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ON SIXTH-ORDER CORRECTIONS
TO THE ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON

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A B S T R A C T

The gauge-invariant contribution to the anomalous magnetic moment of the electron (or muon) from four diagrams has been calculated. The result is infra-red convergent and numerically equal to :

$$(\alpha/\pi)^3 \{ 0.477 \pm 0.023 \}.$$

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The set of Feynman diagrams that has been calculated is shown in the figure. It consists of :

- 1) the "double corner" graph ;
- 2) the corner and self-energy graph (at opposite sides) ;
- 3) the "mirror" image of graph 2) ;
- 4) the symmetric double self-energy graph.

The contribution of this set to the anomalous magnetic moment is gauge invariant and infra-red convergent.

We used parametric representations for the renormalized vertex and self-energy insertions. These representations split naturally into an explicitly infra-red divergent part, which has the form of a renormalization subtraction term, plus a part which is a smooth integral, two-dimensional for the vertex and one-dimensional for the self-energy. In this integral one can put the photon mass λ , equal to zero. Using this split form for both insertions in each graph we can write the anomalous magnetic moment contribution

$$a = a_{dd} + a_{cd} + a_{dc} + a_{cc}$$

where index d stands for infra-red divergent and c for infra-red convergent. Due to the factorization of the infra-red divergent parts a_{dd} is a product of two infra-red divergent factors and the second order anomalous magnetic moment. a_{cd} and a_{dc} are the products of an infra-red divergent part and the finite part of the fourth order anomalous magnetic moment from the graph obtained by "collapsing" the corresponding self-energy or vertex part. Finally, a_{cc} is a multi-dimensional infra-red-convergent integral corresponding to partially renormalized insertions. As the renormalization constants for both vertex and self-energy are linear in $\log \lambda$, a_{dd} will give rise to a quadratic $\log \lambda$ dependence while a_{cd} and a_{dc} only give a linear dependence.

In evaluating a_{cc} we carried out the necessary algebra by hand and checked it by computer using the program SCHOONSCHIP (written by M. Veltman) which also furnished us with part of the FORTRAN deck used in the numerical integration. Subsequently we integrated the parametric integrals numerically using the subroutine LSD¹⁾.

We performed several partial checks on the form of the integrands. Our parametrization allowed us in a simple way to "collapse" either a vertex part or a self-energy part, thus reducing the integral to the finite part of the corresponding fourth order graphs. These checks all agreed with the expectations.

Finally our results are (labelled according to the numbering of the graphs in the Figure)

$$a_1 = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \frac{1}{2} \log^2 \lambda + \left(\frac{67}{24} - \frac{\pi^2}{18} - \frac{1}{3} \pi^2 \log 2 + \frac{1}{2} \zeta(3) \right) \log \lambda \right. \\ \left. + \frac{65}{24} - \frac{5\pi^2}{72} - \frac{5}{12} \pi^2 \log 2 + \frac{5}{8} \zeta(3) - 2.652 \pm 0.016 \right\}$$

$$a_2 = a_3 = \left(\frac{\alpha}{\pi}\right)^3 \left\{ -\frac{1}{2} \log^2 \lambda + \left(-\frac{13}{8} + \frac{\pi^2}{18} + \frac{1}{6} \pi^2 \log 2 - \frac{1}{4} \zeta(3) \right) \log \lambda \right. \\ \left. - \frac{43}{64} + \frac{7\pi^2}{144} + \frac{1}{12} \pi^2 \log 2 - \frac{1}{8} \zeta(3) + 3.041 \pm 0.006 \right\}$$

$$a_4 = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \frac{1}{2} \log^2 \lambda + \left(\frac{11}{24} - \frac{\pi^2}{18} \right) \log \lambda \right. \\ \left. + \frac{5}{48} - \frac{\pi^2}{36} - 3.162 \pm 0.011 \right\}$$

from which we get the sum

$$a = \sum_{i=1}^4 a_i = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \frac{47}{32} - \frac{1}{4} \pi^2 \log 2 + \frac{3}{8} \zeta(3) + 0.268 \pm 0.023 \right\}$$

or

$$a = \left(\frac{\alpha}{\pi}\right)^3 \{ 0.477 \pm 0.023 \}$$

The uncertainty on the numbers is the 90 % confidence limits estimated by the integration routine. The errors from the independent graphs have been added quadratically because of the statistical nature of the integration routine. As in previous calculations we observe large cancellations between the individual gauge-dependent graphs.

Including the present results, 29 of the 72 graphs contributing to the anomalous magnetic moment of the electron have been calculated ²⁾. Of the remaining 43 only 23 are independent due to the mirror symmetry.

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