Quantum Measurement and Fluctuations in Nanostructures

A. Shimizu

Institute of Physics, University of Tokyo, Komaba, Tokyo 153, Japan

Abstract. Measurement and fluctuations are closely related to each other in quantum mechanics. This fact is explicitly demonstrated in the case of a quantum non-demolition photodetector which is composed of a double quantum-wire electron interferometer.

1. Introduction

Recent rapid progress of studies on nanostructures is opening up possibilities of new measuring apparatus using nanostructures. For example, a tiny change of the electric charge in a nano-scale region can be detected through a single-electron-tunneling transistor [1]. Another example is a quantum-wire electron interferometer that works as a quantum non-demolition (QND) photodetector, which measures the photon number without absorbing photons [2]. The functions of these nanostructure devices are hardly accessible by conventional devices, thus make nanostructure devices very attractive.

On the other hand, these devices stimulate studies on a very basic problem of physics—what happens when you measure a quantum system? To discuss this problem the nanostructure devices are useful because they allow microscopic analysis of the measuring devices. As a result, we can clarify close relationships among the measurement error, backactions, and fluctuations. I here demonstrate these things by reviewing our studies on the quantum-wire QND photodetector.

2. Quantum-wire QND photodetector

A schematic diagram of the quantum non-demolition (QND) photodetector [2] is shown in Fig.1. Before going to the full analysis in the following sections, I here give an intuitive, semi-classical description [3] of the operation principle.

The device is composed of two quantum wires, N and W. The lowest sub-band energies (of the \(z\)-direction confinement) \(\epsilon_{N_a}^z\) and \(\epsilon_{W_a}^z\) of the wires are the same, but the second levels \(\epsilon_{N_b}^z\) and \(\epsilon_{W_b}^z\) are different. Electrons occupy the lowest levels only. A \(z\)-polarized light beam hits the dotted region. The photon

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energy $\hbar \omega$ is assumed to satisfy $\epsilon_b^W - \epsilon_a^W < \hbar \omega < \epsilon_b^N - \epsilon_a^N$, so that real excitation does not occur and no photons are absorbed. However, the electrons are excited “virtually” [4], and the electron wavefunction undergoes a phase shift between its amplitudes in the two wires. Since the magnitude of the virtual excitation is proportional to the light intensity [4], so is the phase shift. This phase shift modulates the interference currents, $J_+$ and $J_-$. By measuring $J_\pm$, we can know the magnitude of the phase shift, from which we can know the light intensity. Since the light intensity is proportional to the photon number $n$, we can get information on $n$. We thus get to know $n$ without photon absorption, i.e., without changing $n$; hence the name QND [5]. (More accurate definition of QND will be given in section 7.) In contrast, conventional photodetectors drastically alter the photon number by absorbing photons. Keeping this semi-classical argument in mind, let us proceed to a fully-quantum analysis.

Fig. 1 A quantum non-demolition photodetector composed of a double-quantum wire electron interferometer. (Taken from [2])

3. Quantized light field for a waveguide mode

We assume that the measured light of frequency $\omega$, plane polarized in the $z$ direction, is confined in the $x$ and $z$ directions in a waveguide, propagating in
the y direction with the propagation constant $\beta_\omega$. A normalized mode function $u(r)$ then takes the form

$$u(r) = (0, 0, u(r)), \quad u(r) = v_\omega(x, z) \exp(i\beta_\omega y)/\sqrt{L_y}. \quad (1)$$

Here, $L_y$ is a normalization length, and $v_\omega(x, z)$ is the lateral mode function.

$u(r)$ is normalized as

$$\int \epsilon |u|^2 d^3r = 1, \quad (2)$$

where $\epsilon$ is the dielectric constant. (The permeability is unity at the optical frequency.) The quantized optical electric field in this mode is expressed as

$$\hat{E}(r, t) = \sqrt{2\pi\hbar\omega} \left[ \hat{a} u(r) e^{-i\omega t} + h.c. \right]. \quad (3)$$

The annihilation operator $\hat{a}$ thus defined is the one for a freely propagating waveguide mode. When mirrors are placed at $y = \pm L_y/2$, on the other hand, the measured light is confined in all directions, and $u(r)$ is then given by a superposition of Eq. (1) with $\pm \beta_\omega$. Using such $u(r)$ in Eq. (3), we obtain $\hat{a}$ for the confined mode [9], and $\hat{n} \equiv \hat{a}^\dagger \hat{a}$ then defines the photon number in the confined mode [7].

