EXOTIC REACTIONS IN INCLUSIVE EXPERIMENTS

Chan Hong-Mo and Paul Hoyer ^{*)}

CERN - Geneva

A B S T R A C T

The exoticity condition on the energy dependence of inclusive reactions : $a + b \rightarrow c + X$ is examined in the framework of the "dual-resonance theory". It is found that $ab\bar{c}$ being exotic is sufficient to guarantee weak energy dependence of the cross-section to leading order. The additional condition also suggested of ab being exotic, concerns only terms of higher orders usually neglected in dual phenomenology and is, in any case, insufficient to guarantee the absence of Regge dependence to such orders.

*) On leave from the Department of Theoretical Physics, University of Helsinki.

The purpose of this note is merely to clarify certain points concerning the energy-dependence of some inclusive reactions which have been termed exotic. Most of the facts presented are probably known already to a number of experts, but in view of their immediate applications to phenomenology, it is thought worthwhile to make them generally available.

Consider the inclusive reaction

$$a + b \rightarrow c + X \quad (1)$$

Using an analogue of the optical theorem suggested by Mueller ¹⁾, the distribution of c in (1) is related to the three-body amplitude

$$a + b + \bar{c} \rightarrow a + b + \bar{c} \quad (2)$$

For c in the fragmentation region of b , (i.e., when $s_{b\bar{c}}$ is small), it has been argued ²⁾ that the system $b\bar{c}$ may be considered as a quasi-particle, so that when the quantum numbers of $a\bar{b}\bar{c}$ or the missing mass X are exotic, (2) receives no contribution from normal Regge exchange, leading thus to weak energy dependence of the inclusive cross-section.

Two objections ³⁾ have been raised against the proposal outlined above. These may best be typified by the following examples.

(A) The reaction :

$$\pi^+ p \rightarrow \pi^- + X \quad (3)$$

The quantum numbers of X , namely $(\pi^+ \pi^+ p)$, are indeed exotic, but those of the direct channel $(\pi^+ p)$ are not. Since the direct

channel can, in principle, form resonances, it seems reasonable to expect structure and, hence, Regge energy dependence in the cross-section for (3) also.

(B) The fragmentation vertex :

$$p \rightarrow k^- \quad (4)$$

By applying the criterion of Ref. 2) and the factorization theorem to reactions initiated by $\pi^\pm p$ and $K^\pm$, it can be shown ³⁾ that the Regge contribution to the vertex (4) is identically zero. At first sight, this appears to be an inconsistency ⁴⁾.

These facts led some authors ³⁾ to propose that for the cross-sections of (1) to be weakly energy-dependent, one needs not only that

$$(ab\bar{c}) \text{ is an exotic channel} \quad (5)$$

but also that

$$(ab) \text{ is an exotic channel} \quad (6)$$

All such arguments, however, depend critically on what one calls "duality", for which unfortunately there is no generally accepted definition. For the present discussion, we shall adopt for definiteness the framework of the so-called dual-resonance theory ⁵⁾, although we believe that the result may have more general validity. In this framework, then, we shall show that condition (5) is normally sufficient to guarantee a weak energy-dependence, at least in the fragmentation region where s_{bc} is small. The objections (A) and (B), as well as the condition (6) concern in general only higher order effects which are usually neglected in dual phenomenology. Moreover, if terms of such order were to be considered, then even the additional condition (6) is insufficient to guarantee the absence of Regge energy-dependence.

We first note that, at the tree-diagram level, the problem has already been solved explicitly⁶⁾. It was shown that among the 60 diagrams in the Veneziano amplitude for (2) corresponding to the various permutations of the external lines, only those 18 in which a, b and $\bar{c}$ are adjacent have the right discontinuity which contributes to the cross-section for reaction (1). Thus, if the quantum numbers of $X = (ab\bar{c})$ are exotic, as in (3), the contribution of the dual tree diagram to the energy dependent part of the cross-section vanishes. Moreover, in the fragmentation region of b , diagrams in which b and $\bar{c}$ are not adjacent have vanishing limits. Therefore, in case $b\bar{c}$ is exotic, as in (4), the contribution of the dual tree-diagram to the cross-section is again zero. At this level, then, one sees that condition (5) is indeed sufficient to imply a weak energy dependence.

