increases rapidly away from the forward direction. The typical value (30) of q^2 for this increase, $\approx 0.05 M^2$, is indeed rather smaller than anticipated by one of us (J.S.B.), not much exceeding $m_\pi^2 \approx 0.022 M^2$. It has sometimes been guessed that m_π^2 would be the relevant parameter because

"it has all to do with PCAC, so that everything is determined by the one pion pole". This is fallacious. The direct contribution from one pion exchange between leptons and hadrons is rather small, and actually vanishes when the lepton mass is ignored, as it was ignored here. The situation is analogous to that of the Goldberger-Treiman relation relating the axial form factor at $q^2 = 0$, in muon capture, to the strength of the induced pseudoscalar ; the latter is dominated by the pion pole, but one does not expect the fall-off with q^2 of the axial form factor itself to be governed by m_π^2 . The origin of the large numerical coefficients of the transverse terms in (29) is rather in the strength of weak magnetism $[(\frac{1}{2})^2 = 5.5]$ and in the small value of $q_3^2 (\approx M^2/7)$ for the low lying resonance. The longitudinal part itself, the first term in (27), diminishes rapidly with q^2 because of a factor arising from the assumption that in the centre-of-mass system only the longitudinal component of lepton pair polarization is effective. It may be therefore that the surprisingly strong competition from the transverse part is peculiar to the resonance region, and will be less marked for highly inelastic events. Here we are again frustrated by the absence of monoenergetic neutrino beams, which, together with measurement of secondary lepton energy, would permit separation of different degrees of inelasticity.

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	(N/Z) = 1 "carbon"	N/Z = 126/82 "lead"	N/Z = 126/82 "shadowed"
-----	-----	-----	-----
Longitudinal inelastic			
$(W/M)^2 < 2.13$	0 . 452	0 . 405	0 . 115
$(W/M)^2 > 2.13$	0 . 218	0 . 195	0 . 096
-----	-----	-----	-----
Transverse inelastic			
$(W/M)^2 < 2.13$	0 . 436	0 . 390	0 . 390
$(W/M)^2 > 2.13$	0 . 165	0 . 148	0 . 148
-----	-----	-----	-----
Quasi-elastic			
=====	0 . 833	1 . 275	1 . 275
=====	=====	=====	=====
Total	2 . 104	2 . 413	2 . 024

TABLE 1 : Different contributions to the cross-section per nucleon for events with
 $\theta < 5^\circ$ and muon energy > 1.2 GeV.

	(N/Z) = 1 "carbon"	N/Z = 126/82 "lead"	"lead" "shadowed"
-----	-----	-----	-----
Longitudinal inelastic			
$(W/M)^2 < 2.13$	0 . 125	0 . 112	0 . 032
$(W/M)^2 > 2.13$	0 . 060	0 . 054	0 . 024
-----	-----	-----	-----
Transverse inelastic			
$(W/M)^2 < 2.13$	0 . 036	0 . 032	0 . 032
$(W/M)^2 > 2.13$	0 . 012	0 . 011	0 . 011
-----	-----	-----	-----
Quasi-elastic	0 . 086	0 . 182	0 . 182
=====	=====	=====	=====
Total	0 . 319	0 . 391	0 . 281

TABLE 2 : Different contributions to the cross-section per nucleon for events with
 $\theta < 2^\circ$ and muon energy > 1.2 GeV.

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FIGURE CAPTIONS

- Figure_1 The forward neutrino cross-section averaged over free neutrons and protons.
- a) The inelastic cross-section according to Adler's theorem using $\sigma_{\pi} = 35$ mb for missing mass $W > 2.13 M_p$ and a resonance formula, Eq. (24), for $W < 2.13 M_p$.
- b) The contribution to the inelastic cross-section from $W < 2.13 M_p$.
- c) The elastic cross-section.
- Figure_2 The differential cross-section, Eq. (26), for the forward process in the resonance region for a neutrino energy $4 M_p$ (upper curve) and the result (lower curve) of multiplying by the square of the modulus of the shadow factor, Eq. (5), the potential V being computed in a modified forward scattering approximation. W is the invariant mass of the final hadron system.
- Figure_3 $\sigma(\gamma p \rightarrow \pi^0 p)$ in μb plotted against the invariant mass of the final hadron system W in GeV. The line was computed using the Chew-Low formula, Eq. (15), with σ_{π} given by the resonance formula, Eq. (24), (which we have applied even outside the resonance region). The dots are taken from the fit to the data in Ref. 29).
- Figure_4 The inelastic neutrino cross-section, in our model, for invariant masses of the final hadron system $W < 1.39$ GeV.
- Figure_5 Normalized model of the CERN neutrino spectrum plotted against the neutrino energy in proton mass units.

Figure 6 The inelastic differential cross-section $d\sigma/d \cos \theta$ in our model averaged over free neutrons and protons and over the CERN neutrino spectrum. The muon energy was required to be > 1.2 GeV. The longitudinal contribution (which is liable to shadowing) and the transverse contribution (which is not) are shown separately. The horizontal scale is related to $\cos \theta$ by $\cos \theta = 1.0 - 0.00095 x$.

FIG.1

FIG.2

FIG. 3

FIG.4

FIG.5

FIG. 6