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AXIAL VECTOR NUCLEAR SUM RULES AND EXCHANGE EFFECTS

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The last scientific talk of Amos de-Shalit was the summary of the Heidelberg "International Conference on Nuclear Reactions induced by Heavy Ions".

Listing the problems still open in nuclear physics he mentioned the behaviour of the nucleons inside the nucleus. Quoting his words: "If one has a nucleus and one has nucleons in it, then one asks oneself, The nucleons in the nucleus, do they behave like the free nucleons, or differently?... In other words we want to know, Do they retain their mass? Do they retain their charge distribution? Do they retain their coupling constant to the weak field of beta decay, to the μ mesons?"

This article, which addresses itself to the last question, is a tribute to a great scientist and is dedicated to the memory of a friend.

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ABSTRACT

The nuclear pionic vertex and its relation to the π -nuclear (charge exchange) scattering length are investigated using dispersion techniques. The aim is to extract information on mesonic effects (exchange current effects) in the pionic vertex and in the axial vector current matrix element.

A sum rule is obtained: the sum of pionic vertices between the ground state and all excited nuclear states is connected to an integral of the total π -nuclear cross-section in the (3,3) resonance energy and above. The exchange current effects are shown to be related in a model-independent way to shadow phenomenon in the cross-section.

The Goldberger-Treiman relation extends this result to Gamow-Teller matrix elements (nuclear Adler-Weisberger sum rule). Estimates are given for the renormalization of the axial coupling constant in some nuclei.

1. INTRODUCTION

A classical but notoriously difficult problem in the description of nuclei is the explicit introduction of the mesonic degrees of freedom. The usual description of the so-called exchange effects is made by means of field-theoretical models. Important progress has recently been made along these lines with the introduction of soft-pion techniques¹⁾.

Here we want to introduce a different approach for mesonic effects in the nuclear pionic vertex and in the axial current matrix elements, based on dispersion relations.

The idea is to connect the π -nucleus scattering length to the pionic vertex by such a relation. By this means we can hope to extend our understanding of the scattering of low-energy pions by nuclei to the pionic vertex about which very little is known.

The advantage of this method is that the exchange effects are displayed in a nearly model-independent way, and that they can be related to measurable quantities, namely the pion-nucleus total cross-section. The price to pay for this gain is that the mesonic effects thus described do not refer to a single axial nuclear matrix element but rather to a sum over many nuclear states*). The sum rule involved is reminiscent of the photo-nuclear sum rule of Gell-Mann, Goldberger and Thirring⁵⁾, where the mesonic effects are contained in a dispersive integral over the photon-nucleus total cross-section.

In the nucleon case we know an example where the soft-pion expression for the scattering length also gives the renormalization of the axial coupling constant. It is the famous Adler-Weisberger (AW) sum rule^{6,7)}, which is an alternative form of the soft-pion theorems. It can be derived from the Weinberg-Tomozawa^{8,9)} expression for the isospin odd scattering length for pions of zero mass:

*) Similar sum rules have been formally written by Kim and Primakoff^{2,3)}. However the important cancellation effect discussed in this article has not been discussed, which makes questionable the practical applications made by Kim and Mintz⁴⁾.

$$\left(1 + \frac{\mu_0}{m}\right) a^- = \frac{\mu_0}{8\pi f_\pi^2} = \frac{\mu_0 g^2(0)}{8\pi f_A^2(0) m^2} \quad (1)$$

where

μ_0 ($\mu_0 \rightarrow 0$) and m are the pion and nucleon masses, respectively,

f_π is the decay constant of the pion,

$g(0)$ is the renormalized π -nucleon coupling constant with its form factor, and

$f_A(0)$ is the axial vector coupling constant.

