Frame Dependence of Vector Dominance in the Veneziano Model and a New Test of ρ Universality*

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Certain frame-dependent predictions of vector dominance are studied in terms of simple, parameter-free Veneziano models. It is shown that the transverse helicity-frame differential cross section for $\pi^-p \rightarrow \rho^0 n$ has a sharp forward peak, in accord with recent experiments. Within our approach (which is more general than the Veneziano one) this peak is shown to provide a new test and verification of ρ universality. The ratio ρ_{1-1}/ρ_{11} of $\pi^-p \rightarrow \rho^0 n$ density-matrix elements is also discussed.

I. INTRODUCTION

THERE are two important statements of vector dominance which have created much controversy during the past two years:

(A) The amplitude for $\gamma(\text{isovector}) + p \rightarrow \pi^+ + n$ is proportional to $\pi^- + p \rightarrow \rho^0(\text{transverse}) + n$.

(B) If we denote by $\sigma_{\perp}$ (σ_{11}) the differential cross section for $\pi^\pm$ photoproduction by isovector photons polarized perpendicular (parallel) to the production plane, and by ρ_{mn} the density-matrix elements for $\pi^-p \rightarrow \rho^0 n$, then the following relation holds:

$$\frac{\sigma_{\perp} - \sigma_{11}}{\sigma_{\perp} + \sigma_{11}} = \frac{\rho_{1-1}}{\rho_{11}}. \quad (1)$$

However, both statements (A) and (B) are ambiguous unless the quantization axis for the spin of the ρ meson is chosen. The most important choices so far have been: (i) the helicity frame, (ii) the Jackson frame, and (iii) a frame defined by the condition $\text{Re} \rho_{10} = 0$ (Donohue-Högaasen).¹

With respect to the statement (A), the most prominent feature of $\gamma p \rightarrow \pi^+ n$ is a forward narrow peak (of width $\approx \mu^2$; $\mu = \text{pion mass}$); hence it is essential to determine the frame in which $\pi^-p \rightarrow \rho^0(\text{transverse}) n$ has also such a peak. With respect to (B), it is important to determine the frame where Eq. (1) is expected to be valid, at least in the asymptotic ($s \rightarrow \infty$) limit. Of course, if vector dominance makes sense, the frames for (A) and (B) must coincide.

It must be stressed that the choice of proper frame is a dynamical question and cannot be solved by purely kinematical analysis. An answer can be obtained by

appeal to proper dynamical models, and here we shall use a simple Veneziano model with no free parameters (and no satellite terms). One of the most important conclusions to emerge is that the presence of a sharp forward peak in the transverse helicity-frame differential cross section for $\pi^-p \rightarrow \rho^0 n$ offers a new, hitherto unnoticed, test of ρ -meson universality. Such a peak is supported by recent experimental information²; therefore, our analysis offers a new verification of ρ universality.

II. BASIC KINEMATIC RELATIONS AND FORWARD STRUCTURE IN JACKSON FRAME

We shall denote by $f_{\lambda_N - \lambda_{\bar{N}}, \lambda_\rho}^{\sigma, \sigma_e}$ the parity-conserving t -channel helicity amplitudes (PCTHA), and by $\tilde{f}_{\lambda_N - \lambda_{\bar{N}}, \lambda_\rho}^{\sigma, \sigma_e}$ the singularity-free PCTHA³ (with the Wang kinematic factor extracted); as usual, $\sigma \equiv P(-)^J = \pm$ (=normality) and $\sigma_e \equiv C(-)^J = \pm$ (also well specified in asymptotic PCTHA of $\pi N \rightarrow \rho N$). Then the conspiracy relation

$$\tilde{f}_{01}^{--}(s, 0) = \frac{m^2 - \mu^2}{4M} \tilde{f}_{11}^{++}(s, 0), \quad (2)$$

with M , m , and μ the nucleon, ρ -meson, and pion masses, is known to hold, and will subsequently be of particular interest.

For a possible peak of width $\approx \mu^2$, pion exchange is expected to be responsible. In addition, A_2 and A_1 are exchanged. A_2 satisfies (2) by evasion, and its over-all contribution vanishes in the forward direction. Finally, A_1 (which contributes to f_{10}^{--} and f_{11}^{--}) is expected to be unimportant.

