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DUALITY AND INCLUSIVE PHENOMENOLOGY

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A B S T R A C T

It is suggested that Regge corrections to an energy-independent cross-section for the inclusive process $a+b \rightarrow c+X$ are expected to be absent in general only if both the channels (ab) and $(ab\bar{c})$ are exotic.

Duality for two-body scattering which involves an unstable hadron is discussed.

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In two-body to two-body scattering the absence of resonances in an exotic channel is associated by duality ¹⁾ with an energy-independent imaginary part of the forward scattering amplitude and hence, by the optical theorem, with an energy-independent total cross-section.

Guided by the work of Mueller ²⁾, Chan et al. ^{3),4)} have studied the restrictions duality imposes on inclusive processes, and have given a criterion for when the cross-section for an inclusive process should be energy-independent, in a fragmentation ⁵⁾ limit.

Consider the inclusive process

$$a + b \rightarrow c + X \quad (1)$$

where c is a fragment of the target, b . In the fragmentation limit $S_{ab} \rightarrow \infty$ with S_{bc} fixed, Chan et al. predict that the cross-section $d\sigma/(d^3p_c/p_c^0)$ for reaction (1) will be energy-independent provided that

$$(ab\bar{c}) \quad \text{is an exotic channel.} \quad (2)$$

By using the criterion (2) these authors then proceed to make predictions for the energy behaviour of 72 inclusive processes.

With no resonances in the $(ab\bar{c})$ channel, one might naively expect only a constant contribution to the specific part of the discontinuity considered ²⁾. According to duality ¹⁾, this would be generated by Pomeron exchange, and there should therefore not be any extra contribution with a decreasing Regge behaviour. This neglects, however, the effect of two-body channels which, if not exotic, would readily generate variations in the discontinuity which is being considered. The $(a\bar{c})$ and $(b\bar{c})$ channels are both below threshold in the limiting fragmentation region, which imposes (ab) much above threshold, so relevant effects can come only from the (ab) channel. If one does not require (ab) also to be exotic it will induce variations in the relevant discontinuity. More generally speaking, if two-body channel resonances

exist the absence of three-body resonances is not enough to deduce constancy of this discontinuity. The condition put forward in Ref. 3) cannot be sufficient. Indeed if it were sufficient factorization would make it inconsistent. This is shown in the following. Circumventing the difficulties thus met one obtains another criterion, imposing both (abc) and (ab) exotic in order for the limiting fragmentation property of inclusive reactions to be observable at low energies.

To show that if (abc) being exotic were a sufficient condition for no Regge contribution, it would be an inconsistent criterion one may proceed as follows.

- (i) - Consider $a = \pi^\pm$, $b = p$, $c = K^-$. Here (abc) is exotic in both cases, so that in the Regge description

$$\frac{d\sigma}{(d^3p_c/p_c^0)} \approx f_1(s_{ab}, s_{bc}) + f_2(s_{ab}, s_{bc}) s_{ab}^{\alpha_{\text{Regge}}(0)-1} \quad (3)$$

Sufficiency of criterion (2) gives $f_2 = 0$. But for $a = \pi^\pm$ respectively we have ³⁾

$$f_2^{(\pm)} = f_2^{(p')} \pm f_2^{(p)}$$

so that $f_2^{(\pm)} = 0$ implies that the P' and p contributions separately vanish.

- (ii) - Next consider $a = K^0$, $b = p$, $c = K^-$ and use factorization. Here (abc) is exotic in both cases, so that again criterion (2) leads to $f_2 = 0$. But for $a = K^0$ respectively, given that P' and p decouple by factorization of reaction (i), we have

$$f_2^{(\frac{+}{0})} = f_2^{(\omega)} \pm f_2^{(A_2)}$$

so that $f_2^{(\frac{+}{0})} = 0$ implies that ω and A_2 contributions vanish separately.

(iii) - Finally consider $a = K^-$, $b = p$, $c = K^-$. By factorization of processes (ii) it can be seen that the P' , ρ , ω and A_2 trajectories all decouple in the imaginary part, and hence that $f_2 = 0$ for this process. But for this case $(ab\bar{c})$ is non-exotic.