An unsubtracted threshold dispersion relation for the isospin odd amplitude gives

$$\left(1 + \frac{\mu_0}{m}\right) a^- = \frac{\mu_0 g^2(0)}{8\pi m^2} - \frac{\mu_0}{4\pi^2} \int \frac{d\omega}{\omega} \left[\sigma_+(\omega) - \sigma_-(\omega) \right]_0 \quad (2)$$

where ω is the pion energy, $\sigma_\pm$ is the π^+ and π^- "cross-section" on protons, and the subscript zero indicates that an extrapolation to zero mass is involved: $(\text{Im } f)_{\mu_0=0} = (k/4\pi)\sigma_0$, where f is the πN amplitude. Combining the relations (1) and (2), the AW sum rule follows:

$$f_A^2(0) = 1 + \frac{f_\pi^2}{\pi} \int \frac{d\omega}{\omega} \left[\sigma_+(\omega) - \sigma_-(\omega) \right]_0 \quad (3)$$

Here we want to extend the AW sum rule to the nuclear case. The crucial problem in the application of such a relation, particularly in the nuclear case, is the extent to which one can use the cross-sections for physical pions. In the nucleon case the replacement of $\sigma_0(\omega)$ by the physical cross-section $\sigma(\omega)$ is legitimate (with some kinematical corrections). In the nuclear case, we conclude that this is not so and that this replacement has to be done with caution, especially if one wants to emphasize the many-body contributions to the axial form factor. The relation between the scattering length and the pionic coupling constant is, in fact, more subtle than it appears at first sight.

In the first section, the dispersion relation in the nuclear case is discussed, while in the second section the relevant features of the

scattering pions by nuclei are summarized. The third section contains our conclusions on the axial vector nuclear sum rules and the exchange current effects.

2. DISPERSION RELATIONS FOR THE NUCLEAR CASE

We want to write forward dispersion relations for the scattering of pions by nuclei in a way that is analogous to relation (2).

2.1 For physical pions

Such relations have been discussed by T. Ericson, Formánek and Locher^{10,11)}. They differ from those of the nucleon for the following reasons:

- i) The pion absorption extends the cut below the physical threshold. The lack of data does not permit an exact evaluation of the influence of the absorption on the charge exchange amplitude. The indications are¹⁰⁾ that this influence is small ($\approx 5\%$) and we will neglect it both in the scattering length and in the threshold region of the dispersive integrand.
- ii) The unphysical region has many poles corresponding to excited nuclear states.

The pion energies at these poles are

$$\omega_N = \pm \left[-\frac{\mu^2}{2M} + \epsilon_N \right] \ll \mu$$

where ϵ_N is the excitation energy of the state N, M is the nuclear mass, and μ is the mass of the physical pion. Since $\epsilon_N \ll \mu$, these poles act in the physical region as a single effective pole very close to $\omega = 0$.

The residue of each of these poles is proportional to the square of the coupling constant at the pionic vertex connecting the ground state to the excited state N. The residue is not necessarily positive, its sign depending on the parities of the nuclear states. The resulting residue of the effective pole can then have *a priori* either sign. In the case of ^9Be it was found empirically by Ericson et al.¹⁰⁾ that the residue of this effective pole is in fact very close to the nucleon value f, $f^2 = (\mu/2M)^2 g^2/4\pi = 0.08$. They found $f_{\text{eff}}^2 = 0.07 \pm 0.02$.

The coherent sum of the poles for N neutrons and Z protons taken as free has indeed the property $(f^2)_{\text{eff}} = f^2$ equal to the nucleon value and thus independent of the nuclear size. Ericson et al. conjectured a generalization of their empirical result interpreting it as a reflection of this coherence which they showed to be equivalent to an impulse approximation summed over the excited nuclear states. The limitations of this interpretation will be discussed in Section 4.

Neglecting absorption, we write the threshold forward dispersion relation for the charge exchange amplitude (so as not to burden the formulas we write it for the same isospin case as that of the nucleon. For nuclei with an excess of one neutron the sign of the integral should be reversed. In the case of isospin larger than 1/2, see Refs. 3 and 11 for a general formulation):

$$\left(1 + \frac{\mu}{M}\right) a^- = \frac{2}{\mu} \cancel{f_{\text{eff}}^2} - \frac{\mu}{4\pi^2} \int_0^\infty \frac{dk}{w} \left[\sigma_+(w) - \sigma_-(w) \right] \quad (4)$$

where k is the pion momentum.