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¹ J. T. Donohue and H. Högaasen, Phys. Letters **25B**, 554 (1967).

² J. Scharenguivel *et al.*, Phys. Rev. Letters **24**, 332 (1970).

³ B. Diu and M. Le Bellac, Nuovo Cimento **53A**, 158 (1968). In terms of their notation, $\tilde{f}_{00}^{--} = \tilde{f}_1^{--}$, $\tilde{f}_{10}^{--} = \tilde{f}_2^{--}$, $\tilde{f}_{01}^{++} = \tilde{f}_3^{++}$, and $\tilde{f}_{11}^{++} = \tilde{f}_4^{++}$. As is well known, there is a second conspiracy relation in $\pi N \rightarrow \rho N$: $f_{00}^{--}(s, 0) = i f_{00}^{--}(s, 0)$; in our model this is satisfied by evasion.

With these observations, it turns out to be sufficient to construct Veneziano models for the invariants B_1 , B_2 , and B_3 [in the notation of Ref. 3]; in terms of them the PCTHA are

$$f_{00}^{-+} = \frac{\sqrt{(-t)}}{2Mm\tau} [2m^2sB_1 + (t+m^2-\mu^2)sB_2 + \tau^2B_3],$$

$$\tilde{f}_{01}^{++} = -B_1 \quad (3)$$

$$\tilde{f}_{01}^{-+} = \frac{1}{4}(t+m^2-\mu^2)B_1 + \frac{1}{2}tB_2, \quad \tilde{f}_{11}^{++} = MB_1.$$

and, in our simple model,

$$f_{10}^{--} = f_{11}^{--} = 0.$$

In the expression for f_{00}^{-+} ,

$$\tau^2 \equiv [t - (m + \mu)^2][t - (m - \mu)^2].$$

Clearly, pion exchange contributes to the PCTHA f_{00}^{-+} and f_{01}^{-+} .

It can be seen now very easily (and in fact irrespective of specific model) that the transverse differential cross section for $\pi^-p \rightarrow \rho^0n$ in the Jackson frame ($\equiv \sigma_T^{(J)}$) has no forward peak. This transverse cross section receives contributions only from helicity states with $\lambda_\rho = \pm 1$. Since in the Jackson frame the density-matrix elements $\rho_{mn}^{(J)}$ are directly related to t -channel helicity amplitudes $f_{\lambda_N \lambda_{\bar{N}}, \lambda_\rho \lambda_\pi}$, therefore

$$\rho_{mn}^{(J)} = \sum_{\lambda_N \lambda_{\bar{N}}} f_{\lambda_N \lambda_{\bar{N}}, m0} f_{\lambda_N \lambda_{\bar{N}}, n0}^*.$$

We see that from the PCTHA containing π only f_{01}^{-+} contributes to $\sigma_T^{(J)}$. Clearly, a forward peak will arise in $\sigma_T^{(J)}$ if and only if f_{01}^{-+} has a term with a pole at $t = \mu^2$. However, the amplitude f_{01}^{-+} is sense-nonsense at $\alpha_\pi(t) = 0$ and its residue must contain a nonsense factor $\alpha_\pi(t)$; this cancels the pole at $t = \mu^2$. Thus, our first conclusion is that $\sigma_T^{(J)}$ has no forward peak⁴ and that the Jackson frame is not proper for a test of vector dominance.

III. VENEZIANO MODEL AND FORWARD STRUCTURE IN HELICITY FRAME

With the isospin decomposition

$$B_i = B_i^{(+)}\delta_{\alpha\beta} + \frac{1}{2}B_i^{(-)}[T_\alpha, T_\beta]$$

we proceed in the construction of simple Veneziano models for $B_1^{(-)}$, $B_2^{(-)}$, and $B_3^{(-)}$; the $B_i^{(+)}$ correspond to isospin exchange $I_i = 0$ and do not contribute to $\pi^-p \rightarrow \rho^0n$. In accord with phenomenological fits to backward $\gamma N \rightarrow \pi N$ and duality arguments for $p\bar{p} \rightarrow \pi^+d$ and $\gamma N \rightarrow \pi N$, we shall take an $N_\alpha - N_\gamma$

