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SCALE AND CHIRAL SYMMETRY BREAKING

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A B S T R A C T

The absence of a q -number δ term in the hadronic energy momentum density $\theta_{00} = \theta_{00} + \delta + u$ is compatible with the meson baryon scattering analysis of von Hippel and Kim.

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There has recently been much work ¹⁾⁻⁷⁾ on the possibility that scale and conformal invariance may be useful approximate dynamic symmetries. It is hypothesized that one or more scalar isoscalar mesons may dominate the trace of the hadronic energy momentum tensor. The hadronic energy momentum tensor is usually ¹⁾⁻³⁾ decomposed as follows

$$\theta_{00} = \bar{\theta}_{00} + \delta + u \quad (1)$$

where $\bar{\theta}_{00}$ is chiral invariant and has dimension 4, so that it is also scale invariant, δ is chiral invariant but breaks scale and conformal invariance, and $u = u_0 + cu_8$ ^{*)} violates both chiral and scale invariance and, hopefully, has a unique dimension d . It is well known ^{2),5),7)} that $\langle 0 | \delta | 0 \rangle \neq 0$, but the question arises whether δ can be just a trivial c -number, or must be a q -number with non-zero matrix elements between, for example, baryons. Most authors ¹⁾⁻³⁾ have argued that δ is indeed a q -number, and that it is responsible for the greater part of the masses of all particles except the octet pseudoscalar mesons.

In this note we argue that the absence of a q -number δ term is in fact consistent with all that is known about the breaking of $SU_3 \times SU_3$, including the results of von Hippel and Kim ⁸⁾. The reason for this different conclusion is as follows: previous theoretical ⁹⁾ and phenomenological ⁸⁾ analyses of the matrix elements of the scalar densities u_0 and u_8 have neglected the existence of scalar meson poles, and arguments, Refs. 1)-3), for the existence of an operator δ term have depended on the values of the matrix elements so obtained. However, we show that if δ is just a constant the scalar meson pole terms in the matrix elements of u_0 and u_8 may in fact be large, producing large corrections in their matrix elements. But the preferred value of c is unchanged

^{*)} u_0 and u_8 being scalar-isoscalar fields in a $(3,3^*) + (3^*,3)$ representation of $SU_3 \times SU_3$ ⁹⁾.

and the on-mass shell meson-meson and meson-baryon scattering amplitudes are only slightly altered at threshold by the inclusion of scalar meson exchange graphs. If one takes δ a c-number, and neglects possible scalar meson mixing effects, one can estimate the scalar meson exchange corrections to the meson-baryon scattering length calculations made by von Hippel and Kim⁸⁾. Any of $d=1, 2$ or 3 is consistent with experiment.

This paper is arranged as follows: firstly, previous arguments¹⁾⁻³⁾ for the existence of an operator δ term are reviewed. Secondly, it is shown that the matrix elements of u_0 and u_8 will in general have scalar meson poles. Thirdly, a phenomenological Lagrangian model is exhibited in which $c=-1.25$, δ is a constant, meson-meson and meson-baryon amplitudes are only affected by scalar meson exchanges, and d and the non-pole contribution to $\langle B|u_0|B \rangle$ are arbitrary. Fourthly, the scalar meson exchange corrections to the calculations of von Hippel and Kim are estimated.

Using the standard decomposition¹⁾⁻³⁾ (1), of the energy-momentum tensor one has the "virial theorem"

$$\Theta_{\mu}^{\mu} = (4-d_s) \delta + (4-d)(u_0 + cu_8) \quad (2)$$

If one now assumes δ is a c-number, one has⁷⁾ $\langle 0|\Theta_{\mu}^{\mu}|0 \rangle = 0$, so one has

$$4\delta = -(4-d)\langle 0|(u_0 + cu_8)|0 \rangle$$

and hence

$$\langle B|\Theta_{\mu}^{\mu}|B \rangle = (4-d)\langle B|(u_0 + cu_8)|B \rangle_{\text{conn}} \quad (3)$$