2.2 For zero mass pions

We will now discuss the dispersion relation for zero mass pions which leads to the AW sum rule.

The behaviour of the imaginary amplitude governs the convergence of the dispersive integral. The behaviour at very large energies is not expected to be changed by letting the external pion mass go to zero. It is hence reasonable to assume that the dispersion relation for zero mass pions does not require a subtraction, as is the case for the physical pion.

We have now to choose the way in which we take the zero mass limit. We want this limit to be as similar as possible to the physical case. In that case, we have seen that the poles are located at the energy $\omega_N = \pm [\epsilon_N - (\mu^2/2M)]$. The corresponding pion momentum is $k_N^2 = [\epsilon_N - (\mu^2/2M)]^2 - \mu^2 \approx -\mu^2$. It is to a very good approximation independent of the state N. We want the feature of having a single effective pole with a unique momentum transfer to be preserved in the

zero mass limit. We therefore take the excited states to be degenerate with the ground state $\epsilon_N = 0$ so as to retain the inequality $\epsilon_N \ll \mu_0$ in the limit $\mu_0 \rightarrow 0$.

In a Fermi gas model, the excitation energy ϵ_N is given in terms of Fermi momentum p_F by $\epsilon_N \approx p_F^2/2m$. As we do not want to alter the structure of the nucleus, the Fermi momentum p_F must be kept unchanged. The logical way to make $\epsilon_N = 0$ is therefore by taking the nucleons to be infinitely massive ($m = \infty$), i.e. to take the nuclear medium to be static.

The physical threshold starts when the incident energy allows a real pion and a real nucleus in the intermediate state. In the zero mass limit only the mass of the external pion is taken to zero and not that of the intermediate pion which keeps the physical value¹²⁾. The physical threshold starts then at an incident pion energy $\omega = \mu$. The dispersion relation in the zero mass limit $\mu_0 \rightarrow 0$ is

$$\left(1 + \frac{\mu_0}{M}\right) a^- = \frac{2f_{\pi\pi}^2(0)}{\mu_0} - \frac{\mu_0}{4\pi^2} \int_{\mu}^{\infty} \frac{d\omega}{\omega} \left[\sigma_+(\omega) - \sigma_-(\omega) \right]_0 \quad (5)$$

3. SCATTERING LENGTHS AND π -NUCLEAR SCATTERING

3.1 Scattering at zero energy

The scattering of low-energy pions by nuclei can be represented by an equivalent optical potential which has a local and a non-local part¹³⁾. The local part V_0 arises from the s-wave pion-nucleon scattering. In the first-order approximation*):

$$2\mu V_0(r) = -4\pi \left\{ \left(\frac{a_n + a_p}{2} \right) \rho(r) + \left(\frac{a_n - a_p}{2} \right) \left[\rho_n(r) - \rho_p(r) \right] 2 \frac{t \cdot T}{\omega} \right\} \quad (6)$$

*) We give here the relativistic quantity which enters in the wave equation

$$(\nabla^2 + k^2) \phi(r) = 2\mu V(r) \phi(r) \quad (7)$$

where t and T are the pion and nucleus isospin operators; a_n and a_p are the s-wave scattering lengths of π^- on neutron and protons; ρ_n , ρ_p , and ρ are the neutron, proton, and average density, respectively.

Similarly, the non-local part arises out of the p-wave π -N scattering.

Pion absorption introduces an imaginary part in the potential but this imaginary part plays practically no role in the real part of the scattering length.

Higher-order corrections to the local potential (charge symmetric part) are particularly important since there is a large cancellation in the sum $(a_n + a_p)$, as required by the soft-pion theorems. The main correction is the rescattering of pions with virtual excitation of the nucleus (nuclear polarization). This produces a repulsive interaction with $\frac{1}{2}(a_n + a_p) \approx 0$ replaced by an effective value $b_0 \approx -0.035 \mu^{-1}$. The origin of the charge symmetric local potential is then a many-body effect.

