CM-P00058679

Archives -

Ref.TH.1268-CERN

A CHIRAL INVARIANT DUAL MODEL

R.C. Brower
CERN - Geneva

A B S T R A C T

Unequal intercept dual N point functions are found which satisfy the Adler condition for all the external pion lines. With the no-exotics hypothesis, a chiral invariant model is constructed which, in the linear expansion of the invariants, exactly coincides with the non-linear $\mathbf{6}$ model with $(\frac{1}{2}, \frac{1}{2})$ breaking. Factorization yields a unique π , A_1 , ρ , f^0 and the standard difficulties with tachyons and ghosts. The model can be extended to axial and vector currents with the correct soft couplings, massive kaons, and higher order dual diagrams.

Ref.TH.1268-CERN

11 December 1970

The successful application ¹⁾ of the Veneziano model to processes with four pseudoscalars depends heavily on the satisfaction of the Adler condition for the external pions and the elimination of the $J = 0$ tachyon on the leading trajectory. Neither of these features has been satisfactorily extended to the N point dual model. Here the Adler condition is extended to all pion lines in an N pseudo-scalar amplitude, without destroying the good factorization properties of the equal intercept model. This provides the first opportunity to study a dual model which obeys the full consequences of chiral symmetry.

This letter will sketch the construction of this new Born term, leaving the detailed verification of its properties and discussion of alternative constructions to a forthcoming article.

By writing the four-pion amplitude in the two forms

$$\begin{aligned} & (1 - \alpha_p(s) - \alpha_p(t)) B(1 - \alpha_p(s), 1 - \alpha_p(t)) \\ &= - \frac{\alpha_p(s) \alpha_p(t)}{\alpha_p(s) + \alpha_p(t)} B(-\alpha_p(s), -\alpha_p(t)) \end{aligned} \quad (1)$$

we can consider the Adler zero either to be in a multiplicative ("quark spin") factor or to be intrinsic to the dual function. The latter interpretation appears more attractive for generalization, particularly if one hopes eventually to explain the sum rule $\alpha_p - \alpha_\pi = \frac{1}{2}$.

We apply the Adler condition to a single N point function $B(p_1, \dots, p_N) = B_N(\alpha_{ij})$, with arbitrary trajectory intercepts $\alpha_{ij}(0)$

$$B(p_1, \dots, p_N) \rightarrow 0, \text{ as } p_i^\mu \rightarrow 0 \quad (2)$$

and seek a solution to the constraints imposed on $\alpha_{ij}(0)$. As pointed out by Mandelstam ²⁾, the Adler condition for two soft pions (p_i, p_{i+1}) requires that the dipion p wave coupling $[(p_{i+1} - p_i)_\mu B^\mu]$ be universal ($q_\mu B^\mu = O(q^2)$, $q = p_i + p_{i+1}$). But, in addition, chiral algebra

$$[Q_\alpha^5, Q_\beta^5] = iK \epsilon_{\alpha\beta\gamma} I_\gamma, \quad K=1 \quad (3)$$

for $SU_2 \times SU_2$ [as opposed to $O(3,1)$, $K < 0$ or $E(3)$, $K = 0$], fixes the sign and magnitude of the universal coupling. So even with the over-all coupling g^2 of the dual model, given a real $\rho \pi \pi$ coupling, this sign $(\pm 1, 0)$ provides a new constraint.

A particularly convenient approach to the above constraints [Eqs. (2) and (3)] is to first consider the universality condition for the dipion system (p_i, p_{i+1}) . Referring the reader to Ref. 3) for the details of polology and isospin, the argument is as follows. To avoid a trivial model [i.e., Eq. (3)], we must have some non-zero charges or soft poles (at $q^\mu = p_i^\mu + p_{i+1}^\mu \rightarrow 0$) on the external lines. In $B(p_1, \dots, p_N)$ only the adjacent poles in $(p_{i-1} + p_i + p_{i+1})^2$ and $(p_i + p_{i+1} + p_{i+2})^2$ are available, so the charge must enter on one line $(p_{i-1}$ or $p_{i+2})$ and leave on the other. Finally, in order that these poles are soft and that one soft pole (or charge) is not lost under factorization in α_{ik} , we must have the mass condition

$$\alpha_{i,k}(0) = \alpha_{i+2,k}(0), \quad k > i+2 \quad (4)$$

where $\alpha_{j,j+1}(0) = -m_\pi^2$. It is encouraging to note that Eq. (4) is not only necessary, but sufficient for the existence of a conserved dual vector B^μ , as demonstrated by explicit construction in Ref. 3).