This example shows that if criterion (2) is sufficient, it is not necessary. Since reaction (iii) has resonances in the $(ab\bar{c})$ channel, on the other hand, one expects duality to a Regge trajectory which will in turn give $f_2 \neq 0$ in the asymptotic behaviour. This leads us to suspect that criterion (2) is indeed not sufficient.

The appropriate sufficient condition for energy independence of the cross-section of reaction (1) seems to be

$$\begin{aligned} (ab\bar{c}) & \text{ is an exotic channel} \\ \text{and } (ab) & \text{ is an exotic channel} \end{aligned} \tag{4}$$

One cannot now demonstrate an inconsistency by factorization. Both of these conditions are physically reasonable, since the first ensures that there is no energy dependence in $d\sigma/(d^3p_c/p_c^0)$ due to resonances in X and the second ensures no resonances in the direct channel ^{*}).

We do not expect intuitively that the channels $(b\bar{c})$ or $(a\bar{c})$ need be exotic, because both of these channels are below threshold in the inclusive reaction (1).

^{*}) It is expected that other resonances (of which c may be a decay product) be smeared out in the cross-section, in the same way in which they must be in, for example, the total K^+p cross-section.

Let us adopt criterion (4), for the moment, and study how the predictions of Chan et al.³⁾ are modified. For (ab) to be exotic, we need the projectile to be K^+ or p when the target is a nucleon. It is therefore expected that in general all processes considered by Chan et al. will have $f_2 \neq 0$ except

$$K^+ N \rightarrow \begin{cases} \pi^\pm \\ K^- \\ \bar{p} \end{cases} + X \quad (5)$$

$$p N \rightarrow \begin{cases} \pi^\pm \\ K^\pm \\ \bar{p} \end{cases} + X \quad (6)$$

For the inclusive processes (5) and (6) the cross-section is predicted to be energy-independent.

To understand condition (4) more precisely we proceed as follows. It is clear from the kinematics of the inclusive process that the only resonant effects can come from the (abc^-) or (ab) channels. For the real part of a two-body amplitude, it is not true that the absence of resonances implies the absence of decreasing Regge behaviour. In order to show that this problem does not arise here, we will make precise the relation between the inclusive process and the discontinuities of the three-body amplitude.

Let $T(abc \rightarrow abc)$ be the scattering amplitude for forward $abc \rightarrow abc$ scattering, and let $A_i(ab \rightarrow cX_i)$ be the amplitude for the process (1) to a specific final state cX_i . The cross-section for reaction (1) is

$$d\sigma/(d^3p_c/p_c^0) \approx \sum_i |A_i(s_{ab}^+, s_{b\bar{c}}, s_{a\bar{b}\bar{c}}^+)|^2 \quad (7)$$

where the superscripts mean that we are evaluating A_i at points immediately above the real axis in the physical sheet of the S_{ab} and S_{abc}^- variables. No superscript is needed for S_{bc}^- because this variable is below its threshold and there is no cut.

For large values of S_{ab} we can rewrite (7) up to a constant factor as

$$\begin{aligned}
 S_{ab} \, d\sigma/(d^3p_c/p_c^0) &= \sum_i A_i(S_{ab}^+, S_{bc}^+, S_{ac}^+) A_i(S_{ab}^-, S_{bc}^-, S_{ac}^-) \\
 &= \sum_i A_i(S_{ab}^+, S_{bc}^-, S_{ac}^+) A_i(S_{ab}^+, S_{bc}^-, S_{ac}^-) \\
 &+ \sum_i A_i(S_{ab}^+, S_{bc}^-, S_{ac}^+) \left\{ A_i(S_{ab}^-, S_{bc}^-, S_{ac}^-) \right. \\
 &\quad \left. - A_i(S_{ab}^+, S_{bc}^-, S_{ac}^-) \right\}
 \end{aligned} \tag{8}$$