The s-wave scattering lengths are not sensitive to the presence of the non-local potential. We have investigated numerically the case of light nuclei ($A \lesssim 30$) and found that the suppression of the non-local potential does not affect the scattering length. This conclusion is confirmed by the $1s$ level shifts of pionic atoms (proportional to the s-wave scattering length) which are accounted for by the local potential alone for all measured nuclei (up to ^{23}Na). Here we restrict our considerations to these light nuclei ($A \lesssim 30$) where we have evidence that the scattering length is decoupled from the non-local interaction.

The charge exchange local potential $V_{\text{ch.ex}}$ can be related to the soft-pion result. We have shown in a previous work¹⁴⁾ that the soft-pion expression $L = \mu/8\pi f_\pi^2 = 0.09 \mu^{-1}$ is essentially the Born amplitude $(1/2\pi) \int d^3x V_{\text{ch.ex}}$. This equality is approximate; it is valid to first order in the pion mass. But the higher-order corrections (nuclear polarizability, pion absorption) are small ($< \approx 10\%$); the experiments give a Born amplitude¹⁵⁾ $(1/2\pi) \int d^3x V_{\text{ch.ex}} = 0.092 \mu^{-1}$ which is in very good agreement with the soft-pion value $L = 0.09 \mu^{-1}$.

However, the scattering length itself deviates strongly from the soft-pion expression. The repulsive real potential distorts the pion wave and reduces the pion probability inside the nucleus (the reduction is ≈ 2 for a nucleon number $A \approx 15$ and the reduction becomes more pronounced with increasing A). However, as will be shown, the distortion effect disappears from the dispersion relation.

3.2 Integrals over the physical region

1. For the physical pion we make a decomposition of the cross-section in partial waves

$$\gamma = -\frac{\mu}{4\pi^2} \int_0^\infty \frac{dk}{\omega} (\sigma_+ - \sigma_-) = -\frac{\mu}{4\pi^2} \int_0^\infty \frac{dk}{\omega} [(\sigma_+ - \sigma_-)_{l=0} + (\sigma_+ - \sigma_-)_{l=1} + \dots] \quad (8)$$

We claim that the distortion effect in the amplitude a^- is nearly cancelled by the low-energy s-wave part of the integral in the dispersion relation (4). The argument goes as follows:

The distortion effect can be expressed¹⁶⁾ in a non-relativistic potential theory by a dispersion relation

$$a = a_{\text{Born}} + \frac{1}{2\pi^2} \int d^3k \left[|f^{l=0}(k)|^2 + 3|f^{l=1}(k)|^2 + \dots \right] \quad (9)$$

where f^l is the partial wave amplitude generated by the potential and k is the pion momentum. Only the real local potential V_0 is relevant here, since the amplitude a depends only on V_0 .

In practice, the s-wave amplitude alone accounts for most of the distortion effect of Eq. (9). A numerical estimate in the case of ^9Be shows that the dispersive integral with the s-wave amplitude and with the full amplitude differs by less than 10%. With isospin indices, we therefore can write

$$a^- = a_{\text{Born}}^- + \frac{2}{\pi} \int_0^\infty dk f^{l=0} \left[(f^{l=0})^2 + 2(f^{l=1})^2 \right] = a_{\text{Born}}^- - \frac{1}{4\pi^2} \int_0^\infty dk [\sigma'_+ - \sigma'_-]_{l=0} \quad (10)$$

Here σ' is the cross-section generated by the local potential.

The distortion integral of Eq. (10) is formally identical to the s-wave integral of Eq. (8) if in this last integral we make the pion non-relativistic, $\omega = \mu$. The conditions for the cancellation of the distortion effect to occur are then the following:

- i) the s-wave cross-section $\sigma'_{\ell=0}$ generated by the local optical potential valid at low energies must be equal to the physical s-wave cross-section

$$(\sigma')_{\ell=0} = (\sigma)_{\ell=0} \quad (11)$$

We have neglected the effect of the absorption on the scattering length. The absorptive cross-section at low energies is also omitted in the dispersive integral. The equality (11) then refers to the elastic cross-section which should produce the cancellation.