⁴ In the Jackson (and in the Donohue-Högaasen) frame our conclusions are in agreement with Y. Avni and H. Harari, Phys. Rev. Letters **23**, 262 (1969). However, as it becomes clear in Secs. III and IV, the transverse helicity-frame differential cross section (with the ρ universality ratio $g_{\rho NN}/g_{\rho\pi\pi} = 1$) does have a sharp forward peak.

exchange-degenerate baryon trajectory $\alpha_N(s)$, but a Δ_8 non-exchange-degenerate trajectory $\alpha_\Delta(s)$.

From Eqs. (3), f_{00}^{-+} is the only PCTHA containing B_3 ; it is also the only amplitude containing a pion pole. Then we take

$$B_3^{(-)}(s, t, u) = c_1[B(-\alpha_\pi(t), \frac{1}{2} - \alpha_N(s)) + s \leftrightarrow u] \\ + c_2[B(-\alpha_\pi(t), \frac{1}{2} - \alpha_\Delta(s)) + s \leftrightarrow u \\ - B(\frac{1}{2} - \alpha_N(s), \frac{1}{2} - \alpha_\Delta(u)) - s \leftrightarrow u], \quad (4)$$

where $B(x, y)$ is the Euler B function. It is easily seen that this amplitude has correct $s \rightleftharpoons u$ crossing, corresponds to a nondegenerate α_Δ and for $s \rightarrow \infty$ $t = \text{fixed}$ behaves $\sim s^{\alpha_\pi(t)}$, as (3) demands. Now, near the pion pole ($t \rightarrow \mu^2$),

$$B_3^{(-)} \approx - \frac{2(c_1 + c_2)}{\lambda(t - \mu^2)} = - \frac{2g_{\pi NN}g_{\rho\pi\pi}}{t - \mu^2}, \quad (5)$$

where λ = trajectory slope (common).

The invariant $B_2^{(-)}$, which appears in f_{00}^{-+} and f_{01}^{-+} , is expected to contain a contribution from pion exchange; and since f_{01}^{-+} has no pole at $t = \mu^2$, the simplest form with a nondegenerate Δ is

$$B_2^{(-)}(s, t, u) = b_1[B(1 - \alpha_\pi(t), \frac{1}{2} - \alpha_N(s)) - s \leftrightarrow u] \\ + b_2[B(1 - \alpha_\pi(t), \frac{1}{2} - \alpha_\Delta(s)) - s \leftrightarrow u \\ + B(\frac{1}{2} - \alpha_N(s), \frac{1}{2} - \alpha_\Delta(u)) - s \leftrightarrow u]. \quad (6)$$

Near the nucleon pole ($s \rightarrow M^2$),

$$B_2^{(-)} \approx -(b_1 + b_2)/\lambda(s - M^2). \quad (7a)$$

A straightforward calculation of the one-elementary-nucleon contribution with a coupling⁵

$$g_{\rho NN}\bar{u}(p_2)\gamma_\mu u(p_1)e_\rho^\mu \quad (e_\rho^\mu = \rho \text{ polarization}) \quad (7b)$$

gives

$$B_2^{(-)} = g_{\pi NN}g_{\rho NN}/(s - M^2). \quad (7c)$$

From Eqs. (3), $B_1^{(-)}$ contributes to PCTHA with both $\sigma = +$ and $\sigma = -$; hence, it will contain two parts, say, $B_{1\pm}^{(-)}$. Because of the conspiracy relation (2), these will satisfy

$$B_{1+}^{(-)}(s, t=0) = B_{1-}^{(-)}(s, t=0). \quad (8)$$

Such a relation requires a conspiring pair either of Regge poles or of Regge cuts. Since we know of no simple crossing-symmetric models involving Regge cuts, the only possibility to satisfy (2) [with $\tilde{f}_{01}^{-+}(s, 0) \neq 0$] in the Veneziano approach is in terms of a conspiring parity doublet of poles. In fact, the simplest construction of $B_1^{(-)}$ proceeds by taking a conspirator trajectory $\alpha_{\pi'}$ identical to the pion,

$$\alpha_{\pi'}(t) \equiv \alpha_\pi(t) = \lambda(t - \mu^2),$$

⁵ The model can be extended to include magnetic ρ nucleon coupling, as well. The magnetic coupling contributes only to the invariant amplitudes B_5 , B_6 , and B_8 , which in the t channel are controlled by A_2 and A_1 exchange and thus do not affect the basic structure of $\pi^-p \rightarrow \rho^0n$ at small $-t$.