We can also treat the case where the measured light takes a wavepacket form, for which the “mode function” is given by a superposition of $u(r)e^{-i\omega t}$ over a narrow range of $\omega$. Replacing $u(r)e^{-i\omega t}$ with this mode function in Eq. (3), we obtain $\hat{a}$ for the wavepacket mode [9], and $\hat{n} \equiv \hat{a}^\dagger \hat{a}$ then defines the photon number in the wavepacket [7]. In any case, the number state is defined by $\hat{n}|n\rangle = n|n\rangle$, with $n = 0, 1, 2, \ldots$, and any state vector of light in the mode of interest can be expressed as

$$|\psi_{ph}\rangle = \sum_n a_n |n\rangle, \quad (4)$$

which we assume for the state before the measurement.

Either of the above three cases can be treated in a similar manner in the following discussions. However, since equations become slightly complicated in the wavepacket case, we hereafter assume the former two cases.

4. Single quantum-wire structure

Before going to the full analysis, let us consider the simplified case where the light field interacts with a single electron which is confined in a single quantum-wire structure.

Assuming for simplicity that the confinement potential in the y direction is high enough, we can decompose the y dependence of the electron wavefunction; $\psi_{el}(r, t) = \psi_{el}(x, z, t)Y(y)$. Hence, we hereafter drop the y-subband eigenfunction $Y(y)$ from equations.
The electron is emitted from the source region, which is in the thermal equilibrium (of zero temperature, for simplicity). Hence, no quantum coherence exists between the electron and photons before they interact. To describe this fact, it is convenient to consider that the initial \((t = 0)\) wavefunction of the electron takes a wavepacket form:

\[
\psi_{el}(x, z, t = 0) = e^{ikx}G(x)\varphi_a(z).
\]

Here, \(\varphi_a\) is the eigenfunction of the lowest level (which the electron is assumed to occupy) of the \(z\) subbands, and \(G\) is a localized function. When \(G\) does not change appreciably on the scale of the Fermi wavelength, the detailed form of \(G(x)\) is irrelevant to the following results. We will therefore use the simplified notation like \(|\psi_{el}\rangle = |\varphi_a\rangle\). Combining this with Eq. (4), we write for the initial state vector of the coupled photon-electron system as

\[
|\Psi\rangle = |\psi_{ph}\rangle|\psi_{el}\rangle = \sum_n a_n|n\rangle|\varphi_a\rangle.
\]

Our task is now to investigate its time evolution — I will present here only the final state, i.e., the state after the photon-electron collision.

Let us work in the Schrödinger picture, in which the optical electric-field operator \(\hat{E}(r)\) is given by \(\hat{E}(r, t = 0)\) of Eq. (3). The Hamiltonian of the coupled photon-electron system is given by

\[
H = H_{ph} + H_{el} + H_I, \quad H_I = -e\mathbf{r} \cdot \hat{E}(r),
\]

where \(H_{ph}\) and \(H_{el}\) denote the free-photon and free-electron Hamiltonians, respectively, and \(H_I\) is the photon-electron interaction in the dipole approximation. Since \(\mathbf{u}(\mathbf{r})\) varies on the scale of the photon wavelength, \(\hat{E}(r)\) does not vary appreciably on the scale of the electron Fermi wavelength. As a result, \(H_I\) induces only an “adiabatic change” in the state vector \(|2\rangle\), and we can show that the final state is simply given by \(|2\rangle\)

\[
|\Psi\rangle = \sum_n a_n e^{i\theta_n}|n\rangle|\varphi_a\rangle.
\]