- ii) this equality (11) should hold over the convergence range of the distortion integral (9); and
- iii) this range should be such that the pion does not become relativistic.

This last condition $\omega = \mu$ is well fulfilled. In nuclei, the effective range cuts the integrand of the distortion expression (9) before the pion becomes relativistic.

The first two conditions require more discussion. They imply the following conditions:

- a) The same local optical potential should be valid over the convergence range of the integral. There is evidence that this is fulfilled. Scattering experiments up to an energy of ≈ 80 MeV have been analysed¹⁷⁾ using the same optical potential as the one which fits pionic atom data.
- b) The s-wave amplitude should be dominated by the local potential over this range so that no mixing occurs between the local and non-local interaction. At low energy, as discussed before, this is true; the s-wave amplitude depends only on the local potential. However, when the energy increases we predict from numerical models that the role of the non-local interaction should increase¹⁸⁾. There is a critical energy for which the two interactions should balance (one is repulsive, the other attractive) and the net s-wave amplitude is zero (except for absorption). Beyond the critical energy, the non-local interaction plays the dominant role in the s-wave amplitude.

The critical energy is estimated to be ≈ 45 MeV in ${}^9\text{Be}$ ¹⁸⁾. The total cross-sections for s-waves, calculated with the local interaction only and with the full potential, are equal at low energy and start to deviate strongly when the critical energy is approached, as shown in Fig. 1.

However, the main part ($\approx 70\%$) of the distortion integral is contained in the energy region below the critical energy, and we may assume that the conditions for the cancellation to occur are satisfied. The exact extent to which the cancellation occurs has to be investigated experimentally.

To give a feeling for the magnitudes involved, we plot in Fig. 1 the difference $(\sigma_- - \sigma_+)/\omega$ for ${}^9\text{Be}$, as a function of the pion momentum k . The resonance region has been extracted from the existing data by the authors of Ref. 10. The low-energy part is the s-wave cross-section (elastic part). It has been calculated by Krell¹⁷⁾ with the optical potential valid for π -mesic atoms ($b_{0\text{eff}} = -0.035 \mu^{-1}$). The figure shows that the dispersive integral is not completely dominated by the 3,3 and higher resonances, as in the nucleon case. A sizeable fraction of the integral comes instead from the low-energy region. The reason for the difference lies in the value of $\sigma_- - \sigma_+ = 8\pi a^-(a^- + 2a^+)$ at threshold. For the nucleon, the isospin symmetric amplitude a^+ is nearly zero, consistent with the soft-pion requirements. On the other hand, for a nucleus a^+ is large and increases with the nucleon number. The difference $\sigma_- - \sigma_+$ at low energies thus increases with A , as do the distortion effects. An experimental confirmation of the predicted behaviour of the cross-section is very desirable.

To summarize this section, we give as indicative numbers the contributions to the two terms of the dispersion relation in the ${}^9\text{Be}$ case:

scattering length (μ^{-1})		= pole contribution + dispersive integral			
Born app.	distortion	threshold region	3,3 resonance	above	?
0.09	-0.03	-0.02	-0.06		

Due to lack of data we have not included the energy region above the 3,3 resonance. This region has presumably a certain cancelling effect in the dispersion integral, as in the nucleon case. With the numbers given we find $f_{\text{eff}}^2 = 0.07$ (in agreement with Ref. 10) close to the nucleon value $f^2 = 0.08$

2. For the zero mass pion.

When the pion mass goes to zero the s-wave πN interaction vanishes since it is proportional to the pion mass. Therefore the local part $2\mu V_0$ which enters in the wave equation (7) also vanishes. The s-wave π -nuclear interaction then arises solely from the non-local interaction. The threshold component of the total cross-section that exists for the physical pion is absent here. This is the main difference between the cross-sections in the two cases.