Physically, the result quoted above can be understood in terms of the Veneziano amplitudes for the exclusive processes from which the cross-section of (1) is built up. In order to contribute appreciably to the fragmentation region of b , i.e., to yield fast backward c in the c.m. system, a diagram must have b adjacent to $\bar{c}$. Furthermore, to have resonance effects in the ab channel as suggested in (A), the diagram must also have a and b adjacent, as shown in Fig. 1(a). This is not allowed, however, in either of the examples (3) and (4).

Following Di Giacomo et al.⁷⁾, if one assumes that adjacent lines be approximated by a bundle of resonances, one notes that Fig. 1(a) is of order g^2 , where g is the universal coupling parameter in the Veneziano model. Summing over all resonance states in the bundle X then yields again the result of the six-point function quoted above, which thus corresponds to the cross-section of (1) to lowest order in g , namely g^4 .

Consider now exclusive processes of the next higher order, namely the diagrams of Fig. 1(b), (c) and (d) with two superparticle lines inserted. Summation over the resonance states now yields the

so-called twisted loop diagram of order g^6 , which, according to Veneziano folklore, corresponds to diagrams with Pomeron exchanges as shown in Fig. 2(b),(c) and (d).

Inasmuch as one believes that the tree-diagrams of the Veneziano model are already a fair approximation to reality at low energies, one ought to consider higher order terms in g^2 as small corrections. One would then claim that Pomerons are weakly coupled compared to proper Reggeons, and that their exchanges become important only when enhanced at high energy by their slower energy-dependence. This hypothesis has some experimental justification in that asymptotic values of total cross-sections for non-exotic channels are much smaller than their values near threshold. In particular, in the case of $\pi\pi$ interactions, the hypothesis forms the basis of Chew and Snider's multiperipheral bootstrap⁸⁾. It has also been made implicitly in many (if not most) applications of dual models to phenomenology⁹⁾.

In this scheme then, Fig. 2(a) is expected to dominate when the incoming energy s_{ab} is small. When s_{ab} is large, however, as in the present case, Fig. 2(b) becomes appreciable also because of its slower energy dependence. If the quantum numbers of $ab\bar{c}$ happen to be exotic, then Fig. 2(a) vanishes. In that case, the fragmentation distribution is dominated by Fig. 2(b) which is independent of energy. There are indeed energy dependent corrections to the distribution from Fig. 2(c) and (d). However, so long as one remains in the fragmentation region where $s_{b\bar{c}}$ is small in absolute value, the $b\bar{c}$ vertex in both Fig. 2(c) and (d), involving the Pomeron, is small compared to the Regge vertex in Fig. 2(b), yielding thus only weak energy-dependence for the over-all inclusive cross-sections. Notice that $s_{b\bar{c}}$ is actually negative. Hence the terms under consideration correspond to Pomeron exchange below its physical threshold. The neglect of such terms is common practice in dual phenomenology, a close analogue of which being found in the analysis by Lovelace of $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$ ⁹⁾.

Finally, we note that if one insists on taking such higher order terms into consideration, then even condition (6) is insufficient to guarantee the absence of Regge energy dependence. Indeed,

ab exotic implies the absence in Fig. 1 of diagrams with a and b adjacent. However, this still leaves Fig. 1(c) or 2(c), which contains a contribution from proper Regge exchange.

The arguments given above, although reasonable and consistent with our present knowledge of dual theories, must nonetheless be regarded as only tentative. It is, for example, extremely difficult to give a general estimate of the relative magnitude of those terms we labelled "second order". For lack of anything better, we suggest that a crude measure of g^2 is given by (i) the width of low-mass resonances relative to the slope of the trajectory α' , and (ii) the asymptotic value of σ_{total} as compared with its value near threshold for non-exotic channels. If so, the Pomeron coupling could be weaker than normal Regge couplings by as much as one order of magnitude.