where \(\theta_n = \zeta n + \text{terms independent of } n\). Here, \(\zeta\) is an effective coupling constant which is a function of the structural parameters such as the effective mass \(m^*\) and the wire width:

\[
\zeta = \frac{2\pi\hbar\omega|\langle\varphi_b|ez|\varphi_a\rangle|^2/\Delta}{\hbar^2 k/m^*} \int_{-\infty}^{\infty} |u(x, y_0, z_0)|^2 dx,
\]

where \(y_0, z_0\) denote the center position of the wire (which extends along the x axis), and

\[
\Delta \equiv \epsilon_b - \epsilon_a - \hbar\omega
\]
is the detuning energy. \((\epsilon_a\text{ and } \epsilon_b\text{ are the first and the second subband energies.})\)

We see that the final state acquires \(n\)-dependent phase shift, \(\theta_n\). If we could measure \(\theta_n\) we would be able to know the photon number \(n\). However, for the single quantum-wire structure as we are assuming in this section, there is no way to measure \(\theta_n\). In the case of \(a_n = \delta_{n,n_0}\), for example, \(\theta_{n_0}\) is the absolute phase of the wavefunction, which is not a physical quantity, and thus is unable to observe. We therefore see that we could not measure \(n\) if we used a single-wire structure.

5. Double quantum-wire structure

We now turn to the case of Fig.1; a double-wire structure composed of narrow (N) and wide (W) quantum wires. As before, suppose that an electron wavepacket is emitted from the source. As it proceeds towards the positive x direction, the electron wave is split into two, and the state vector of the coupled photon-electron system becomes

\[
|\Psi\rangle = |\psi_{ph}\rangle|\psi_{el}\rangle = \sum_n a_n|n\rangle(|\varphi^N_a\rangle + |\varphi^W_a\rangle)/\sqrt{2}, \tag{12}
\]

where \(\varphi^N_a(z)\) and \(\varphi^W_a(z)\) denote the lowest-subband eigenfunctions of the N and W wires, respectively.

Similarly to Eq. (8), the final state is shown to be \[2\]

\[
|\Psi'\rangle = \sum_n a_n|n\rangle(e^{i\theta^N_n}|\varphi^N_a\rangle + e^{i\theta^W_n}|\varphi^W_a\rangle)/\sqrt{2}. \tag{13}
\]

where \(\theta^N_W = \zeta_{N,W}n + \text{terms independent of } n\). Here, \(\zeta_N\) and \(\zeta_W\) are the effective coupling constants of the N and W wires, respectively, which are given by Eq. (10) with \(\phi_a, b \rightarrow \phi^N_{a,b}\), \(\Delta \rightarrow \Delta_{N,W}\), and \(y_0, z_0 \rightarrow y^N_{0}, z^N_{0,0}\).

Unlike the absolute phase in Eq. (8), we can measure the relative phase \(\theta^N_n - \theta^W_n\) in Eq. (13) by the method described in the next section. The relative phase is given by

\[
\theta^N_n - \theta^W_n = gn \tag{14}
\]

where \(g \equiv \zeta_N - \zeta_W\) is an overall effective coupling constant. Since the intersubband transition energy is higher in the N wire than in the W wire, the detuning energies have opposite signs: \(\Delta_N > 0, \Delta_W < 0\), as seen from Eq. (11). This results in \(\zeta_N > 0, \zeta_W < 0\), hence \(g = |\zeta_N| + |\zeta_W| \neq 0\). (Typically, \(\zeta_W \simeq -\zeta_N\), so that \(g \simeq 2\zeta_N\).) Measurement of the relative phase thus provides us with the knowledge about \(n\) (see Eq. (17) below).