In the energy region around and above the 3,3 resonance the differences are smaller. At high energies we expect this difference to be very small. Thus apart from the threshold region, the main difference comes from the 3,3 resonance region.

In the nucleon case the difference in that region is of a kinematic nature¹²⁾. The imaginary part of the p-wave amplitude for zero-mass pions is enhanced compared to that for the physical pions by a factor $(q/\bar{q})^2$, the ratio of the square of the momenta for the physical and for the zero-mass pion. On the other hand, the p-wave cross-section which originates in the nucleon pole is reduced, for the zero-mass pion, by a factor $[K(0)]^2$, the squared form factor of the π nucleon coupling constant.

In the absence of a reliable description of the multiple scattering of pions in nuclei in the resonance energy region, it is difficult to assert how much these changes are reflected in the nucleus. But as these changes are small we may assume that they do not modify the multiple scattering aspects. The shadow effects of the cross-section will be taken to be the same for the zero mass and for the physical pion.

4. CONCLUSIONS

We can now draw the following four conclusions:

i) Our first conclusion refers to the universality of the effective pole in the dispersion relation for the physical pion.

We eliminate from both sides of the dispersion relation (4) the many-body effects of the scattering length. Once this is done, there remains on the left-hand side the Born amplitude which is a universal quantity. On the right-hand side we have the effective coupling constant plus a truncated integral $\mathcal{Y}'$ with the low-energy part removed

$$a_{\text{Born}} = \frac{\mu}{8\pi f_{\pi}^2} = \frac{2 f_{\pi}^2}{\mu} + \mathcal{Y}' \quad (12)$$

The many-body effects of the effective coupling constant are then contained in this truncated integral. Although the experimental information on the scattering of pions by nuclei is very scarce, we know that there are shadow effects in the cross-section. The total cross-section for pions on nuclei of isospin zero is not additive but scales empirically as $A^{0.83}$ from ${}^4\text{He}$ to ${}^{16}\text{O}$ ¹⁰). The reduction factor of the cross-section due to shadow effects is thus $A^{0.83}/A$. In the absence of any other experimental information, we assume that the same reduction factor applies to the charge exchange cross-section. The integral $\mathcal{Y}'$ is reduced in magnitude, and the effective coupling constant is therefore smaller than that of the nucleon.

In ${}^9\text{Be}$ for instance, we estimate $f_{\text{eff}}^2 \approx 0.07$. The small departure from the nucleon value ($f^2 = 0.08$) is less than the accuracy of the existing data. Ericson et al. give for ${}^9\text{Be}$: $f_{\text{eff}}^2 = 0.07 \pm 0.02$.

The deviation of the effective pole from universality reflects the occurrence of exchange effects. In the absence of these effects the pion source $j_{\pi}(x)$ would be a sum of one-body operators, which reduces the non-relativistic limit to

$$j_{\pi}^a(x) = \sum_i \frac{g}{2m} \underline{\sigma}_i \cdot \underline{q} \delta(\underline{x} - \underline{x}_i) \tau_i^a \quad (13)$$

and the residue of the effective pole would then be*)

$$\begin{aligned} g_{\pi}^2 &= -\frac{1}{8\pi} \sum_N \langle 0 | J_{\pi}^+ | N \rangle \langle N | J_{\pi}^- | 0 \rangle - \text{c.t.} \\ &= -\frac{1}{8\pi} \frac{g^2}{4m^2} \sum_N \langle 0 | \sum_i \sigma_i \cdot \underline{q} e^{i\underline{q} \cdot \underline{x}_i} \tau_i^+ | N \rangle \langle N | \sum_j \sigma_j \cdot \underline{q} e^{-i\underline{q} \cdot \underline{x}_j} \tau_j^- | 0 \rangle - \text{c.t.} \end{aligned} \quad (14)$$

with a momentum transfer $\underline{q}$, $\underline{q}^2 = -\mu^2$ and $q_0 = 0$.