and relation (8) valid for all t (not just $t=0$). Then

$$\begin{aligned} B_{1+}^{(-)}(s,t) &\equiv B_{1-}^{(-)}(s,t) \equiv B_1^{(-)}(s,t) \\ &= a_1[B(1-\alpha_\pi(t), \tfrac{1}{2}-\alpha_N(s)) - s \leftrightarrow u] \\ &\quad + a_2[B(1-\alpha_\pi(t), \tfrac{1}{2}-\alpha_\Delta(s)) - s \leftrightarrow u] \\ &\quad + B(\tfrac{1}{2}-\alpha_N(s), \tfrac{1}{2}-\alpha_\Delta(u)) - s \leftrightarrow u, \end{aligned} \quad (9)$$

which corresponds to a conspirator choosing nonsense at $\alpha_\pi(t)=0$. For $s \rightarrow M^2$ we find by comparing the Veneziano and the one-elementary-nucleon contributions

$$B_1^{(-)} \approx -\frac{a_1+a_2}{\lambda(s-M^2)} = -\frac{g_{\pi NN}g_{\rho NN}}{s-M^2}. \quad (10)$$

With (5), (7), and (10) the constants c_1+c_2 , b_1+b_2 , and a_1+a_2 are expressed in terms of the πN , $\rho\pi$, and ρN couplings.

It is straightforward now to calculate the limits of the above Veneziano expressions for $s \rightarrow \infty$ and t fixed. Equations (4) and (5) give

$$B_3^{(-)}(s,t) \rightarrow g_{\pi NN}g_{\rho\pi\pi}K_\pi(s,t)(\mu^2-t)^{-1}, \quad (11)$$

where

$$K_\pi(s,t) \equiv \Gamma(1-\alpha_\pi(t))(1+e^{-i\pi\alpha_\pi(t)})(\lambda s)^{\alpha_\pi(t)}. \quad (12)$$

Equations (6), (7a), (7c), (9), and (10) give

$$-B_1^{(-)}(s,t) \rightarrow B_2^{(-)}(s,t) \rightarrow g_{\pi NN}g_{\rho NN}s^{-1}K_\pi(s,t). \quad (13)$$

Expressions (3) of the PCTHA directly give the longitudinal and transverse differential cross sections in the Jackson frame:

$$\begin{aligned} \sigma_L^{(J)} &= \sum_{\lambda_N, \lambda_{\bar{N}}} |f_{\lambda_N \lambda_{\bar{N}}, 00}|^2 \\ &= \frac{(-t)}{4M^2 m^2 \tau^2} |2m^2 s B_1^{(-)} \\ &\quad + (t+m^2-\mu^2)s B_2^{(-)} + \tau^2 B_3^{(-)}|^2 \quad (14) \\ \sigma_T^{(J)} &= \sum_{\lambda_N, \lambda_{\bar{N}}, \lambda_\rho=\pm 1} |f_{\lambda_N \lambda_{\bar{N}}, \lambda_\rho 0}|^2 \\ &= \frac{s^2}{4M^2 \tau^2} [\tau^2 |B_1^{(-)}|^2 \\ &\quad + |(t+m^2-\mu^2)B_1^{(-)} + 2tB_2^{(-)}|^2], \end{aligned} \quad (15)$$

and with (13),

$$\sigma_T^{(J)} \rightarrow \frac{g_{\pi NN}^2 g_{\rho NN}^2}{4M^2 \tau^2} |K_\pi(s,t)|^2 [\tau^2 + (t+\mu^2-m^2)^2].$$

We thus verify that $\sigma_T^{(J)}$ is a smooth function of t and has no forward peak.