The correctness or otherwise of our predictions can ultimately be judged only by direct comparison with experiment which is unfortunately not yet available at present. To our knowledge, only two pieces of indirect evidence exist which are both in support of our considerations :

(A) The fragmentation distributions of $p \rightarrow \pi^-$ for $p_{\text{lab}} < 1\text{GeV}$, are available from the following bubble chamber experiments :

$$K^+ + p \rightarrow \pi^- + x \quad {}^{10)} \quad 12.7 \text{ GeV} \quad (7)$$

$$K^- + p \rightarrow \pi^- + x \quad {}^{11)} \quad 9 \text{ GeV} \quad (8)$$

$$\pi^+ + p \rightarrow \pi^- + x \quad {}^{10)} \quad 7 \text{ GeV} \quad (9)$$

$$\pi^- + p \rightarrow \pi^- + x \quad {}^{10)} \quad 25 \text{ GeV} \quad (10)$$

In order to compare the four reactions better, we shall define invariant distributions normalized by the corresponding asymptotic σ_{total} for the direct channel, thus,

$$f_K^* = \left(\omega \frac{d^3 \sigma}{d p^3} \right)_{Kp} / [\sigma_T(Kp)]_\infty \quad (11)$$

$$f_\pi^* = \left(\omega \frac{d^3 \sigma}{d p^3} \right)_{\pi p} / [\sigma_T(\pi p)]_\infty \quad (12)$$

Now reaction (7) satisfies both conditions (5) and (6) and is thus expected by previous arguments to receive a contribution only from Pomeron exchange, i.e., Fig. 2(b). Factorization and the constancy of the Pomeron contribution then imply that $f_{K^+}^*$ represents the Pomeron contribution also for all the three other reactions at all energies. Next, reactions (8) and (10) satisfy neither (5) nor (6), while (9) satisfies only (5). Thus the differences

$$\Delta f_K^* = f_{K^-}^* - f_{K^+}^*$$

and

$$\Delta f_\pi^* = f_{\pi^-}^* - f_{\pi^+}^*$$

represent the "first-order" Regge contribution to reactions (8) and (9), corresponding to Fig. 2(a), while the difference

$$\Delta' f^* = f_{\pi^+}^* - f_{K^+}^*$$

represents a "second order" term corresponding to Fig. 2(d). The experimental values of Δf_K^* and Δf_π^* taken at the experimental energies shown are respectively around 100 p.c. and 50 p.c. of $f_{K^+}^*$ (10), (11). When scaled by the assumed $1/\sqrt{s}$ dependence to 7 GeV, they are both approximately 100 p.c. of $f_{K^+}^*$. On the other hand,

$\Delta'f^*$ at 7 GeV/c is consistent with zero and is given an upper limit of ~ 10 p.c. by the experimenters¹⁰⁾. In our language, this means that the "second order" term, Fig. 2(d) is down by at least one order of magnitude from those terms of the "first order" represented by Fig. 2(a).

(B) The fragmentation distributions $p \rightarrow K^\pm$ are available from the classic counter experiment of Allaby et al., at 19.2 GeV¹²⁾ :

$$P + P \rightarrow K^+ + X \quad (13)$$

$$P + P \rightarrow K^- + X \quad (14)$$

Both of these reactions satisfy conditions (5) and (6), with (14) having in addition $a\bar{c}$ and $b\bar{c}$ exotic. In the notation of Fig. 1 and 2 then, reaction (13) receives contributions from (b) and (c), which are of order g^6 in the cross-section. To reaction (14), however, only diagrams with three super-particle states inserted can contribute. These correspond to double loop diagrams and are of higher order, namely g^8 . One finds that the experimental cross-section for (14) is lower than that for (13) by a factor varying between 5 and 300 in the range measured ($-0.5 < p_{||}^L < 0.2$, $p_{\perp} < 0.70$).

