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LIGHT-CONE ANALYSIS OF INCLUSIVE
ELECTROMAGNETIC PROCESSES

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A B S T R A C T

The scaling behaviours of the processes $e^-e^+ \rightarrow (h_1 + \dots + h_n) + \text{anything}$ and $e(\mu) + p \rightarrow e(\mu) + (h_1 + \dots + h_n) + \text{anything}$ are studied assuming that multiple products of operators have light-cone singularities analogous to those proposed by Brandt and Preparata for the product of two electromagnetic currents.

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The experimental confirmation ¹⁾ of Bjorken's ²⁾ conjecture that the structure functions of deep inelastic electroproduction should scale has been discussed in several theoretical frameworks - notably parton models ^{3),4)}, dual resonance models ⁵⁾ and operator products near the light cone ⁶⁾. Since these models are equally successful in describing the SLAC data, it is important to investigate their applicability to other processes and try to discriminate between them. The deep annihilation process $e^-e^+ \rightarrow p + \text{anything}$ has been studied in parton models and in the dual resonance model, with differing results ⁷⁾. Drell and Yan and Landshoff and Polkinghorne ⁸⁾ have also studied the inclusive process $e(\mu) + N \rightarrow \text{hadron} + \text{anything}$ using parton models, and obtained similar scaling predictions.

In this note it is suggested how a natural extension of the light-cone expansion ideas of Brandt and Preparata ⁶⁾ may be used to study the scaling behaviour of inclusive electromagnetic processes such as $e^-e^+ \rightarrow (h_1 + h_2 + \dots + h_n) + \text{anything}$ or $e(\mu) + N \rightarrow e(\mu) + (h_1 + \dots + h_n) + \text{anything}$ ^{*}). To do this it is first shown that the behaviours of these processes in a certain limit are related to certain light-cone singularities of semi-time-ordered multiple operator products. It is then suggested following Brandt and Preparata ⁶⁾ that the singularity structure of these operator products may be

$$P(\phi_1(x_1), \dots, \phi_n(x_n), \phi_{n+1}(0)) \underset{x_1^2 \rightarrow 0}{\sim} (x_1^2)^{-r} \sum_m x_1^{\alpha_1} \dots x_1^{\alpha_m} \tilde{O}_{\alpha_1 \dots \alpha_m}^m(x_2, \dots, x_n, 0) \quad (1)$$

In order to estimate the strength of the c number light-cone singularity $(x_1^2)^{-r}$ it is further suggested that the dimensions of the multi-local operators appearing in the expansion (1) are given by the sums of dimensions of local operators.

From these assumptions it follows that the structure functions ⁷⁾ $\bar{W}_1$ and $\nu \bar{W}_2$ of the process $e^-e^+ \rightarrow h + \text{anything}$ scale in the way suggested by Drell, Levy and Yan. Predictions for

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^{*}) The h_i being hadrons : π , K, p, etc.

the process $e(\mu) + N \rightarrow e(\mu) + h + \text{anything}$ are similar to those of the parton model in their common range of validity. To develop the light-cone approach these two processes are discussed in some detail : further generalizations are straightforward.

In the one-photon exchange approximation the partial cross-section for the process $e^-e^+ \rightarrow h + \text{anything}$ can be expressed ⁷⁾ in terms of the tensor

$$\overline{W}_{\mu\nu} = \sum_n \langle 0 | J_\mu(0) | h(p), n \rangle \langle n, h(p) | J_\nu(0) | 0 \rangle \times (2\pi)^4 \delta^4(q - p - p_n) \quad (2)$$

which can be written in terms of two structure functions :

$$\overline{W}_{\mu\nu} = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \overline{W}_1(q^2, \nu) + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) \overline{W}_2(q^2, \nu) \quad (3)$$

where q = momentum of incoming photon, p = momentum of h , p_n = momentum of unobserved hadrons, $\nu = p \cdot q$. Using the LSZ reduction technique on the matrix elements in (2) one finds in the case where h is a boson ^{*)}