For an N pion amplitude, we must impose the mass constraint (4) for each i . By repeated application of Eq. (4), all odd trajectories are constrained to be the pion, and all even trajectories to fall into two classes $[\alpha_p(0) = \alpha_r(0) + c + d, \quad \alpha_s(0) = \alpha_r(0) - c + d]$

for even and odd i . Hence the general solution ^{*}) is

$$\alpha_{ij}(0) = \frac{c}{2} [(-)^i + (-)^j] + \frac{d}{2} [1 + (-)^{i+j}] - m_\pi^2 \quad (5)$$

The crucial observation is that the Adler condition (2) can also be satisfied if we take $d = 0$, $c \neq 0$. Contrary to "expectations" (i.e., $c = 0$, $d = \frac{1}{2}$), one needs two types of even trajectories, displaced equal distances above and below the pion.

To observe the Adler zero, we express our amplitude ^{**}), $B^+(p_1, \dots, p_N) = B(\alpha_{ij})$ with $c = \alpha_p(0) - \alpha_\pi(0) > 0$, $d = 0$,

$$B^+ = \int dV \prod_i \pi [(z_{i+1} - z_i)^{-c(-)^i + m_\pi^2 - p_i^2 - p_{i+1}^2} (z_{i+2} - z_i)^{m_\pi^2 - p_{i+1}^2}] \times \prod_{i>j} \pi (z_j - z_i)^{-2p_j \cdot p_i} \quad (6)$$

in Koba-Nielsen ⁴) variables, where dV is the Möbius invariant volume element, $\prod_i [dz_i / (z_{i+1} - z_i)] (z_a - z_b)(z_b - z_c)(z_c - z_a) / dz_a dz_b dz_c$. For $p_1^2 \rightarrow 0$, the zero comes from the vanishing of the integral,

$$\int_{z_{i-1}}^{z_{i+1}} dz_i (z_{i+1} - z_i)^{-c(-)^i - 1} (z_i - z_{i-1})^{c(-)^i - 1}$$

^{*}) To avoid ambiguity, we define $\alpha_{ij} = \alpha_{ji}$ to be the trajectory in the channel $(p_i + p_{i+1} + \dots + p_{j-1})^2 \equiv (p_j + p_{j+1} + \dots + p_{i-1})^2$, and we take all indices modulus N . Also note that no solution exists for an odd number of pions, except the equal intercept case that violates the Adler condition.

^{**}) In terms of invariants s_{ij} , there is no explicit off-mass shell dependence (p_1^2) . The intrinsic dependence through s_{ij} is sufficient to give the off-shell Adler conditions and the m_π^2 dependence of the σ model.

The striking feature of this function B^+ is that it differs so slightly from the equal intercept case. Only the measure of the integrand is modified by the invariant factor $\prod_i (z_{i+1} - z_i)^{-c(-)^i}$. This allows the trajectories to be conveniently specified by a dual diagram (see the Figure), where the "quarks" carry a new quantum $\sigma = \pm \frac{1}{2}$ (a pseudospin). For $B^+(p_1, \dots, p_N)$, the i th quark running from p_{i-1} to p_i has $\sigma = \frac{1}{2}(-)^i$, and the (ij) trajectory is displaced from the pion by the sum of the pseudospins in that channel times c . When the diagram is rotated by one unit ($p_i \rightarrow p_{i+1}$), $B^+(p_1, \dots, p_N)$ goes into a new function $B^-(p_1, \dots, p_N)$ defined by Eq. (6) with $c \rightarrow -c$. Each can be easily factorized in the four-dimension oscillator formalism ⁵⁾.

A DUAL σ MODEL

It is natural to make the no-exotics assumption, since it has led to interesting results in both dual and chiral models. We construct the N pion amplitude,

$$A_N = \frac{g^{N-2}}{2} \sum_{\text{per}} \text{Tr}(\tau_1 \dots \tau_N) [B^+(p_1, \dots, p_N) + B^-(p_1, \dots, p_N)] \quad (7)$$

by summing over all non-cyclic and non-anticyclic permutations.

For $N = 4$, this reduces to $B[-\alpha_\sigma(s), -\alpha_\rho(t)] + B[-\alpha_\rho(s), -\alpha_\sigma(t)]$, and with the sum rule $\alpha_\rho - \alpha_\sigma = \frac{1}{2}$ (or $\alpha_\sigma = \alpha_\rho - 1$) to the single beta function $B[-\alpha_\rho(s), -\alpha_\rho(t)]$. Also for $N = 4$, the spin factor $\alpha_\rho(t) [\alpha_\rho(s)]$ can be inserted to kill the tachyon in $B^+(B^-)$, and with the sum rule ($\alpha_\rho - \alpha_\sigma = \frac{1}{2}$) we return precisely to the Lovelace amplitude (1). However, no direct generalization of this procedure to higher N appears possible, even though $B^\pm$ have tachyons in only half of the even channels.