At zero momentum transfer one has the left-hand side $= M_B$. One may now argue^{2),3)} that the phenomenological analysis of von Hippel and Kim⁸⁾ suggests $\langle N|(u_0 + cu_8)|N \rangle \simeq 0$, and hence that Eq. (3) cannot hold. Alternatively¹⁾, one may apply Eq. (3) to two members of the baryon octet, e.g., the nucleon and Ξ , and eliminate $(4-d)$, obtaining

$$\frac{\langle N|\Theta_{\mu}^{\mu}|N \rangle}{\langle \Xi|\Theta_{\mu}^{\mu}|\Xi \rangle} = \frac{m_N}{m_{\Xi}} = \frac{\langle N|(u_0 + cu_8)|N \rangle_{\text{conn}}}{\langle \Xi|(u_0 + cu_8)|\Xi \rangle_{\text{conn}}} \quad (4)$$

One may then argue that Eq. (4) is incompatible with $\langle B|u_0|B\rangle \simeq 215 \text{ MeV}$ as suggested by von Hippel and Kim.

However, it is easy to see that $\langle 0|u_{0,8}|\sigma\rangle \neq 0$ in general, in a spontaneously broken conformal symmetry in which a scalar meson σ most plausibly the $\epsilon(700)$, dominates matrix elements of the divergence of the scale current $\partial^\mu S_\mu = \theta^\mu_\mu$. Consider the two-point function:

$$\int d^4x \langle 0|T(\partial^\mu S_\mu(x), u_i(0))|0\rangle \quad (5)$$

Making a low energy theorem by partial integration:

$$\begin{aligned} \int d^4x \langle 0|T(\partial^\mu S_\mu(x), u_i(0))|0\rangle &= -\langle 0|[S, u_i(0)]|0\rangle \\ &= i d \langle 0|u_i|0\rangle \end{aligned} \quad (6)$$

From which we see that $\langle 0|u_i|0\rangle = 0$ if and only if the spectral function (5) = 0. Hence, $\langle 0|u_i|0\rangle \neq 0$ implies $\langle 0|u_i|\sigma\rangle \neq 0$. In all spontaneously broken $SU_3 \times SU_3$ theories $\langle 0|u_0|0\rangle \neq 0$, and in general $\langle 0|u_8|0\rangle \neq 0$. Hence, in general, $\langle 0|u_0|\sigma\rangle$ and $\langle 0|u_8|\sigma\rangle \neq 0$, and matrix elements of u_0 and u_8 have scalar meson poles. Since previous theoretical and phenomenological analyses of these matrix elements have neglected pole contributions, it is not a priori clear that (3) does not hold in nature.

In fact one can easily see that the pole terms in matrix elements of u_0 and u_8 may be substantial if δ has no q-number part. This is because the coupling of σ to other particles is proportional to their masses (mass² if bosons) in a conformal invariant theory. Consider the following chiral $SU_3 \times SU_3$ Lagrangian ¹⁰⁾ as a pedagogical example:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} D_\mu M_i D^\mu M_i + i \bar{B}_i \gamma_\mu D^\mu B_i \\ & - m_0 \bar{B}_i B_i + (iF' f_{ijk} + D' d_{ijk}) \bar{B}_i \gamma_\mu \gamma_5 B_j D^\mu M_k \\ & - (u_0 + cu_8) \end{aligned} \quad (7)$$

where the M_i are pseudoscalar meson fields

B_i are baryon fields

D^μ are covariant derivatives

$(u_0 + cu_8)$ contains meson baryon as well as purely meson terms.

The purely meson part of $(u_0 + cu_8)$ has the low order expansion:

$$(u_0 + cu_8)_{mm} = -\frac{3}{2} \bar{m}^2 f^2 + \frac{1}{2} m_\pi^2 \pi^2 + \frac{1}{2} m_K^2 K^2 + \frac{1}{2} m_8^2 \eta^2 + \dots \quad (8)$$

where $\bar{m}^2 = \frac{1}{3}(2m_K^2 + m_\pi^2)$, and the meson baryon terms start:

$$(u_0 + cu_8)_{bm} = (E \delta_{jk} + D d_{ojk}) \bar{B}_j B_k + c(iF f_{ijk} + D d_{ijk}) \bar{B}_i B_k + \dots \quad (9)$$

Where E , D and F are parameters whose values can be estimated phenomenologically. From the observed baryon masses one has $F \simeq (m_N - m_{\Xi})/\sqrt{3}$, $D \simeq \frac{\sqrt{3}}{2}(M_{\Xi} - M_{\Lambda})$. It is clear that the non-derivative meson-baryon interaction terms are related to the values of D , E and F . The results of von Hippel and Kim⁸⁾ correspond to $E + \sqrt{(2/3)}D \simeq 215$ MeV.