Using closure over the intermediate nuclear states we find with expression (14) that f_{eff}^2 is a universal quantity

$$f_{\text{eff}}^2 = \left(\frac{\mu}{2m} \right)^2 \frac{g^2}{4\pi} = f^2$$

independent of the nuclear wave function. The deviations of the effective coupling constant from universality thus display the exchange effects in a model-independent way.

Since the shadow effect is more pronounced in heavier nuclei, the effective coupling constant should decrease with A . For ^{17}O for instance, we estimate $f_{\text{eff}}^2/f^2 \approx 0.85$.

ii) By PCAC and the generalized Goldberger-Treiman relation, we can extend this first conclusion on the many-body effects of the pionic vertex to those of the axial current at a fixed momentum transfer $t = \mu^2$.

The generalized Goldberger-Treiman relation connects the pionic vertex to the axial current matrix element

$$g_{\pi} \langle 0 | J_{\pi}^a | N \rangle = -i q_{\lambda} \overline{\langle 0 | J_A^{\lambda a} | N \rangle} \quad (15)$$

where $q = p_N - p_0$ is the momentum transfer, J_A is the axial vector current, and the bar means that the pion pole has been removed.

*) By c.t. we denote the crossed term arising from the operators in reversed order.

The conclusions about the sum rules for the pionic vertex can thus be extended to the axial current.

According to the previous result of pole universality the sum

$$\sum_N q_\lambda q_\mu \langle 0 | J_A^{\lambda+} | N \rangle \langle N | J_A^{\mu-} | 0 \rangle - \text{c.t.}$$

would be a universal quantity $f_A^2 \mu^2$ in the absence of exchange effects. The exchange effects reduce this sum by a factor (given in the first conclusion) smaller than unity which decreases with increasing nucleon number.

iii) The third conclusion refers to the exchange effects on the pionic vertex at zero momentum transfer. It is obtained from the dispersion relation (5) for zero mass pion. On the left-hand side of relation (5) the scattering length is given by the universal soft-pion expression $a^- = \mu_0 / 8\pi f_\pi^2$.

In the limit $\mu_0 \rightarrow 0$ the nuclear scattering length is the coherent sum of the scattering lengths of the individual nucleons, and no multiple scattering effects occur^{*)}.

The many-body effects of the effective pole are then contained in the dispersive integral. We have assumed that the shadow effects in the cross-sections are not modified by letting the external pion mass go to zero, and we then take the magnitude of the dispersive integral to be reduced below the nucleon value by the same empirical factor $A^{0.83}/A$ as in the case of the physical pion. The exchange effects therefore reduce the effective coupling constant. The estimated reductions are given in the next conclusion.

iv) Finally we use the Goldberger-Treiman relation to obtain a sum rule for the Gamow-Teller matrix elements. The following sum S

*) We should at this point mention that the soft-pion expression remains valid in spite of the existence of parity doublets. In principle, the existence of nuclear excited states of opposite parity, and degenerate with the ground state, invalidate the soft-pion expression. The resulting modification is small¹⁹⁾. It originates in the Fermi motion of the nucleons. But in the static limit $m \rightarrow \infty$ no modification occurs.

$$S = \lim_{\substack{|q| \rightarrow 0 \\ (q_0=0)}} (i q_\lambda)(i q_\mu) \langle 0 | J_A^{\lambda+} | N \rangle \langle N | J_A^{\mu-} | 0 \rangle - \text{c.t.}$$

has the universal limit $|q|^2 f_A^2$ in the absence of exchange effects.

The presence of exchange effects reduces this limit by the amounts shown in Table 1:

Table 1

Reduction factor for the results of the sum rule
(GT matrix elements squared)

^9Be	0.91
^{15}O	0.87
^{17}O	0.86

The case of ^3He has not been considered in spite of its great intrinsic interest because i) the scaling law in $A^{0.83}$ has not been tested and is likely to be invalid in such a light system, ii) the shadow terms are small and their effect is comparable to the effects that were neglected in the scattering length (pion absorption, nuclear polarization).