$$\begin{aligned} \overline{W}_{\mu\nu} &= (m_h^2 - p^2)^2 \int d^4x d^4y e^{-ip(x-y)} \langle 0 | T^*(J_\mu(0), \phi(x) | n \rangle \langle n | T^*(\phi^+(y), J_\nu(0)) | 0 \rangle \\ &\quad \times (2\pi)^4 \delta^4(q - p - p_n) \\ &= (m_h^2 - p^2)^2 \int d^4x d^4y d^4z e^{-ip(x-y)} e^{iqz} \\ &\quad \times \langle 0 | T^*(J_\mu(z), \phi(x)) T^*(J_\nu(0), \phi^+(y)) | 0 \rangle \end{aligned} \quad (4)$$

Thus the structure functions $\overline{W}_1$ and $\overline{W}_2$ of the process $e^-e^+ \rightarrow h + \text{anything}$ are related to a Fourier transform of the vacuum expectation value of a semi-time-ordered product of four operators. If one defines

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^{*)} This and subsequent steps proceed analogously if h is a fermion. ϕ is an interpolating field for h : e.g., $(1/f_\pi m_\pi^2) \partial_\mu A_\mu^\pi$ for a pion.

$$F_{\mu\nu}(z, p) \equiv \int d^4x d^4y e^{-ip(x-y)} \langle 0 | T^*(J_\mu(z), \phi(x)) T^*(J_\nu(0), \phi^+(y)) | 0 \rangle \times (M_h^2 - p^2)^2$$

then

$$\bar{W}_{\mu\nu}(q, p) = \int d^4z e^{iqz} F_{\mu\nu}(z, p) \quad (5)$$

Clearly the scaling behaviour of $\bar{W}_1$ and $\bar{W}_2$ as $q^2, \nu \rightarrow \infty$ is governed by the singularity of $F_{\mu\nu}(z, p)$ as $z^2 \rightarrow 0$, and hence by the $z^2 \rightarrow 0$ singularity structure of

$$T^*(J_\mu(z), \phi(x)) T^*(J_\nu(0), \phi^+(y)) \quad (6)$$

The question arises of what the light-cone singularity structure of the multiple operator product (6) is likely to be. Brandt and Preparata ⁶⁾ have proposed that for a product of two fields ^{*)} :

$$P(A(x), B(0)) \underset{x^2 \rightarrow 0}{\sim} (x^2)^{-r} \sum_m x^{\alpha_1} \dots x^{\alpha_m} O_{\alpha_1, \dots, \alpha_m}^m(0) \quad (7a)$$

It is natural to suppose a simple generalization

$$P(A_1(x_1), \dots, A_n(x_n), A_{n+1}(0)) \underset{x^2 \rightarrow 0}{\sim} (x^2)^{-r} \sum_m x^{\alpha_1} \dots x^{\alpha_m} \tilde{O}_{\alpha_1, \dots, \alpha_m}^m(x_1, \dots, x_n, 0) \quad (7b)$$

where the $\tilde{O}_{\alpha_1 \dots \alpha_m}^m(x_2, \dots, x_n, 0)$ are multi-local operators. In order to estimate the strength of the light-cone singularity $(x^2)^{-r}$ in (7b) one needs to know the scale dimensions of multi-local operators. We suggest that the lowest dimensional multi-local operators may have

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*) P indicates any type of product - e.g., time-ordered, Wick, etc.

the same dimensions as products of the lowest dimensional local operators. Since one believes ⁹⁾ that one knows which are the local operators of lowest dimension, one can now estimate the leading light-cone singularities in expansions of the form (7b).

In the case of the quadrilocal operator (6), one is interested in the lowest dimensional tri-local operators which could appear in an expansion. Since the tri-local operators should have singularities corresponding to external particles, in order to contribute on-mass shell in (5), one expects them to involve the ϕ field. Analogously to the energy-momentum tensor $\Theta_{\mu\nu}$ in the light-cone expansion of two currents, one would expect tri-local products

$$P(\Theta_{\mu\nu}(0), \phi(x), \phi^+(y))$$

to contribute to the leading light-cone singularity in an expansion of the relevant operator product (6). This singularity would then be $(z^2)^{-1}$.