Let us calculate now the longitudinal and transverse differential cross sections ($\sigma_{L,T}^{(H)}$) in the helicity frame, where the density-matrix elements are directly related with s -channel helicity amplitudes (SHA) $M_{\lambda_\pi \lambda_N, \lambda_\rho \lambda_{N'}}$. In our model, with only $B_1^{(-)}$, $B_2^{(-)}$, $B_3^{(-)} \neq 0$, the

SHA with $\lambda_\rho=0$ are as follows:

$$\begin{aligned} M_{0\frac{1}{2}, 0\frac{1}{2}} &= 0, \\ M_{0-\frac{1}{2}, 0\frac{1}{2}} &= \frac{(-t)^{1/2}}{2Mm} [sB_2^{(-)} + (t+m^2-\mu^2)B_3^{(-)}]. \end{aligned}$$

Hence the longitudinal differential cross section is

$$\begin{aligned} \sigma_L^{(H)} &= \sum_{\lambda_N \lambda_{N'}} |M_{0\lambda_N, 0\lambda_{N'}}|^2 \\ &= \frac{(-t)}{4M^2 m^2} |sB_2^{(-)} + (t+m^2-\mu^2)B_3^{(-)}|^2. \end{aligned} \quad (16)$$

The transverse helicity-frame differential cross section is readily obtained by using $\sigma_L^{(H)} + \sigma_T^{(H)} = \sigma_L^{(J)} + \sigma_T^{(J)}$. Thus from (14)–(16),

$$\sigma_T^{(H)} = \frac{1}{2M^2} [| -sB_1^{(-)} + tB_3^{(-)} |^2 + | tB_3^{(-)} |^2]. \quad (17)$$

Replacing the asymptotic expressions (11) and (13),

$$\begin{aligned} \sigma_T^{(H)} &= g_{\pi NN}^2 g_{\rho\pi\pi}^2 \frac{|K_\pi(s,t)|^2}{2M^2} \\ &\quad \times \left[\left| \frac{g_{\rho NN}}{g_{\rho\pi\pi}} + \frac{t}{\mu^2-t} \right|^2 + \frac{|t|^2}{(\mu^2-t)^2} \right]. \end{aligned} \quad (18)$$

So far we did not impose any relation on the ρN and $\rho\pi$ couplings. The universality of ρ meson demands that the ratio

$$R \equiv g_{\rho NN}/g_{\rho\pi\pi} \quad (19)$$

be equal to unity. Then

$$\begin{aligned} \sigma_T^{(H)} &= g_{\pi NN}^2 g_{\rho\pi\pi}^2 \frac{1}{2M^2} |K_\pi(s,t)|^2 \\ &\quad \times \left[\frac{\mu^4}{(t-\mu^2)^2} + \frac{t^2}{(t-\mu^2)^2} \right]. \end{aligned} \quad (20)$$

The term $\mu^4(t-\mu^2)^{-2}$ does produce now a sharp forward peak (of width $\sim \mu^2$). The results of a calculation at pion energies 2.7, 4.1, 8.0, and 11.2 GeV are given in Fig. 1 (solid curves). For comparison, the fits to data from a careful experiment of the Rutherford-Purdue collaboration² are also given (dashed curves); the agreement is good.

The same cross sections are plotted against charged-pion photoproduction data in the forward direction (Fig. 2); clearly, the model reproduces experiment very well.

IV. TEST OF ρ -MESON UNIVERSALITY

Let us now see what happens if we vary the ratio (19) away from the universality value $R=1$. From

FIG. 1. Solid curves: differential cross sections $\sigma_T^{(H)}$ at pion energies $p_0=2.7, 4.1, 8.0$, and 11.2 GeV predicted by our (parameter-free) Veneziano model Eq. (20). Dot-dashed curves: same at $p_0=8.0$ GeV for values of the ratio $R=g_{\rho NN}/g_{\rho\pi\pi}=0.5, 2$, and 5 . Dashed curves: Rutherford-Purdue fits to experimental data (Ref. 2). The cross sections are always normalized at $t=-\mu^2$.