It is clear that a sum rule does not have a full predictive power for the individual matrix element. But let us assume that the exchange effects can be simply described by a renormalization of the axial coupling constant $f_A \rightarrow f'_A$ (which should be the case in nuclear matter)

$$q_\lambda J_A^{\lambda+} = \sum_i g'_A \vec{\sigma}_i \cdot \vec{q} \tau_i^+$$

The renormalization effects are then independent of the particular transition. The exchange effects of nuclear physics for an individual transition would then be the shadow effects calculated with the sum rule. In that case f'^2_A/f_A^2 should be given by the numbers of Table 1.

The measured β decay rates for the transitions $^{15}\text{O} \rightarrow ^{15}\text{N}$ and $^{17}\text{F} \rightarrow ^{17}\text{O}$ have given indications of the existence of renormalization effects²⁰⁾. The axial coupling constant is estimated from these two transitions to be reduced below the nucleon value, $f_A'^2/f_A^2 = 0.85$ and 0.83 , respectively. These numbers should be compared to those of Table 1: 0.87 and 0.86 , and the agreement is quite encouraging. However, these transitions do not exhaust the sum rule. As we do not expect these nuclei to be really representative of nuclear matter, other contributions to the sum rule should be measured to confirm or invalidate the simple renormalization picture.

In the case of the physical pion we had to limit ourselves to light nuclei to ensure the decoupling of the scattering length and the non-local interaction. No such problems arise here, and it is quite interesting to speculate about heavier nuclei. The shadow effect of the cross-section should be more pronounced and we expect the scaling law to become $A^{2/3}$. With such a law the complete shadow is quickly attained (for $A = 100$ $A^{2/3}/A = 0.21$ and the $\pi^\pm$ cross-sections are about equal). When the shadow is complete, f_A' tends towards the limit value 1 ($f_A'^2/f_A^2 = 0.65$), and the renormalization of the axial coupling constant by the pion disappears.

To summarize our work we have studied here the many-body effects of the pionic vertex. We have shown that they are decoupled from those of the scattering length. When the pion mass is changed from zero value to its physical value, the ratio of the scattering length to the pion mass (a^-/μ) is drastically reduced by many-body effects. These are not reflected in the pionic vertex, they are instead cancelled in the dispersion relation by the low-energy part of the dispersion integral. Any attempt to relate the many-body effects of the pionic vertex directly to those of the scattering length without consideration of these cancelling effects will fail. We believe²¹⁾ that this omission is at the origin of the findings of Kim and Mintz⁴⁾ of very large renormalization effects for the induced pseudoscalar form factor of the transition $^6\text{Li} \rightarrow ^6\text{He}$ in μ capture.

The complete independence between the pionic vertex and the s-wave amplitude is well known in the nucleon case. The low energy s- and p-wave pion-nuclear scattering have different physical description. The

p-wave originates in the pion pole and cannot be described by the same Hamiltonian as the s-wave. It would indeed be surprising to find that they are linked in light nuclei, since the s- and p-wave π nuclear amplitudes originate in the s- and p-wave π nucleon amplitudes, respectively.

We have obtained sum rules for the axial vector matrix elements in the time-like momentum transfer region at $t = \mu^2$ and $t = 0$. In the simple renormalization picture they lead to the estimates of the renormalization of the axial coupling constant and of the coupling constant of the pionic vertex in the time-like region. We have shown that the renormalization factor is size-dependent and is less than unity. It is expected to vary little with momentum transfer.

Finally, we should remark that the conclusions of this article are semi-quantitative. Those connected to physical pions could easily become quantitative if the relevant data were available. It is surprising to discover how scarce are the experimental data. We believe that the time is ripe for experiments.

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Figure captions

Fig. 1 : $(\sigma_- - \sigma_+)/\omega$ for ${}^9\text{Be}$ versus pion momentum. The solid line at high energies is taken from Ref. 10 and is based on experimental data.

The curves at low energies are purely theoretical and refer to the elastic s-wave cross-section. They have been calculated with the π -nuclear potential from π -mesic atoms

-·-·-·-· with local potential only

----- with full potential (local and non-local).

FIG.1