One can easily make $\mathcal{L}$ conformally invariant apart from the chiral symmetry breaking term $(u_0 + cu_8)$ and a c -number δ term. As pointed out in Ref. 6), as the meson fields M_i obey the non-linear transformation law

$$[Q_i^5, M_j] = i f_{ij}(M) \quad (10)$$

they must have dimension 0. One may take the baryon fields to have the canonical dimension $3/2$. Introducing ^{4),5)} a Goldstone field σ with the anomalous transformation law

$$i [S, \sigma(x)] = x_\mu \partial^\mu \sigma(x) + \frac{1}{b} \quad (11)$$

the $SU_3 \times SU_3$ invariant part of $\mathcal{L}$ can easily be made conformal invariant:

$$\begin{aligned} \tilde{\mathcal{L}}_0 = & \frac{1}{2} D_\mu M_i D^\mu M_i e^{2b\sigma} + i \bar{B}_i \gamma_\mu D^\mu B_i \\ & - m_0 \bar{B}_i B_i e^{b\sigma} + (i F' f_{ijk} + D' d_{ijk}) \bar{B}_i \gamma_\mu \gamma_5 B_j D^\mu M_k \end{aligned} \quad (12)$$

To give $(u_0 + cu_8)$ dimension d , one multiplies the meson-meson terms $(u_0 + cu_8)_{MM}$ by $e^{db\sigma}$, the meson-baryon terms $(u_0 + cu_8)_{BM}$ by $e^{(d-3)b\sigma}$. Because of the constant term in (8), $(u_0 + cu_8)$ now contains a term $-\frac{3}{2} \bar{m}^2 f^2 db\sigma$. Hence also

$$\langle 0 | u_0 | \sigma \rangle = -\frac{3}{2} \bar{m}^2 f^2 db \quad (13)$$

To cancel the term in u_0 linear in σ , we add to the conformal and chiral invariant part of the Lagrangian a term

$$-\left(\frac{3}{8} d \bar{m}^2 f^2\right) e^{4b\sigma} \quad (14)$$

In order to make $\mathcal{L} \rightarrow 0$ as $x \rightarrow \infty$, and to get $\langle 0 | \theta_\mu^\mu | 0 \rangle = 0$, we also subtract a c-number δ term

$$\delta = -(d-4) \frac{3}{8} \bar{m}^2 f^2 \quad (15)$$

The Lagrangian

$$\tilde{\mathcal{L}} = \tilde{\mathcal{L}}_0 + \frac{3}{8} d \bar{m}^2 f^2 e^{4b\sigma} - \left[(d-4) \frac{3}{8} \bar{m}^2 f^2 + (u_0 + cu_8)_{MM} e^{d b \sigma} + (u_0 + cu_8)_{BM} e^{(d-3)b\sigma} \right] \quad (16)$$

is chiral and conformal invariant apart from a chiral and conformal violating term $-(u_0 + cu_8)$ of dimension d , and a c -number δ term ^{*)}.

It is clear that the conformal Lagrangian (16) has the same meson-baryon contact terms as the original Lagrangian (7), but amplitudes will also include σ exchanges. Matrix elements of u_0 will also contain pole terms: taking $-\langle 0 | u_0 | \sigma \rangle = \frac{3}{2} m^2 f^2 d b$ [Eq. (13)] and from (16) the σ BB coupling

$$-b [m_0 + (m_B - m_0)(d-3)] \quad (17)$$

we have

$$\langle B | u_0 | B \rangle_{\text{conn}} = \left(E + \sqrt{\frac{2}{3}} D \right) + \frac{b [m_0 + (m_B - m_0)(d-3)] \left(\frac{3}{2} \bar{m}^2 f^2 d b \right)}{m_\sigma^2 - t} \quad (18)$$

