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ASYMPTOTIC BEHAVIOUR OF THE PARTIAL CROSS-SECTIONS

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ABSTRACT

It is shown that the model recently proposed by Hwa, in which the cross-sections for the production of a given number of particles approach a constant asymptotically and generate a multiplicity that grows without limit, cannot satisfy "decent" scaling. It is also pointed out that the relation between the asymptotic behaviour of cross-sections with a given number of particles and cross-sections with a given number of charged particles in a multiperipheral model is very sensitive to the details of the exchanged isospin structure.

A recent cosmic ray experiment on high energy p-p interaction ¹⁾ has strongly supported the hypothesis of a logarithmic growth with the energy of the average multiplicity of the produced particles, in agreement with the prediction of the multiperipheral model (MPM). On the basis of the MPM one also expects, at least in some approximations ²⁾, a Poisson-like distribution to hold.

The simple Poisson distribution

$$P(n, s) = \frac{\sigma(n, s)}{\sigma_{\text{tot}}(s)} = \frac{(\bar{n}(s))^n}{n!} e^{-\bar{n}(s)} \quad (1)$$

with $\bar{n} = g \ln s$ predicts that the cross-sections with a given number of produced particles decrease to zero asymptotically with a power of s that is determined by the coefficient of the logarithmic growth of the multiplicity.

The same data that support the logarithmic behaviour of $\bar{n}$ are not, however, in good agreement with (1): for instance, whereas the average number of charged particles seems to behave like

$$\bar{n}_{\text{ch}} \approx .7 \ln s$$

a behaviour with energy of the four-prong cross-section of the form

$$(\ln s)^4 s^{-.7}$$

is ruled out ³⁾: a best fit to the available data in which the n_{ch} prong cross-section is parametrized as

$$\sigma(n_{\text{ch}}, s) = c_n (\ln s)^{n_{\text{ch}}} s^{-p}$$

seems to indicate ⁴⁾ values of p of the order of 0.35, i.e., half the value expected if the $\sigma(n_{\text{ch}}, s)$ were to obey (1).

Actually the measured cross-sections for the production of four or more charged particles are compatible with a constant asymptotic behaviour ³⁾, and on this basis Hwa ⁵⁾ has recently proposed a model in which $\lim_{s \rightarrow \infty} \sigma(n, s) = c_n$. The logarithmic growth of $\bar{n}$, together with the asymptotic constancy of σ_{tot} , is obtained assuming $\sigma(n, s) = (1/n^2) \theta(f(s) - n)$ with

$$\lim_{s \rightarrow \infty} f(s) s^{-a} = \text{const} \quad a > 0 \quad (2)$$

The physical basis of this model relies on the dominance of double diffractive dissociation processes in the production amplitudes. On the same basis, the limiting distribution hypothesis had been previously formulated ⁶⁾; therefore an interesting problem is to show the possibility of constructing a model that satisfies (2), and in which the one-particle distribution scales, i.e.,

$$\lim_{s \rightarrow \infty} E \frac{d^3 \sigma}{d^3 p} = f(x, p_T^2) \quad (3)$$

with $x = 2p_T/\sqrt{s}$ as usual. We will say that a model has "decent" scaling if (3) holds and a (s and x independent) constant M exists such that ⁷⁾

$$\int E \frac{d^3\sigma}{d^3p} d^2p_T < M$$

at all s . This problem has been solved in an affirmative way by Hwa ⁵⁾. He would like to show that (2) is incompatible with the "decent" scaling of the two-particle inclusive reactions $A+B \rightarrow a+b+X$, and, in particular, that if (2) holds it is impossible to find an M such that

$$\int E_a E_b \frac{d^6\sigma}{d^3p_a d^3p_b} d^2p_a^a d^2p_b^b < M \quad (4)$$

uniformly in s , x_a and x_b . In fact, if this happens

$$\int \frac{d^6\sigma}{d^3p_a d^3p_b} d^3p_a d^3p_b < M \ln^2 s$$

On the other hand, the same integral equals

$$\sum_n n(n-1) \sigma(n, s)$$

which behaves like s^a in the model under consideration.