Since \(\zeta_N \neq \zeta_W\) (i.e., \(g \neq 0\)) is essential to the above discussion, we also see that a double-wire structure composed of identical quantum wires would not work as a photodetector. Hence, the use of double-wire structure composed of non-identical wires is essential to the present QND photodetector.
6. Measurement of the relative phase

We can measure the relative phase $\theta_N^N - \theta_W^W$ in Eq. (13) by composing an electron interferometer. In Fig. 1, a simple interferometer is employed: The two phase-shifted components of Eq. (13) is superposed at the “mode converter”, which is composed of a thin barrier of 50 % transmittance. The mode converter plays the same role as the beam splitter does for the optical beam: the input electron waves are superposed, so that the state vector evolves into

$$|\Psi''\rangle = \sum_n a_n |n\rangle (C_{n+}|\varphi_+\rangle + C_{n-}|\varphi_-\rangle),$$

(15)

where $|\varphi_{\pm}\rangle$ are the traveling modes of the two output channels, and

$$C_{n\pm} = \left[ e^{i(\theta_N^N + \theta_0)} \pm e^{i(\theta_W^W - \theta_0)} \right] / 2.$$  

(16)

Here, the additional phase angle $\theta_0$ is a function of the structural parameters, such as the height and thickness of the barrier, of the mode converter.

We measure the intensities of the output electron waves as the interference currents, $J_+$ and $J_-$. Equations (15) and (16) yield

$$\langle J_{\pm}\rangle \propto \sum_n |a_n|^2 |C_{n\pm}|^2 = \frac{1}{2} \sum_n |a_n|^2 [1 \pm \cos(gn + \theta_0)] = \frac{1}{2} [1 \pm \langle \cos(gn + \theta_0) \rangle].$$

(17)

When the mode converter is designed in such a way that $\theta_0 = -\pi/2$, for example, this relation yields $\langle J_+\rangle - \langle J_-\rangle \propto \langle \sin gn \rangle$. We can therefore measure $n$ by measuring $J_{\pm}$.

7. QND property

As we will see in section 9, we need many electrons to reduce the measurement error. The many-electron versions of Eqs. (12), (13) and (15) are obtained by taking their Slater determinant for the electron part. (Here, each electron state must of course be different in either of spin, or the center position of the wavepacket, etc.) Since we measure $J_{\pm}$ of such a many-electron state, the state vector after the measurement is “reduced” to an eigenstate of the many-electron $J_{\pm}$. (See also section 9.) For the reduced state vector, only the photon part is of our interest. When $N_{\pm}$ electrons are found in the $\pm$ channels, the photon state after the measurement is found to be

$$|\psi_{ph}''(N_+,N_-)\rangle = \left[ P(N_+,N_-)/\binom{N}{N_+} \right]^{-1/2} \sum_n a_n (C_{n+})^{N_+} (C_{n-})^{N_-} |n\rangle,$$

(18)
where $P(N_+, N_-)$ is the probability of finding $N_+$ electrons in the $\pm$ channels (for a given $N = N_+ + N_-), and is evaluated to be

$$P(N_+, N_-) = \binom{N}{N_+} \sum_n |a_n|^2 |C_{n+}|^{2N_+} |C_{n-}|^{2N_-}. \quad (19)$$

In other words, with the probability of $P(N_+, N_-)$ the post-measurement photon state becomes $|\psi_{ph}''(N_+, N_-)\rangle$. Let us confirm the QND property using these equations.

We first consider the case where the initial photon state is a number state; $|\psi_{ph}\rangle = |n_0\rangle$. Since $a_n = \delta_{n,n_0}$ in this case, we find from Eqs. (18) and (19) that $|\psi_{ph}''(N_+, N_-)\rangle = |n_0\rangle$. That is, when the pre-measurement photon state is a number state, the post-measurement state becomes the same number state — no change occurs by the measurement either in the photon number or in the state vector! Hence the name a QND photodetector [5].

We next consider the general case where the initial photon state is given by Eq. (4) with $a_n$ being arbitrary. In this case, the general requirement of quantum mechanics requires some changes in the state vector. Otherwise, the uncertainty principle, for example, would be broken. (See the next section.) Therefore, even when you use a QND detector the state vector of the measured system must be changed [5, 2, 8]. Indeed, the post-measurement photon state, Eq. (18), is clearly different from the initial state. In particular, the photon-number distribution after the measurement is

$$\langle n|\psi_{ph}''(N_+, N_-)\rangle|^2 = \frac{|a_n|^2 |C_{n+}|^{2N_+} |C_{n-}|^{2N_-}}{\sum_m |a_m|^2 |C_{m+}|^{2N_+} |C_{m-}|^{2N_-}}, \quad (20)$$

which is different from that before the measurement, $|\langle n|\psi_{ph}\rangle|^2 = |a_n|^2$.