The analysis of the scaling behaviour of the structure functions $\bar{W}_1(q^2, \nu)$ and $\bar{W}_2(q^2, \nu)$ of Eq. (3) then proceeds analogously to the case of electroproduction. One finds

$$\begin{aligned} \bar{W}_{\mu\nu} \sim & (q_\mu q_\nu - g_{\mu\nu} q^2) \bar{V}_1(q^2, \nu) \\ & + (\nu^2 g_{\mu\nu} - \nu(p_\mu q_\nu + p_\nu q_\mu) + q^2 p_\mu p_\nu) \bar{V}_2(q^2, \nu) \end{aligned} \quad (8)$$

$$\text{as } q^2, \nu \rightarrow \infty, \text{ where } \left. \begin{aligned} \bar{V}_1 &\sim \nu^{-1} \bar{f}_1(\nu/q^2) \\ \bar{V}_2 &\sim \nu^{-2} \bar{f}_2(\nu/q^2) \end{aligned} \right\} \text{ as } q^2, \nu \rightarrow \infty.$$

Since $\bar{W}_1 = + q^2 \bar{V}_1 - \nu^2 \bar{V}_2$ and $\bar{W}_2 = q^2 \bar{V}_2$ it follows that $\bar{W}_1$ and $\nu \bar{W}_2$ both scale. Hence the light-cone results agree with the parton model results of Drell and Yan ⁷⁾.

As a check on the assumption (7b) and the assumption on the dimensions of multi-local operators one may note that $\bar{W}_1$ and $\bar{W}_2$ contribute to the structure functions W_1 and W_2 for the deep inelastic scattering of a time-like photon: these have the structure

(9)

Conventional light-cone expansions ⁶⁾ would suggest that W_1 and $\sqrt{\nu} W_2$ should scale. If the light-cone singularity in the product (6) were stronger than that suggested by our ansatz (e.g., if there were multi-local operators with dimensions lower than those postulated) then $\bar{W}_1$ and $\sqrt{\nu} \bar{W}_2$ would increase as $q^2, \nu \rightarrow \infty$. Hence cancellations between the different terms in (9) would be required for the scaling of W_1 and $\sqrt{\nu} W_2$ to be recovered. The complexity of the discontinuity equation (9) is the reason why the operator product expansions of Brandt and Preparata and of Frishman have not been applicable to the process $e^- e^+ \rightarrow h + \text{anything}$.

The ansatz (7b) for the light-cone singularities of multiple operator products can be extended to other processes, e.g., $e^- e^+ \rightarrow (h_1 + \dots + h_n) + \text{anything}$ and $e(\mu) + N \rightarrow e(\mu) + (h_1 + \dots + h_n) + \text{anything}$. One must reduce into the matrix elements analogous to (2) all the fields corresponding to final state particles. The current-current light-cone $[z^2 \rightarrow 0 \text{ in (4)}]$ will then be relevant in the limit where $q^2, q \cdot p_i$ ($i = 1 \dots n$) and in the case of electroproduction $q \cdot p_t$ all $\rightarrow \infty$ at the same rate, while the $p_i \cdot p_j$ and $p_i \cdot p_t$ are kept fixed. In the case of electroproduction this corresponds to final state hadrons being fragments of the target nucleon. ^{*)}

As an example consider the process $e(\mu) + N \rightarrow e(\mu) + h + \text{anything}$. Following Drell and Yan and Landshoff and Polkinghorne ⁸⁾ we introduce integrated structure functions ω_1 and ω_2 ^{**) :}

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*) p_t = momentum of nucleon target.