The above argument can be trivially extended to the m particle inclusive reactions to show that if a condition analogous to (4) is imposed in a theory with constant asymptotic cross-sections, the decrease of c_n with n must be faster than n^{-m} ; however, Eq. (4) is sufficient to prevent the possibility of having a growing multiplicity generated by this mechanism.

Coming back to the MPM, we want now to examine whether the relation between $\bar{n}/\ln s$ and the power of the asymptotic vanishing of the partial cross-sections holds in general in the MPM, independently of the exchanged isospin structure.

In the hypothesis that the bulk of the produced particles are pions (or in general $I=1$ objects), several models have been examined in a recent paper by Schwimmer and the author ⁸⁾. The first model (I model) assumes the dominance of $I=1$ exchanges along the MP chain, the second (A model) assumes the alternate exchange of $I=1$ and $I=0$. A third model (H model), which we considered mainly for its simple factorizable structure, assumes the dominance of $I=\frac{1}{2}$ exchanges. All the information about particle distributions are contained in a function $S(g_c, g_o, s)$ such that the probability of having an event with n_c charged particles and n_o neutral ones is given by :

$$P(n_c, n_o, s) = \sigma(n_c, n_o, s) \sigma_{ht}^{-1}(s) =$$

$$= \frac{g^{m_c + m_o}}{m_c! m_o! S(g, g, s)} \frac{d^{m_c}}{d g_c^{m_c}} \frac{d^{m_o}}{d g_o^{m_o}} S(g_c, g_o, s) \Big|_{g_c = g_o = 0}$$

The function S turns out to be in the three models ⁹⁾:

$$S^H(g_c, g_o, s) = s^{g_o/3} \left[\cosh \left(\frac{2}{3} g_c \ln s \right) - 1 \right] \quad (5)$$

$$S^I(g_c, g_o, s) = g_c \ln s \int_0^1 e^{(u g_o \ln s)^{1/2}} \left(\frac{1-u}{u} \right)^{1/2} I_1 \left([2u(1-u)]^{1/2} g_c \ln s \right) du$$

$$\sim (\text{when } s \rightarrow \infty) (g_c \ln s)^{1/2} \int_0^1 \exp \left\{ \left(\frac{1}{2} g_o u + [2u(1-u)]^{1/2} g_c \right) \ln s \right\} (1-u)^{3/4} du \quad (6)$$

$$S^A(g_c, g_o, s) = \left[\frac{g_o^2 + 2g_c^2}{3} \ln^2 s \right]^{1/4} I_{1/2} \left[\left(\frac{g_o^2 + 2g_c^2}{3} \right)^{1/2} \ln s \right] \quad (7)$$

The multiplicities are given, in all models, by

$$\langle n_{ch} \rangle = 2 \langle n_o \rangle = \frac{2}{3} g \ln s$$

and the asymptotic behaviour of $P(n_c, n_o, s)$ is given (apart from logarithms) by $S(g, g, s)^{-1}$, which behaves as s^{-g} in the three models ⁹⁾; therefore the expected equality between the ratio $(\langle n_{ch} \rangle + \langle n_o \rangle) / \ln s = g$ and the power of the asymptotic vanishing of $P(n_c, n_o, s)$ holds. However, the cross-sections with a given number of prongs are given by ⁸⁾

$$C(m_c, s) = \sum_{m_o} \sigma(m_c, m_o, s) =$$

$$= \sigma_{tot} \frac{g^{m_c}}{m_c!} S^{-1}(g, g, s) \frac{d^{m_c}}{d g_c^{m_c}} S(g_c, g_o, s) \Big|_{\substack{g_c = 0 \\ g_o = g}}$$

and are in the three models :