However, the unique property of a QND detector can be seen by considering an ensemble of many equivalent systems [5, 2, 8] — such an ensemble has been very frequently used (either explicitly or implicitly) in discussions on quantum physics [12]. For each member in the ensemble, the above equations can be applied. We can therefore calculate the density operator $\hat{\rho}$ of the ensemble as follows. Here, I will present the density operator traced over the electron degrees of freedom, $\hat{\rho}_{ph} = \text{Tr}_{el}[\hat{\rho}]$, which is of our principal interest. Before the measurement, all members have the same state vector of Eq. (4), hence

$$\hat{\rho}_{ph} = \sum_{m,n} a_m a_n^* |m\rangle\langle n|, \quad (21)$$

and the photon-number distribution over the ensemble, $\text{Prob}(n)$, is simply given by $\text{Prob}(n) = |a_n|^2$. After the measurement, on the other hand, a member in the state of Eq. (18) is found in the ensemble with the probability of Eq. (19). Therefore, the photon density operator (for a given $N = N_+ + N_-$) becomes

$$\hat{\rho}_{ph}'' = \sum_{N_+} P(N_+, N_-) |\psi_{ph}''(N_+, N_-)\rangle\langle \psi_{ph}''(N_+, N_-)|$$
\[
\sum_{n,m} a_m a^*_n |m\rangle \langle n| \left( \frac{1}{2} e^{i\zeta_N (m-n)} + \frac{1}{2} e^{i\zeta_W (m-n)} \right)^N
\]

(22)

and the distribution after the measurement is

\[
\text{Prob}''(n) = \sum_{N_+} P(N_+,N_-) |\langle n|\psi''_{ph}(N_+,N_-)\rangle|^2
\]

\[
= \sum_{N_+} \binom{N}{N_+} |a_n|^2 |C_{n+}|^{2N_+} |C_{n-}|^{2N_-}
\]

\[
= |a_n|^2 (|C_{n+}|^2 + |C_{n-}|^2) = |a_n|^2,
\]

(23)

where use has been made of Eqs. (16), (19) and (20). We thus find that the photon-number distribution over the ensemble is unchanged. In this sense the QND photodetector is said to cause no change in the “statistical distribution” of the photon number, or, to cause no “backaction” on the photon number \([5, 2, 8]\). In particular, we find from Eq. (23) that the final state has the same average and variance of \(n\) as the initial state:

\[
\langle n \rangle_{\text{final}} = \langle n \rangle_{\text{init}}, \quad \langle \delta n^2 \rangle_{\text{final}} = \langle \delta n^2 \rangle_{\text{init}}.
\]

(24)

This is in a sharp contrast with conventional photodetectors, which drastically alter the photon-number distribution by absorbing photons.

Note that all the above results referred to either the initial or the final state. It can be shown that \(\text{Prob}(n)\) does change during the measurement, i.e., during the photon-electron collision \([2, 8]\). The absence of change is claimed only for the post-measurement state, and this suffices to claim the QND property \([2, 8]\).

This is in a sharp contrast with previous QND photodetectors \([5]\), which were claimed to cause no change of \(\text{Prob}(n)\) throughout the measurement. This fact demonstrates that the operation principle of the present QND photodetector is much different from the previous ones. We recently developed a general theory which clarifies the physics of various types of QND measurement \([8]\).

8. Backaction noise generated by the measurement

We have seen that our QND photodetector causes no backaction on the measured variable — the photon number \(n\), in the sense of Eqs. (23) and (24).

On the other hand, we expect from the uncertainty principle that the detector must cause some backaction on the phase \(\phi\) — the conjugate variable of \(n\) — of the photon field \([5, 2]\).