**) We denote $p_t \cdot q = \nu$, $p \cdot q = \nu_p$, $p \cdot p_t = K$.

$$\begin{aligned}
 \mathcal{W}_{\mu\nu} &= \int d^4p \delta^{(+)}(p^2 - m_h^2) \delta(p_i \cdot q - \nu) \delta(p \cdot p_i - \mathcal{K}) \times \\
 &\quad \sum_n \langle N(p_i) | J_\mu(0) | h(p), n \rangle \langle n, h(p) | J_\nu(0) | N(p_i) \rangle (2\pi)^4 \delta^4(p_i + q - p - p_n) \\
 &\equiv - (g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \mathcal{W}_1(\nu, \nu_p, q^2, \mathcal{K}) \\
 &\quad + (p_\mu^i - \frac{\nu}{q^2} q_\mu) (p_\nu^i - \frac{\nu}{q^2} q_\nu) \mathcal{W}_2(\nu, \nu_p, q^2, \mathcal{K})
 \end{aligned}
 \tag{10}$$

As in the deep annihilation case (4) one has

$$\begin{aligned}
 \mathcal{W}_{\mu\nu} &= \int d^4p \delta^{(+)}(p^2 - m_h^2) \delta(p_i \cdot q - \nu) \delta(p \cdot p_i - \mathcal{K}) \times \\
 &\quad \int d^4x d^4y d^4z e^{iqz} e^{-ip(x-y)} \langle N(p_i) | T^*(J_\mu(z), \phi(x)) T^*(J_\nu(0), \phi^\dagger(y)) | N(p_i) \rangle
 \end{aligned}$$

and as before the scaling behaviours of the structure functions $\mathcal{W}_1$ and $\mathcal{W}_2$ are determined by the $z^2 \rightarrow 0$ light-cone singularity of the operator (6). The light-cone analysis proceeds as in the earlier case except that instead of (5) one must study the behaviour at q^2 , $\nu_p, \nu \rightarrow \infty$ and $\mathcal{K}$ fixed of objects of the form

$$\int d^4z e^{iqz} G(z, p, p_i)
 \tag{11}$$

Note this is one of the limits studied in the parton model : there $\mathcal{K} \rightarrow \infty$ was also examined. The behaviour of objects (11) in this limit has been studied by Frishman et al. ¹⁰⁾ and by Müller and Rühl ¹¹⁾. The light-cone singularity dominates except possibly on a set of measure zero. Introducing $\mathcal{U}_1$ and $\mathcal{U}_2$ by analogy with the $\bar{V}_1$ and $\bar{V}_2$ of Eq. (8), one finds

$$\begin{aligned}
 \mathcal{W}_{\mu\nu} &\sim (q_\mu q_\nu - g_{\mu\nu} q^2) \mathcal{U}_1(\nu, \nu_p, q^2, \mathcal{K}) \\
 &\quad + (\nu^2 g_{\mu\nu} - \nu (p_\mu^i q_\nu + p_\nu^i q_\mu) + q^2 p_\mu^i p_\nu^i) \mathcal{U}_2(\nu, \nu_p, q^2, \mathcal{K})
 \end{aligned}$$

where

$$\begin{aligned} U_1 &\sim \nu^{-2} f_1(\nu/q^2, \nu_p/q^2, \kappa) \\ U_2 &\sim \nu^{-3} f_2(\nu/q^2, \nu_p/q^2, \kappa) \end{aligned}$$

Hence the structure functions νW_1 and νW_2 of Eq. (10) should scale.

These scaling behaviours are similar to those expected by Drell and Yan and by Landshoff and Polkinghorne⁸⁾ in one of the limits they studied in the parton model. The limit $\kappa \rightarrow \infty$, ν/κ fixed which they also studied does not seem to be accessible in the light-cone approach unless one makes assumptions about the behaviour as $\kappa \rightarrow \infty$ of objects of the form (11).

Finally some random observations : it is desirable that the validity of the expansion (1) should be examined in models¹²⁾ previously used to study light-cone expansions of the products of two operators. Naturally we find no trace of the factorizing properties of W_1 and W_2 suggested by Drell and Yan⁸⁾. The generalization of the above analysis to processes with more than one final state hadron detected is straightforward. The scaling behaviours of inclusive electroproduction structure functions should be discernible in the deep inelastic muon scattering experiment planned to start at N.A.L. in 1972.

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