We turn now to the properties of our model, with the value of $\alpha_\rho - \alpha_\pi$ determined empirically.

1. - Spectrum

In the pionic channels there are four leading trajectories (for $q_\pm \bar{q}_\pm$), analogous to the singlet and triplet L excitation series of the quark model. In a given even pion channel, the highest trajectory α_ρ (for $q_+ \bar{q}_+$) occurs only in one term (B^+ or B^-), so we have a non-degenerate tachyon (t , quantum numbers of the σ), ρ , f^0 , etc. The alternation in isospin is the usual consequence of the symmetry of the trace, $\text{Tr}(\tau_1 \tau_2 \dots \tau_K) = (-)^K \text{Tr}(\tau_K \dots \tau_2 \tau_1)$. Again taking into account the symmetry of the trace, we can separate the two $q_+ \bar{q}_- \pm q_- \bar{q}_+$ trajectories giving a unique π , H ($I=0, B$), etc., and an $I^G=0^-$ pion (p), A_1 , etc. Finally the $q_- \bar{q}_-$ trajectory α_σ has a σ , ρ' , $f^{0'}$, etc. Since we can factorize with four-dimensional oscillators ⁵⁾, we have daughter trajectories for each leading trajectory with the usual exponential level density, and the typical abundance of ghosts.

The two unphysical states (t and p) are nonsense members of their L series (with ρ and A_1) in a spin $\frac{1}{2}$ quark model, and therefore they indicate the same failure of our embryonic spin to remove a $J=0$ state.

2. - Low energy theorems

If we expand the four-point function in s , t and m_π^2 , we obtain $(2m_\pi^2 - s - t)/F_\pi^2$ for $g^2 = (F_\pi^2 \Gamma(c) \Gamma(-c))^{-1}$, which is precisely the Weinberg ⁶⁾ expansion for the coefficient of $\text{Tr}(\tau_1 \tau_2 \tau_3 \tau_4)$. Hence the sign of our universal constant is correct. Indeed, it has been shown by Osborn ⁷⁾ that in the non-linear σ model with $(\frac{1}{2}, \frac{1}{2})$ symmetry breaking, the N pion amplitude can be expressed as

$$\sum_{\text{per}}' \text{Tr}(\tau_1 \tau_2 \dots \tau_N) F_N(s_{ij}, m_\pi^2) \quad (8)$$

where F_N depends only on the non-exotic invariants, s_{ij} and the mass m_π^2 , and by itself satisfies the Adler condition. On the basis of Ellis's ⁷⁾ uniqueness argument, we can prove that our amplitude B^+ coincides exactly with F_N when B^+ is expanded linearly (except for pion poles which are counted as inverse powers) in s_{ij} and m_π^2 .

Hence our model is a "dual σ model", with higher mass poles replacing the contact terms. Not only do we have all the low energy theorem, but also the first order $(\frac{1}{2}, \frac{1}{2})$ symmetry breaking in m_π^2 of the σ model. To complete the comparison, the free parameter $c = m_\sigma^2 - m_\pi^2$ is our measure of the spontaneous breaking of the true $SU_2 \times SU_2$ symmetric solution. In the limit $c \rightarrow 0$, we recover this solution with parity doublets.

3. - Sum rules

Usually, chirality is taken to include sum rules first derived by the $p = \omega$ technique. But these sum rules in general ⁸⁾ follow from the low energy theorems, plus dispersion relations, plus suppression of exotics. So we automatically satisfy them. In particular, we have a zero width saturation (with ghosts) of the algebraic relations of Weinberg ⁸⁾

$$[X_a(\omega), X_b(\lambda)] = i\epsilon_{abc} I_c, \quad [X_a(\omega), [X_b(\lambda), m^2]] \propto \delta_{ab}$$

Moreover, since the present model is more general (any t, m_π^2 covariant, crossing symmetric, etc.), a study of this "solution" may suggest further algebraic relations that one could then impose with the proviso of no ghosts to obtain more realistic results.

4. - Currents

The vector current [Ref. 3] with the correct $q^\mu \rightarrow 0$ behaviour is proportional to the lowest vector meson, so lends itself to pole saturation, but the precise structure for general q^μ is not known. Similarly, with pole dominance, we can construct an axial current

$$A_{\alpha}^{\mu}(q) = i \langle 0 | J_{\alpha}^{\mu}(0) | p_2, \dots, p_N \rangle,$$

$$A_{\alpha}^{\mu}(g) = \frac{g^{\mu} F_{\pi}}{m_{\pi}^2 - g^2} \frac{m_{\pi}^2}{g^2} A_N(g, \alpha) + \frac{m_A^2 F_{\pi}}{m_A^2 - g^2} \left(g^{\mu} - \frac{g^{\mu} g_2}{g^2} \right) B_{A_1}^{\nu}(g, \alpha) \quad (9)$$

+ higher sing.

