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PION SCATTERING BY NUCLEI IN THE REGION OF THE (3,3) RESONANCE

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A B S T R A C T

The salient features of pion scattering by nuclei around $E_{\pi} \simeq 180$ MeV are discussed within the frame of a simple model. It is found that the experimentally observed phenomena are essentially due to strong absorption and are rather insensitive to details of the scattering mechanism.

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The most striking feature in pion scattering by nuclei is a downward shift in the position of the $(3,3)$ resonance: while the total cross-section for πN scattering ¹⁾ peaks around 180 MeV in the laboratory (and with a width $\Gamma_R \simeq 110$ MeV) the total cross-section for pion scattering by ^{12}C shows a broad maximum at $E_\pi \simeq 145$ MeV ²⁾. The origin of this shift seems unclear. Nevertheless, the application of Glauber's multiple scattering formalism ^{3),4),5)}, and a calculation ⁶⁾ with a Kisslinger type non-local potential ^{7),8)} reproduce the total and differential cross-sections for pion scattering by ^{12}C satisfactorily. This result is quite surprising since none of these models is expected to work at the $(3,3)$ resonance. We believe, therefore, that a simple, rather model independent, mechanism is at work behind the observed phenomena. The aim of this letter is to present a model which may give some insight into the salient features of pion scattering by nuclei.

We describe the nucleus as a slab of matter with constant density ρ , containing non-interacting nucleons. We neglect all effects due to the binding of the nucleons as well as absorptive processes which lead to the emission of two nucleons. The pion is assumed to traverse this slab of matter in straight line ray optics without any reflection at the nuclear surface. The system is then completely characterized by the refractive index $n(E)$ (as a function of the pion kinetic energy E) or the momentum $K(E)$ of the pion inside the nucleus. In the absence of nucleon correlations $K(E)$ is a solution of the dispersion relation

$$K^2(E) + \mu^2 - 4\pi\rho \int^{\pi N} (K(E), E) = \omega^2 \quad (1)$$

and

$$n(E) = K(E)/k \quad ; \quad k^2 = \omega^2 - \mu^2. \quad (2)$$

Here μ denotes the mass of the pion, k its momentum in vacuo, and $f^{\pi N}(k, E)$ is the πN forward scattering amplitude as a

function of k and E in the laboratory system. For the observable πN scattering amplitude, k and E are related by the "energy shell" $k^2 = \omega^2 - \mu^2$. Since Eq. (1) requires a knowledge of $f^{\pi N}$ off the energy shell, a clear separation of how $f^{\pi N}$ depends on k and E is essential. We confine ourselves to p wave scattering in the neighbourhood of the $(3,3)$ resonance, assume resonance dominance and, for simplicity, take the amplitude to be averaged over spin and isospin. Then

$$f^{\pi N}(k, E) = \frac{k^2 C}{E - E_R + i \Gamma(E)/2} \quad (3)$$

This form is suggested by p wave scattering from a separable potential of short range as well as by the static model for πN scattering⁹⁾. The characteristic p wave form is the quadratic dependence in the momentum k multiplied by a resonance function dependent on E . We take the position E_R of the resonance to be constant, while the width is assumed to vary such that the shape of the experimental $\pi^+ - p$ cross-section¹⁾ is reproduced. The momentum dependence of C in Eq. (3) is dominated by the form factor of the nucleon. Assuming C to be independent of momentum should be a good approximation, as long as we do not continue too far off the energy shell. We evaluate the constant C from the optical theorem at the position of the $(3,3)$ resonance

$$\sigma_{tot}^{\pi N}(E_R) = \frac{4\pi}{k_R} \text{Im} f^{\pi N}(k_R, E_R) \quad (4)$$

where $\sigma_{tot}^{\pi N}$ is the proper isospin average for $N=Z$ nuclei

$$\sigma_{tot}^{\pi N} = \frac{1}{2} (\sigma_{tot}^{\pi^+ p} + \sigma_{tot}^{\pi^+ n}) = \frac{2}{3} \sigma_{tot}^{\pi^+ p} = 137 \text{ mb.} \quad (5)$$

With the help of Eqs. (1)-(4) the refractive index is obtained as

$$n_K^2(E) - 1 = - \frac{\Delta}{E - E_R + \Delta + i\Gamma(E)/2} \quad ; \Delta = -4\pi\rho C > 0. \quad (6)$$