To demonstrate this, we consider the case where the initial photon state is a coherent state \(|\xi\rangle\), for which

\[
a_n = e^{-|\xi|^2/2} \xi^n / \sqrt{n!},
\]

(25)
which yields \( \langle n \rangle_{\text{init}} = \langle \delta n^2 \rangle_{\text{init}} = |\xi|^2 \), and the phase fluctuations are evaluated to be \( \langle \delta \phi^2 \rangle_{\text{init}} \simeq 1/4|\xi|^2 \) for large \( |\xi| \) [10]. (In the large-\( |\xi| \) limit, in particular, both \( \langle \delta n^2 \rangle_{\text{init}}/(\langle n \rangle_{\text{init}})^2 \) and \( \langle \delta \phi^2 \rangle_{\text{init}} \) tend to zero, and \( |\xi| \) approaches 3, and finally Fig. 2 (f) for large \( |\xi| \).) It is convenient to introduce “quadrature variables,” \( \hat{a}_1 \) and \( \hat{a}_2 \), which correspond to the amplitudes of the cosine and sine parts of the optical field [10];

\[
\hat{a}_1 \equiv (\hat{a} + \hat{a}^\dagger)/2, \quad \hat{a}_2 \equiv (\hat{a} - \hat{a}^\dagger)/2i. \tag{26}
\]

The above fluctuations in \( n \) and \( \phi \) are translated into fluctuations of these variables as \( \langle \delta a_1^2 \rangle_{\text{init}} = \langle \delta a_2^2 \rangle_{\text{init}} = 1/4 \). Therefore, in the \( a_1-a_2 \) plane a coherent state can be represented as a circular “cloud” [10], which schematically visualizes the fluctuations, as shown in Fig. 2 (a). In this diagram, \( n \) corresponds to the square of the radial distance from the origin, and \( \phi \) to the azimuthal angle [10]. Fluctuations in \( n \) and \( \phi \) are therefore represented by the spread of the cloud in the radial and azimuthal directions, respectively.

It is instructive to consider first the case of identical wires, for which \( \zeta_N = \zeta_W (\equiv \zeta) \). In this case Eqs. (22) and (25) yield

\[
\hat{\rho}_{\text{ph}}'' = \sum_{n,m} a_n^* a_m e^{iN\zeta(m-n)} |m\rangle\langle n| = |e^{iN\zeta}|^2 \langle e^{iN\zeta}, \tag{27}
\]

where, as before, \( N \) denotes the number of colliding electrons. Thus, the identical wires just induce the phase rotation in the parameter \( \zeta \), and the final photon state is the same coherent state as the initial state except for this unimportant phase rotation. This is illustrated in Fig. 2 (b) for the case of \( N = 1 \).

For non-identical wires, on the other hand, \( \zeta_N \neq \zeta_W \), and \( \hat{\rho}_{\text{ph}}'' \) can no longer be factorized in such a simple form. In particular, off-diagonal terms, \( \langle m|\hat{\rho}_{\text{ph}}''|n\rangle \) with \( m \neq n \), are significantly reduced with increasing \( N \). This leads to phase randomization because the quantum-mechanical phase is a measure of the off-diagonal coherence. In fact, we can show for large \( |\xi| \) that [2]

\[
\langle \delta \phi^2 \rangle_{\text{final}} = \langle \delta \phi^2 \rangle_{\text{init}} + \delta \phi_{BA}^2, \quad \delta \phi_{BA}^2 \simeq N g^2/4, \tag{28}
\]