Though by no means conclusive, these evidences are a fair indication that our arguments may be correct.

Two remaining points need to be clarified.

(i) The assumption that the fragmentation vertices of Fig. 2(c) and (d) are small compared with those of Fig. 2(a) and (b) is expected to be valid only for small $s_{b\bar{c}}$. When $s_{b\bar{c}}$ becomes large also, say for example in the double-Regge region, then the slower energy-dependence of Pomeron-exchanges in (c) and (d) will favour them over the vertices with normal Regge exchange in (a) and (b). Notice, however, that $s_{b\bar{c}}$ in the laboratory frame depends only on

p_c , the momentum of c and not on the incoming energy. There is thus always a finite range of p_c for which our arguments are expected to be valid. Practically, we suggest that $|p_c| \lesssim 1 \text{ GeV (lab.)}$ is a reasonable criterion.

(ii) The condition (5) implies a weak energy dependence only to leading order. For those reactions where the fragmentation vertex is zero to leading order such as $p \rightarrow K^-$, this "weak" dependence may represent a sizeable percentage variation with energy.

ACKNOWLEDGMENTS

We have benefitted from discussion with many colleagues both at Stony Brook and at CERN, only some of whom can be mentioned here by name : V. Alessandrini, D. Amati, R. C. Brower, J. Ellis, J. Finkelstein, P.H. Frampton, M. Jacob, C. Quigg, C-I Tan, R.E. Waltz and J.M. Wang. Not all of them, however, agree with our viewpoint presented here.

FOOTNOTES AND REFERENCES

- 1) A.H. Mueller, Phys.Rev. D2, 2963 (1970).
- 2) Chan Hong-Mo, C.S. Hsue, C. Quigg and J.M. Wang, Phys.Rev. Letters 26, 672 (1971).
- 3) Chung-I Tan, private communication ;
J. Ellis, J. Finkelstein, P. H. Frampton and M. Jacob, CERN preprint, TH. 1316 (1971).
- 4) Exoticity of the direct channel implies a real amplitude, but the converse is in general not true. Indeed, it is well known already in two-body collisions that duality plus factorization can lead to real amplitudes for non-exotic reactions, e.g., $K^- p \rightarrow \pi^- \Sigma^+$. What may appear inconsistent in (B) is thus not that a non-exotic channel such as $K^- p \rightarrow K^- + X$ can have flat cross-sections, but that the fragmentation vertex $p \rightarrow K^-$ under Regge exchange is identically zero.
- 5) For reviews see e.g.,
Chan Hong-Mo, Proc.Roy.Soc.London A 318, 379 (1970) ;
V. Alessandrini, D. Amati, M. Le Bellac and D. Olive, CERN preprint, TH 1160 (1970) ;
G. Veneziano, Kiev Conference (1970).
- 6) C.E. De Tar, Kyungsik Kang, Chung-I Tan and J.H. Weis, Brown MIT preprint (1971).
- 7) A. Di Giacomo, S. Fubini, L. Sertorio and G. Veneziano, Phys. Letters 33B, 171 (1970) ;
G. Veneziano, Princeton Institute preprint (1971), and
Talk at the Tel Aviv Conference on Duality, Israel (1971).
- 8) G.F. Chew, T. Rogers and D.R. Snider, Phys.Rev. D2, 765 (1970).
- 9) See e.g., C. Lovelace, Phys.Letters 28B, 264 (1968) ;

- 9) See e.g., C. Lovelace, Phys.Letters 28B, 264 (1968) ;
B. Petersson and N.A. Törnqvist, Nuclear Phys. B13, 629
(1969).
- 10) M.S. Chen et al., University of Rochester preprint (submitted
to Amsterdam Conference) (1971).
- 11) See e.g., C. Quigg, invited paper for the Pasadena Meeting
on Hadron Physics at Intermediate Energies (1971),
quoting data from Stony Brook of M. Foster et al.
- 12) J.V. Allaby et al., CERN Report, 70-12 (1970).

FIG.1

FIG.2