Fig. 1 (dot-dashed curves), it is clear that $R=0.5$ or $R=5$ is in strong disagreement with the Rutherford-Purdue results; for $R=5$ the forward peak disappears; for $R=0.5$ (and for all $R<0.5$, as we find) the resulting shape does not resemble at all the fits to data. $R=2$ appears not to be excluded, but its trend at $-t \gtrsim \mu^2$ is not supported; this is best seen when the corresponding cross section is plotted against the photoproduction data (Fig. 2); at $-t \gtrsim \mu^2$, $R=2$ falls definitely below the experimental points. Clearly, the precise shape of $\sigma_T^{(H)}$ is very sensitive to the exact value of the ratio (19). We conclude that, within our approach, the existence of a sharp peak in $\sigma_T^{(H)}$ is a new test (and verification) of the ρ universality, in addition to other well-known tests.

Apart from the Regge-Veneziano factor $K_\pi(s,t)$, the form (20) of $\sigma_T^{(H)}$ results also in the Born-approximation model [elementary pion+nucleon exchange with a $\rho N\bar{N}$ coupling as in (7b)].^{6,7} Near $t=0$ it is $K_\pi(s,t) \approx 2$ so that a similar test of ρ universality can be carried in the Born model, as well; clearly, the procedure of the test and the verification is more general than the Veneziano model.⁸ However, it is well known that for $-t \gtrsim \mu^2$ the Born model fails completely (see in Fig. 2 the plot of the gauge-invariant Born model for $\gamma p \rightarrow \pi^+ n$), whereas the Regge-Veneziano form (20) leads to reasonable results for much larger $-t$, as well.

Finally, we can calculate the shape of the transverse differential cross section in the Donohue-Högaasen frame. It follows that again $\sigma_T^{(D-H)}$ is a smooth function of t , with no structure whatsoever near $t=0$.

Before we close this section, notice that with expressions (11) and (13) and with $R=1$, Eq. (16) gives for the helicity-frame longitudinal cross section

$$\sigma_L^{(H)} = \frac{1}{4M^2} g_{\pi NN}^2 g_{\rho\pi\pi}^2 |K_\pi(s,t)|^2 \frac{(-t)m^2}{(t-\mu^2)^2}.$$

This vanishes in the forward direction. However, it increases very fast with $(-t)$ so that at $t \approx -\mu^2$ exceeds $\sigma_T^{(H)}$ by more than an order of magnitude. The corresponding $\pi^- p \rightarrow \rho^0 n$ differential cross section ($\sim \sigma_T^{(H)} + \sigma_L^{(H)}$) significantly exceeds the experimental values. To account correctly, exchange of A_2 , which contributes significantly for $-t \gtrsim \mu^2$, as well as of A_1 must be taken into account, and this is in accord with detailed analyses of charged-pion photoproduction.⁹ However, it is beyond the purpose of the present paper.

V. RATIO ρ_{1-1}/ρ_{11}

It is straightforward to calculate the ratio $\rho_{1-1}^{(H)}/\rho_{11}^{(H)}$ in the model of Sec. III. The SHA with $\lambda_\rho = \pm 1$ are as follows:

$$\begin{aligned} M_{0, \frac{1}{2}, \pm 1 \frac{1}{2}} &= 0, \\ M_{0, -\frac{1}{2}, 1 \frac{1}{2}} &= (1/\sqrt{2}M)(-sB_1 + tB_3), \\ M_{0, -\frac{1}{2}, -1 \frac{1}{2}} &= -(1/\sqrt{2}M)tB_3. \end{aligned}$$

⁹ E. Argyres, A. P. Contogouris, C. S. Lam, and S. Ray (unpublished) contains a detailed study of $\gamma N \rightarrow \pi N$. See also M. Ahmad, Fayyazuddin, and Riazuddin, Phys. Rev. Letters **23**, 504 (1969).

⁶ C. Cho and J. Sakurai, Phys. Letters **30B**, 119 (1969).

⁷ D. Horn and M. Jacob, Nuovo Cimento **56A**, 63 (1968).