The same operator ($L-L_0$ of Gliozzi, Thorn and Kaku), which gives conservation for our vector current ($q^{\mu} \approx 0$), also removes the apparent singularity in (9) at $q^2 = 0$,

$$A_N(g, p_2, \dots, p_N) = g_{\mu} B_{A_1}^{\mu}(g, p_2, \dots, p_N) + O(g^2)$$

and re-expresses the Adler condition in terms of the "Ward identities" of our dual amplitude.

If we assume higher $J = 0$ poles decouple as $m_{\pi}^2 \rightarrow 0$ (or pion-pole dominance), we have for $m_{\pi}^2 = 0$ a conserved axial with a Goldstone boson, $A_{\alpha}^{\mu} \approx -q^{\mu} F_{\pi} A_N / q^2$ for $q^2 \approx 0$. We emphasize that this is only the first (but necessary) step to constructing currents away from $q^{\mu} \approx 0$. Pole saturation essentially gives factorization for single current amplitudes, but in a dual theory it is factorization of the two current amplitudes that gives the difficult constraint ³⁾.

5. - SU₃ symmetry

Replacing the τ 's by λ 's in (7), we obtain an $SU_3 \times SU_3$ (or to be precise a $U_3 \times U_3$) model. If trajectories with λ quarks are raised by $m_K^2 - m_{\pi}^2$, amplitudes with massive K's result, which still obey the Adler condition for the external pion lines. Of course, many difficulties, including the degeneracy $m_{\eta}^2 = m_{\pi}^2$, persist.

6. - High orders

Just as for our Born term, the only change for higher order dual diagrams is in the measure of integration. For example, the planar loop has the new factor

$$\prod_{i=1}^{N-1} (z_{i+1} - z_i)^{\frac{1}{2}(1-\epsilon_i)} (z_i - T(z_N))^{\frac{1}{2}\epsilon}$$

where T is the projective transformation ⁹⁾ associated with the loop. The new factors will allow the Adler condition to hold order by order. The new quantum number **6** operates in higher loops only to exclude an odd number of external pions on a given quark loop.

Remarks

It must be admitted that the chiral properties are expected as soon as the Adler condition is satisfied, although as exemplified by the spin factor $\text{Tr}(\gamma_{5\cancel{p}_1} \gamma_{5\cancel{p}_2} \dots \gamma_{5\cancel{p}_N})$ the quadratic zeros of $E(3)$ are a danger. Generally, duality and chirality not only appear compatible, but often interdependent, so we should expect that, as new dual amplitudes appear, other chiral invariant models will be found.

There are many difficulties of the present model one would like to overcome, but the most flagrant (tachyons, no nucleons, no odd number of unnatural parity couplings, etc.) are apparently related to the lack of a basic spin $\frac{1}{2}$ constituent. Thus the role of our pseudospin in giving chirality and its close analogue with S·L coupling is intriguing. Perhaps this is a step in the right direction.

I wish to thank V. Alessandrini, D. Amati, J. Ellis and S. Fubini for many illuminating discussions. Also I am grateful for the hospitality I enjoyed at the Center for Theoretical Physics, M.I.T. and at CERN where this research was performed.

REFERENCES

- 1) C. Lovelace - Phys.Letters 28B, 265 (1968) ;
J. Shapiro - Phys.Rev. 179, 1345 (1969).
- 2) S. Mandelstam - Phys.Rev. 168, 1884 (1968).
- 3) R. Brower and J. Weis - Phys.Rev. 188, 2495 (1969) ; Phys.Rev.D
(Dec. 15, 1970).
- 4) Z. Koba and H.B. Nielsen - Nuclear Phys. B10, 633 (1969).
- 5) S. Fubini, D. Gordon and G. Veneziano - Phys.Letters 29B, 679 (1969).
- 6) S. Weinberg - Phys.Rev.Letters 17, 616 (1966).
- 7) H. Osborn - Nuovo Cimento Letters 2, 717 (1969) ; Erratum 2, 135
(1970) ;
J. Ellis - Nuclear Phys. B21, 217 (1970).
- 8) S. Weinberg - Phys.Rev. 177, 2604 (1969).
Sum rules dependent on detailed assumptions about the currents
(e.g., space-space commutators) are not included in this
concept of chirality.
- 9) C. Lovelace - Phys.Letters 32B, 703 (1970) ;
M. Kaku and Loh-ping Yu - L.R.L. Preprint UCRL-20054 (Aug. 13, 1970) ;
V. Alessandrini - CERN Preprint TH. 1215 (1970).

FIG.