$$C^H(n_c, s) = \left(\frac{2}{3} g \ln s\right)^{n_c} / (n_c! [\cosh(\frac{2}{3} g \ln s) - 1]) \rightarrow s^{-\frac{2}{3}} g$$

$$C^I(n_c, s) = \left(\frac{g \ln s}{\sqrt{2}}\right)^{n_c} \frac{1}{n_c!} \frac{\phi(\frac{n_c}{2}, n_c+1, \frac{1}{2} g \ln s)}{S^I(g, g, s)} \rightarrow s^{-\frac{1}{2}} g^{10)}$$

$$C^A(n_c, s) = \left(\frac{g \ln s}{\sqrt{3}}\right)^{n_c/2} \frac{1}{(n_c/2)!} \frac{I_{(n_c-1)/2}\left(g \frac{\ln s}{\sqrt{3}}\right) - I_{(n_c+1)/2}\left(g \frac{\ln s}{\sqrt{3}}\right)}{I_{-1/2}(g \ln s) - I_{1/2}(g \ln s)} \\ \rightarrow s^{-g(1-1/\sqrt{3})}$$

We see therefore that in the simple factorizable H model the relation between the growth of $\langle n_c \rangle$ and the asymptotic vanishing of $C(n_c, s)$ is preserved, whereas in the other two models the vanishing of $C(n_c, s)$ is substantially weaker than expected (in particular the A model, with $g \simeq 1$, gives $s^{-0.42}$ to be compared with the $s^{-0.35}$ suggested by the experimental data). The reason for this behaviour can be easily seen by examining, e.g., Eq. (6). The behaviour of $P(n_c, n_0, s)$ is determined by the asymptotic behaviour of $S(g, g, s)$, which comes from the integration region $u \simeq 2/3$, where the integral behaves like s^g . However, in the summation over n_0 we gain a factor $S(0, g, s)$, the asymptotic behaviour of which is determined by the $u \simeq 1$ region, where the integrand behaves like $s^{g/2}$.

Without proposing to take too seriously the above-mentioned model, we would like to conclude that the slowness of the decrease of the cross-sections with given number of prongs cannot yet be considered incompatible with a MPM in which a realistic isospin structure is taken into account. We also conclude that the measure of inclusive-exclusive cross-sections of the type of $C(n_c, s)$ [i.e., cross-sections for processes in which the number of the particles of some kind is fixed, but other particles can be present in arbitrary number] is not suitable for providing model independent information about the behaviour of the genuine exclusive cross-sections.

REFERENCES AND FOOTNOTES

- 1) L.W. Jones et al., Phys.Rev.Letters 25, 1679 (1970).
- 2) G.F. Chew and A. Pignotti, Phys.Rev. 176, 2112 (1968).
For a discussion of the approximations involved, see :
L. Caneschi, Nuclear Phys. (in press);
A.H. Müller, Phys.Rev. D4, 150 (1971).
- 3) K.N. Ericson, University of Michigan report No. HE-70-4.
- 4) D.E. Lyon, private communication.
- 5) R.C. Hwa, Phys.Rev.Letters 26, 1143 (1971).
- 6) J. Benecke, T.T. Chou, C.N. Yang and E. Yen, Phys.Rev. 188, 2159 (1969).
- 7) The statement that the asymptotic limit in one variable of a function of two variables is a function of the other variable only is rather trivial. For the physical relevance of the scaling hypothesis, it is therefore necessary to assume that a value $\bar{s}$ exists (and hopefully is not too large), such that, for $s > \bar{s}$, $E(d^3\sigma/d^3p)$ depends on x but not explicitly on s . If this is the case, (4) obviously holds.
- 8) L. Caneschi and A. Schwimmer, Phys.Rev. D3, 1588 (1971).
- 9) The couplings in the A model have been redefined by a factor $(\sqrt{3})^{-1}$ with respect to the notations of Ref. 8).
- 10) Equation (25) of Ref. 8) contains a misprint and the factor s^{-g^2} should actually read $s^{-g^2/2}$.