⁸ A similar test can be carried out in a model with a non-conspiring Regge pion plus a conspiring pair of Regge cuts of magnitude estimated via duality and finite energy sum rules from s - and u -channel baryon states. For the equivalent model for $\gamma p \rightarrow \pi^+ n$, see J. D. Jackson and C. Quigg, Phys. Letters **29B**, 236 (1969).

FIG. 2. Solid curves: properly normalized differential cross sections $\sigma_T^{(H)}$ as in Fig. 1 [= the $\gamma p \rightarrow \pi^+ n$ differential cross section of Eq. (26)] plotted against the $\gamma p \rightarrow \pi^+ n$ data of R. Diebold, SLAC Report No. SLAC-Pub-673, 1969 (unpublished). Dot-dashed curve: $\sigma_T^{(H)}$ calculated from Eq. (18) with $R=2$ at $p_0=8.0$ GeV (properly normalized). Dot-dot-dashed curve: the gauge-invariant Born model.

Our model gives $\rho_{-1-1} \neq \rho_{11}$ so that instead of ρ_{1-1}/ρ_{11} we should actually consider the ratio $\rho_{1-1}/\frac{1}{2}(\rho_{11} + \rho_{-1-1})$. We find

$$\frac{\rho_{1-1}^{(H)}}{\frac{1}{2}(\rho_{11}^{(H)} + \rho_{-1-1}^{(H)})} = \frac{2(-t)\mu^2}{t^2 + \mu^4}. \quad (21)$$

To make a theoretical prediction about the asymmetry ratio $(\sigma_{\perp} - \sigma_{11})/(\sigma_{\perp} + \sigma_{11})$, we must construct an equivalent model for forward photoproduction of charged pions. In the CGLN¹⁰ notation for the invariant amplitudes of $\gamma N \rightarrow \pi N$, the simplest model is as follows.⁹ For the PCTHA containing pion exchange,

$$\begin{aligned} \tilde{f}_{01}^{-+}(s, t) &= A^{(-)} + tB^{(-)} \\ &= \beta_1 \left[\frac{B(-\alpha_{\pi}(t), \frac{1}{2} - \alpha_N(s))}{1 - \alpha_{\pi}(t) - \alpha_N(s)} - s \leftrightarrow u \right] \\ &\quad + \beta_2 [B(1 - \alpha_{\pi}(t), -\alpha_{\Delta}(s)) - s \leftrightarrow u \\ &\quad + B(\frac{1}{2} - \alpha_N(s), \frac{1}{2} - \alpha_{\Delta}(u)) - s \leftrightarrow u]. \end{aligned} \quad (22)$$

A straightforward comparison to the gauge-invariant Born model determines β_1 and β_2 and leads to the following limit for $s \rightarrow \infty$, $t = \text{fixed}$ ⁹:

$$A^{(-)} + tB^{(-)} = \frac{1}{2} \lambda e g_{\pi NN} \frac{\mu^2 + t}{\mu^2 - t} K_{\pi}(s, t). \quad (23)$$

For our purpose, the simplest model proceeds with $C^{(-)} = D^{(-)} = 0$, so that the PCTHA $\tilde{f}_{11}^{++}$, which

contains the conspirator π' , has the following Veneziano representation:

$$\begin{aligned} \frac{1}{M} \tilde{f}_{11}^{++}(s, t) &= A^{(-)} = a_1 [B(1 - \alpha_{\pi'}(t), \frac{1}{2} - \alpha_N(s)) - s \leftrightarrow u] \\ &\quad + \alpha_2 [B(1 - \alpha_{\pi'}(t), \frac{1}{2} - \alpha_{\Delta}(s)) - s \leftrightarrow u \\ &\quad + B(\frac{1}{2} - \alpha_N(s), \frac{1}{2} - \alpha_{\Delta}(u)) - s \leftrightarrow u]. \end{aligned} \quad (24)$$

With $\alpha_{\pi'}(t) = \alpha_{\pi}(t)$, the photoproduction conspiracy relation $\tilde{f}_{01}^{-+}(s, 0) \sim \tilde{f}_{11}^{++}(s, 0)$ gives⁹

$$A^{(-)} \xrightarrow{s \rightarrow \infty} \frac{1}{2} \lambda e g_{\pi NN} K_{\pi}(s, t) \quad (t = \text{fixed}). \quad (25)$$