The first term in Eq. (8) can now be summed over X_i to give $\text{Disc}_{abc}^- T(abc \rightarrow abc)$. The curly bracket in the second term is $\text{Disc}_{ab} A_i(ab \rightarrow cX_i)$, evaluated on the unphysical sheet of S_{abc}^- reached by crossing all branch lines associated with all the open X channels : that is, up to constant factors ^{*)},

$$\begin{aligned}
 S_{ab} \, d\sigma/(d^3p_c/p_c^0) &= \text{Disc}_{abc}^- T(S_{ab}^+) \\
 &+ \sum_i A_i(S_{ab}^+, S_{ac}^+) \text{Disc}_{ab} A_i(S_{ac}^-)
 \end{aligned} \tag{9}$$

^{*)} We note that Eq. (9) is exact for all the kinematic region of the inclusive process (1), including for example the pionization limit.

We know of no a priori reason for cancellations to occur in general between terms in the sum on the right-hand side of Eq. (9), although we cannot rule out such a possibility.

Even if each discontinuity is associated with a Pomeron contribution, some more complicated variation could still be expected from the sum. We nevertheless assume that only resonances could yield a specific Regge behaviour.

We can now see from Eq. (9) that if (ab) is an exotic channel the Regge contributions must cancel in each term of the sum. If in addition (abc) is exotic then the Regge contributions cancel in $\text{Disc}_{abc} T$ also. Thus given these two conditions [criterion (4) above] we expect $f_2 = 0$ in the Regge formula for $d\sigma/(d^3p_c/p_c^0)$. This agrees with the intuitive reasoning given previously.

Note that in order to derive the result we did not need to apply duality to T but only to the discontinuity in Eq. (9). In fact some care is necessary to avoid inconsistencies in discussing duality for the full three-body amplitude, and since these problems occur already in two-body scattering with unstable particles we conclude this note with a brief discussion of this point.

Consider the unitarity relation ⁶⁾

$$\begin{aligned}
 & \text{Disc} \left(\begin{array}{c} a \\ \text{---} \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} \right) \\
 &= \begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} \\
 &+ \left(\begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} + \begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} + \begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} \right) \\
 &+ \left(\begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} + \begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} + \begin{array}{c} a \\ \text{---} \text{---} \end{array} \text{---} \begin{array}{c} a \\ \text{---} \text{---} \end{array} \right) \quad (10)
 \end{aligned}$$

Suppose now that the channels (ab) , (bc) , (ac) [and hence (abc)] are exotic. Then the Regge contributions to all terms on the right-hand side of Eq. (10), except to the three Z graphs in the final term, are expected to cancel by duality. On the other hand we cannot draw a planar duality diagram for the amplitude $abc \rightarrow abc$, so we might be led (wrongly) to say that the left-hand side of Eq. (10) has no Regge contribution. This leads to a contradiction because we can go to a kinematic region where channel $(ab\bar{c})$ is physical (as in the fragmentation limit of $ab \rightarrow cX$) and then deduce that the $(ab\bar{c})$ discontinuity has no Regge contribution, independent of whether $(ab\bar{c})$ is or is not an exotic channel.

The resolution of this difficulty is to say that the planar diagram of the Figure contributes a resonant imaginary part to $T(abc \rightarrow abc)$ scattering, provided that $(ab\bar{c})$ is non-exotic, since we have a momentum transfer variable which is timelike even in the physical region of the s channel process.

This is interesting because exactly the same effect can take place in two-body scattering amplitudes, when unstable particles occur in the external legs. Here the absence of a planar duality diagram for the "natural ordering" of the external legs again does not necessarily imply that the Regge contribution is purely real in the physical region, even if duality is exact. As a simple example, the Regge part of the amplitude $A(\pi^+ e^+ \rightarrow \pi^+ e^+)$ may have a non-vanishing imaginary component which arises from the cross-channel singularities (π pole, 3π cut). Even though they affect the physical amplitude, they are not relevant for a discussion of exchange degeneracy at negative momentum transfers, as the kinematical conditions set by an inclusive reaction require a positive momentum transfer.

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FIGURE CAPTION

Duality diagram for $abc \rightarrow abc$, with channels (ab),
(bc), (ac) exotic.

FIG.1