where, as before, \( g \equiv \zeta_N - \zeta_W \). The physical origin of this backaction noise, \( \delta \phi_{BA}^2 \), is sketched in Fig. 2 (c)-(f), where for simplicity \( \zeta_N = -\zeta_W (\equiv \zeta) \) is assumed. When one electron collides with the photons, the electron amplitudes in the two wires simultaneously cause rotations of angles \( \zeta_N = \zeta \) and \( \zeta_W = -\zeta \), as shown in Fig. 2 (c). As a result, the photon state is split into two clouds. When one more electron joins the game, each cloud is again split into two, and the photon state becomes as Fig. 2 (d). Similarly, we get Fig. 2 (e) for \( N = 3 \), and finally Fig. 2 (f) for large \( N \). This banana-like state is a graphical representation of \( \hat{\rho}_{\text{ph}}'' \), Eq. (22). As compared with the initial state (a), we see that the final state (f) indeed has larger phase fluctuations (which correspond to the azimuthal distribution), while the magnitude of the photon-number fluctuations (the radial distribution) remains the same. Comparison
Fig. 2 (a) When the initial photon state is a coherent state, it can be represented as a circular “cloud” in the $a_1$-$a_2$ plane. (b) When two quantum wires are identical, the photon state rotates by $\zeta$ after the collision with an electron in the wires. (c)-(f) When two quantum wires are non-identical, on the other hand, the photon state is drastically deformed. (c), (d), (e) and (f) represent the photon state after the collision with one, two, three and many electrons, respectively. In (b)-(e) a large value of $\zeta$ is assumed in order to make the diagrams vivid, whereas realistic small $\zeta$ is assumed in (f).
between (b) and (f) demonstrates that the pair of non-identical wires, for which \( \zeta_N \neq \zeta_W \), is the very origin of the backaction noise, \( \delta \phi_{BA}^2 \).

9. Measurement error

A principal postulate of quantum mechanics is that when “ideal measurement” is performed the state vector of the measured system is reduced to an eigenstate of the measured variable. For photon-number measurement, for example, ideal measurement would lead to the post-measurement density operator of the form,

\[
\hat{\rho}_{ph}^{ideal\ meas} = \sum_n |a_n|^2 |n\rangle\langle n|.
\]  

(29)

Actual measuring devices, however, are non-ideal in two points: (i) they would destroy (demolish) the photon state by, say, absorbing photons, and (ii) they have a finite measurement error. Therefore, the density operator (or the state vector) will be reduced to another form. Theory and experiment of such non-ideal measurement have been attracting much attention recently [7, 8, 11].

The present QND photodetector is a good example to understand the physics of non-ideal measurement. Although the QND photodetector does not absorb photons, it may be non-ideal because of a finite measurement error, and the state vector would not be reduced completely. Let us examine this subject, as well as the origin of the measurement error.

Measurement consists of a series of physical interactions which occur among many degrees of freedom in the measured system and the measuring apparatus [12]. In our case, the interactions consist of that between photons and electrons, that between the electrons and ammeters which measure \( J_\pm \), that between the ammeters and a recorder which records the values of \( J_\pm \), and so on. The key to treat non-ideal measurement is the fact that among these interactions we can (almost always) find an interaction process which can be approximated as ideal measurement—for such a process we can apply the above principal postulate of quantum mechanics, and everything can then be evaluated (at least in principle) [8]. Note that we do not need any additional postulate; we can treat non-ideal measurement within the standard framework of quantum mechanics [8].

In our case, we have assumed in section 7 that the measurement of the electronic current, \( J_\pm \), is ideal. As a result, the state vector of the coupled photon-electron system is reduced to an eigenstate of \( J_\pm \). The photon part of the reduced state vector is shown in Eq. (18), and the reduced density operator in Eq. (22). The photon number \( n \) is estimated from the measured values of \( J_\pm \) through Eq. (17). Since we get information on \( n \), the whole process can be called measurement of \( n \). That is, we measure \( n \) through ideal measurement of \( J_\pm \). This measurement of \( n \) is non-ideal because it has a finite measurement error. In fact, since \( J_\pm \) are quantum interference currents, they have finite quantum fluctuations [13]–[18], which make the estimation of \( n \) ambiguous.
(This is a quite general result for quantum interference devices, as shown in \cite{18, 19}.) Namely, the fluctuations of $J_\pm$ give rise to a finite error in measurement of $n$, and the present QND device works as a non-ideal measuring device of $n$. As a result, the post-measurement photon state is not completely reduced to an eigenstate of $n$, as explicitly seen from Eq. (22), which shows that $\hat{\rho}'_{ph} \neq \hat{\rho}_{\text{ideal meas}}$. As $N$ is increased $\hat{\rho}'_{ph}$ approaches $\hat{\rho}_{\text{ideal meas}}$. We thus expect that the measurement error decreases with increasing $N$. This is indeed the case; the measurement error is evaluated to be $\delta n^2_{err} = 1/g^2 N$. (30)