where the initial and final particles are identical. Note that if the chiral symmetry breaking quantity $(M_B - M_0)$ is small, the pole term in (18) may be large. Using ⁶⁾

$$m_\sigma^2 = \frac{(bf)^2}{2} d(4-d)(2m_\pi^2 + m_\pi^2) \quad (19)$$

which can be deduced by examining the coefficient of the σ^2 term in (16) one can check that at zero momentum transfer

$$\langle B | u_0 | B \rangle_{\text{conn}} = \frac{m_0 + (d-3)(m_B - m_0) + (4-d)(E + \sqrt{\frac{2}{3}} D)}{(4-d)} \quad (20)$$

^{*)} Arguments given by some authors ^{1), 2)} determining d on theoretical grounds are incorrect. This point will be discussed in CERN preprint TH.1253 by J. Ellis, P.H. Weisz and B. Zumino.

Note the large difference between (20) and the value

$$\langle B | u_0 | B \rangle_{\text{conn}} = (E + \sqrt{\frac{2}{3}} D)$$

obtained by neglecting the σ pole term. Since in the Lagrangian (16) $\langle 0 | u_8 | 0 \rangle = 0$,

$$c \langle B | u_8 | B \rangle_{\text{conn}} = c \langle B | u_8 | B \rangle_{\text{non pole}} = m_B - m_0 - (E + \sqrt{\frac{2}{3}} D) \quad (21)$$

It is easy to see from Eqs. (20) and (21) that whatever the values of E , D , F and d , the equations (3) and (4) will hold. Hence, there is no a priori contradiction between the value of $(E + \sqrt{2/3} D) \sim 215$ MeV suggested by the analysis of von Hippel and Kim ⁸⁾, and Eqs. (3) and (4).

In order to make a more detailed comparison ^{*)} of the theory with δ a c-number and the results of von Hippel and Kim, we should consider a conformal modification of an $SU_3 \times SU_3$ Lagrangian of the type discussed by Honerkamp ¹¹⁾. A Lagrangian of the Honerkamp type has $f_K/f_\pi \neq 1$, and can be made to give the soft meson theorems of von Hippel and Kim. To conformalize it, one can give the $SU_3 \times SU_3$ invariant part of the Lagrangian a dimension 4, and the $SU_3 \times SU_3$ breaking part of the Lagrangian a dimension d , in just the same way we adopted for the simple model (7). As in the earlier conformal model (16) the $M_i M_i \sigma$ coupling will be

$$\frac{b}{2} \partial_\mu M_i \partial^\mu M_i \sigma - \frac{b d}{2} m_i^2 M_i M_i \sigma$$

which gives the off-shell coupling constant

$$g_{\sigma M_i M_i} (p_\sigma^2 = 0, p_{M_i}^2 = m_i^2, p_{M_i}^2 = m_i^2) = - \frac{(d-2)}{2} b m_\pi^2 \quad (22)$$

Also one has the same σ_{BB} coupling constant, see Eq. (17),

^{*)} We will neglect scalar meson mixing effects, which cannot yet be estimated.

$$-b [m_0 + (m_B - m_0)(d-3)]$$

as before. The σ mass formula (19) becomes ⁷⁾ for $c = -1.25$ ⁸⁾

$$m_\sigma^2 = b^2 d(4-d) \left[m_K^2 (2f_K^2 - f_\pi^2) + \frac{m_\pi^2}{2} (3f_\pi^2 - 2f_K^2) \right] \quad (19a)$$

As in the simple model, the only change in the soft meson theorems of von Hippel and Kim comes from a σ exchange graph, which can be calculated using (17), (19a) and (22) to be

$$(i) \quad \frac{m_\pi^2 [m_0 + (m_N - m_0)(d-3)]}{m_K^2 \left[2\left(\frac{f_K}{f_\pi}\right)^2 - 1 \right] + \frac{m_\pi^2}{2} \left[3 - 2\left(\frac{f_K}{f_\pi}\right)^2 \right]} \left(\frac{2-d}{d(4-d)} \right) \quad (23)$$

for forward πN scattering.