In this model the differential cross section for $\gamma p \rightarrow \pi^+ n$ is

$$\begin{aligned} \sigma &= \frac{1}{32\pi} (|A^{(-)}|^2 + |A^{(-)} + tB^{(-)}|^2) \\ &= \frac{\lambda^2 e^2 g_{\pi NN}^2}{64\pi} |K_{\pi}(s, t)|^2 \frac{t^2 + \mu^4}{(t - \mu^2)^2}, \end{aligned} \quad (26)$$

i.e., exactly proportional to $\sigma_T^{(H)}$ of Eq. (20). Furthermore it is well known that $\sigma_{\perp}$ (σ_{11}) receives contribution only from $+$ ($-$) normality exchange, so that the asymmetry ratio

$$\begin{aligned} \frac{\sigma_{\perp} - \sigma_{11}}{\sigma_{\perp} + \sigma_{11}} &= \frac{(|A^{(-)}|^2 - |A^{(-)} + tB^{(-)}|^2)}{(|A^{(-)}|^2 + |A^{(-)} + tB^{(-)}|^2)} \\ &= 2(-t)\mu^2(t^2 + \mu^4)^{-1} \end{aligned}$$

¹⁰ G. F. Chew, M. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. 106, 1345 (1957).

is identical to the ratio given in Eq. (21). Since our Veneziano models for both $\pi N \rightarrow \rho N$ and $\gamma N \rightarrow \pi N$ are asymptotic, the conclusion is that, at least at small $|t|$, with increasing energy, the two ratios are expected to tend to the same value.¹¹

Finally, it is readily seen that the ratio ρ_{1-1}/ρ_{11} in the Jackson or Donohue-Högaasen frame is completely

¹¹ The same conclusion is obtained in the Born model (Ref. 6). Note that the present experimental data for $(\sigma_1 - \sigma_{11})/(\sigma_1 + \sigma_{11})$ at 3.4 GeV and for $\rho_{1-1}^{(H)}/\rho_{11}^{(H)}$ at 4 GeV [R. Diebold and J. Poirier, Phys. Rev. Letters **22**, 255 (1969)] show a significant difference between the two ratios. This may be due to the following reasons: (i) finite ρ mass corrections important at relatively low s , (ii) nonleading terms in s left out in our asymptotic expressions, and (iii) significant experimental error in the present results for $\rho_{1-1}^{(H)}/\rho_{11}^{(H)}$.

different from (27). Thus, in agreement with other considerations^{6,12} our final conclusion is that the helicity frame offers the only meaningful choice for the definition and test of vector-dominance relations.

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Towards Inclusion of Spin in the Generalized Veneziano Model for $K^- p \rightarrow \pi^- \pi^+ \Lambda$

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By combining the $U(6,6)$ supermultiplet scheme with the Veneziano model, we are able to remove some of the arbitrariness of the Petersson-Törnqvist model. The model is applied to $K^- p \rightarrow \pi^- \pi^+ \Lambda$. The results are encouraging.

INTRODUCTION

RECENT studies¹ of three-particle production data using a model proposed by Petersson and Törnqvist have had considerable success. In this model a reaction is described by an amplitude of the form

$$A = K \sum_{i=1}^{12} B_i.$$

The sum is over the 12 five-point Veneziano terms corresponding to noncyclic permutations of the external lines. K is a kinematic factor which, for processes with over-all abnormal parity, is taken as

$$K = \epsilon_{\alpha\beta\gamma\delta} P_1^\alpha P_2^\beta P_3^\gamma P_4^\delta,$$

where the P_i are any four of the external particle momenta. This kinematic factor takes no account of the spin or isospin of the external particles. In this paper we take an amplitude of the form

$$A = \sum_{i=1}^{12} F_i B_i^i.$$

Following the suggestion of Mandelstam² and the recent work of Delbourgo and Rotelli,³ we extend the latter's prescription for including spin and isospin in

FIG. 1. The permutations remaining after excluding those involving exotic meson exchanges.

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