This result can be understood as follows. As the effective coupling $g$ is increased, the flow of information from the light field to the electrons is increased, hence $\delta n^2_{err} \propto 1/g^2$. On the other hand, we get to know the photon number by measuring the electron phase shift. To measure the phase shift, however, we need many electrons because of the number-phase uncertainty principle (of electron waves) \cite{13, 18}. This results in $\delta n^2_{err} \propto 1/N$. It was shown in Refs. \cite{18} that similar discussions can be applied to most quantum interference devices, and their fundamental limits have been derived \cite{19}.

Interestingly, if we multiply $\delta n^2_{err}$ by the backaction noise $\delta \phi^2_{BA}$, Eq. (28), we get a constant; $\delta n^2_{err} \delta \phi^2_{BA} \simeq 1/4$, whereas the number-phase uncertainty principle (of a light field) gives $\delta n^2_{err} \delta \phi^2_{BA} \geq 1/4$ \cite{10}. This means that the present device is a very effective measuring device in the sense that it extracts the information on the measured variable $n$ with the minimum cost of the backaction noise in the conjugate variable $\phi$.

10. Summary

I have analyzed a quantum non-demolition (QND) photodetector composed of a double quantum-wire electron interferometer, which measures the photon number $n$ without absorbing photons (more precisely, without changing distribution of $n$). It is shown that \textbf{(i)} If we used a single-wire structure, or if we used a double-wire structure composed of two identical wires, we could not get information on the photon number. It is therefore essential to use a double-wire structure composed of non-identical wires. \textbf{(ii)} Such a double-wire structure, on the other hand, is the very origin of the backaction noise, which appears as an increase of quantum fluctuations of the phase of the light field. \textbf{(iii)} The QND photodetector works as a non-ideal measuring device because it has a finite measurement error. As a result, the photon state is not completely reduced to an eigenstate of $n$. \textbf{(iv)} The measurement error, $\delta n^2_{err}$, comes from quantum fluctuations of electrical currents in the interferometer. Because of this fluctuation, we need many electrons to measure the phase shift which is induced by the light field. As a result, $\delta n^2_{err} \propto 1/N$, where $N$ denotes the number of colliding electrons. \textbf{(v)} The error is also inversely proportional to an effective
coupling constant $g^2$ between photons and electrons. The coupling constant is a function of the structural parameters of the quantum wires. (vi) The measurement error and the backaction noise is closely related: $\delta n_{err}^2 \delta \phi_{BA}^2 \simeq 1/4$. Namely, the backaction noise is proportional to $Ng^2$ and is also a function (through $g^2$) of the structural parameters of the wires.

These results demonstrate close relationships between measurement and fluctuations, and not only shed light on the physics of quantum measurement, but also suggest fundamental limitations and possibilities of nanostructure devices.

References

[7] See, e.g., C.W. Gardiner, Quantum Noise (Springer-Verlag, Berlin, 1991). Note that this reference uses SI units, whereas the present paper is working in cgs Gauss units.
[9] More precisely, the relation between the old and new $\hat{a}$’s are expressed as a unitary transformation, and the commutation relations are preserved.


[19] Reference [18] assumed that the coherence length in reservoirs, $\ell^c_{res}$, is short, whereas Refs. [16] assumed that $\ell^c_{res}$ is long (which was implicitly assumed by assuming the perfect Fermi distribution in reservoirs.) The noise formula in the general case, which interpolates between the two limiting cases, was given in Eq. (21) of Ref. [17], which shows that finite $\ell^c_{res}$ induces an “emission noise” in addition to the “granularity noise” derived in [16]. However, concerning the fundamental limits of quantum interference devices, which were derived in [18], the limits depend only on the granularity noise, hence apply to any case irrespective of the length of $\ell^c_{res}$. To break the limits, one must resort to well-designed many-body correlations among electrons, as discussed in [13].