Three points are notable about the expression (23) for σ exchange. The first is that it depends only on the dimension of the chiral and conformal symmetry breaking term, and not on the actual mass or coupling constant b of the particle σ . The second point is that the σ exchange contribution in fact vanishes if $d=2$, because of the form (22) of the $\pi\pi\sigma$ coupling. The third point is that the σ exchange contribution to KN scattering is of the same form as (23) with m_π^2 replaced in the numerator by m_K^2 .

After detailed considerations von Hippel and Kim ⁸⁾ concluded that low energy theorem calculations of the following meson-baryon scattering lengths were likely to be most accurate:

$$(\pi N)_{1/2}, (\pi N)_{3/2}, (KN)_0, (KN)_1, (\bar{K}N)_1$$

We will try to fit the experimental values for these quantities by modifying von Hippel and Kim's analysis to include σ exchange, and keeping $+ \langle N | u_0 | N \rangle_{\text{non pole}} = (E + \sqrt{2/3} D)$ free to be fixed later.

In terms of $R = ((E + \sqrt{2/3} D) - 0.215) / 0.215$, and of d , using the numerical values of other quantities in (23) ($f_K/f_\pi = 1.26$, $f_\pi = 95$ MeV) corrections to the theoretical scattering lengths of von Hippel and Kim are:

for $(\pi N)_{\frac{1}{2}, \frac{3}{2}}$ and $(\pi N)_{\frac{3}{2}, \frac{3}{2}}$:

$$-0.03R + 0.05 \left(\frac{2-d}{d(4-d)} \right) [1.01 - 0.01(d-3) + 0.23(d-4)R] \quad (24)$$

for $(KN)_{0,1}$ and $(\bar{K}N)_1$:

$$-0.14R + 0.52 \left(\frac{2-d}{d(4-d)} \right) [1.01 - 0.01(d-3) + 0.23(d-4)R] \quad (25)$$

It can be seen from the table that the theoretical values of von Hippel and Kim are in perfect agreement with experiment for $(KN)_{0,1}$ and $(\bar{K}N)_1$. For this reason, and because the corrections to these scattering lengths are most sensitive to the value of R , we set the correction term (25) equal to zero

$$0.14R = 0.52 \left(\frac{2-d}{d(4-d)} \right) [1.01 - 0.01(d-3) + 0.23(d-4)R] \quad (26)$$

For each value of d , Eq. (26) gives a preferred value of R . Since the σ exchange contribution to the scattering lengths vanishes for $d=2$, clearly in this case $R=0$ is possible, i.e., the von Hippel and Kim value for $E + \sqrt{2/3} D = +\langle N | u_0 | N \rangle_{\text{non-pole}}$ is consistent with $d=2$. Using the values of R calculated using (26) for $d=1$ and $d=3$, one can calculate the corrections to the von Hippel and Kim scattering lengths for $(\pi N)_{\frac{1}{2}, \frac{3}{2}}$ by using Eq. (24). The results are tabulated below, and it can be seen that any of $d=1, 2$ or 3 is consistent with experiment.

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Process	Scattering lengths (Fermi)		Correction to theoretical scattering lengths		
	Experimental	Theoretical (von Hippel and Kim)			
			d = 1	d = 2	d = 3
$(\pi N)_{\frac{1}{2}}$	0.18 ± 0.02	0.20 ± 0.01	-0.01	0.00	+0.01
$(\pi N)_{\frac{3}{2}}$	-0.11 ± 0.01	-0.14 ± 0.01	-0.01	0.00	+0.01
$(KN)_0$	-0.00 ± 0.04	-0.01 ± 0.01	0.00	0.00	0.00
$(KN)_1$	-0.29 ± 0.02	-0.28 ± 0.01	implies	implies	implies
$(\bar{K}N)_1$	-0.12 ± 0.02	-0.13 ± 0.04	$R = +0.4$	$R = 0$	$R = -0.7$

The figures in the first two columns are taken from von Hippel and Kim ⁸⁾. The figures in the last three columns are calculated using the solutions of Eq. (26) substituted into Eqs. (24) and (25).

R E F E R E N C E S

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