We note the resonant form of the refractive index known in optics as Sellmeier's dispersion formula. The resonance in the nuclear medium is shifted downward by an amount $\Delta (\simeq 30-40 \text{ MeV})$ from the resonance energy for free nucleons ^{*}). Since in practice $|n_K^2 - 1| < 1$, cf., Fig. 1, we can approximately write Eq. (6) as

$$n_K - 1 \simeq - \frac{1}{2} \frac{\Delta}{E - E_R + \Delta + i\Gamma(E)/2} \quad (7)$$

^{*}) One may wonder how Eq. (6) is modified by nuclear correlations. A nuclear Lorentz-Lorenz effect ⁸⁾ amounts to the replacement of $f^{\pi N}$ in Eq. (3) by the effective value

$$f_{\text{eff}}^{\pi N} = \frac{f^{\pi N}}{(1 + \xi 4\pi\rho f^{\pi N}/k^2)} = \frac{k^2 C}{E - E_R - \xi\Delta + i\Gamma(E)/2}. \quad (3a)$$

This gives a refractive index

$$\{n_K^2(E) - 1\}_{LL} = - \frac{\Delta}{E - E_R + (1 - \xi)\Delta + i\Gamma(E)/2}. \quad (6a)$$

Thus, the form (6) is preserved but with a modified resonant shift. The parameter $\xi = 1/3$ for complete short range anti-correlations between all nucleons ⁸⁾ and $\xi = 1/8$ for pure Pauli correlations ¹⁰⁾. Since no new qualitative effect is introduced we will not discuss this point further.

If the refractive index is derived assuming $f^{\pi N}$ to be a function of the energy E alone, we find

$$n_k^2 - 1 = \frac{-\Delta}{E - E_R + i\Gamma(E)/2} \quad (8a)$$

$$n_k - 1 \approx -\frac{1}{2} \frac{\Delta}{E - E_R + i\Gamma(E)/2} \quad , \quad (8b)$$

i.e., a resonant expression without a shift. A closely related expression is the refractive index for the optical limit of Glauber's model ¹¹⁾

$$n_G - 1 = -\frac{1}{2} \frac{\Delta}{E - E_R + i\Gamma(E)/2} \quad , \quad (9)$$

which is exactly Eq. (8b). In Fig. 1 the refractive indices $n_K(E)$ and $n_G(E)$ are compared. Their behaviour as a function of the energy is rather similar. We note especially:

- i) The imaginary part of the refractive indices is of the order of 0.3-0.5 in the energy region of interest. For $E=180$ MeV the wave number of the pion is $k=1.5 \text{ fm}^{-1}$. Therefore, the mean free path for 180 MeV pions in nuclei is of the order of 2 fm, and is small compared to the nuclear dimensions ($2R=6.4 \text{ fm}$ ⁹⁾ in ^{12}C). Therefore, the nucleus is strongly absorbing for pions between 150 and 250 MeV. This observation is also supported by the experimental values of the total cross-section which are not too far from the geometrical limit $2\pi R^2$ (see Fig. 2). Therefore, we view the phenomenon of π nucleus scattering in the (3,3) resonance region mainly as a diffraction or strong absorption phenomenon.
- ii) The scattering amplitude for pion-nucleus scattering is given in our model by

$$F(\eta) = \frac{ik}{2\pi} \int d^2b e^{i\vec{q} \cdot \vec{b}} \left[1 - e^{i(n(E)-1)kL(\underline{b})} \right] \quad (10)$$

The integration over $\underline{b}$ covers the shape of the nucleus in the impact parameter plane and $L(\underline{b})$ is the length of the trajectory through the nucleus at a certain impact parameter $\underline{b}$. In the calculations shown in Figs. 2 and 3 the nucleus is considered a uniform sphere, hence $L(\underline{b}) = \sqrt{R^2 - b^2}$ with $R = 3.2 \text{ fm}$ ¹²⁾. For $E \simeq E_R - \Delta$ the refractive index $n_K(E)$ and for $E = E_R$ the refractive index $n_G(E)$ pass through $\text{Re} n(E) = 1$. For these energies the scattering amplitude Eq. (10) is purely imaginary at all angles. Therefore, the minima in the differential cross-sections at these energies and in their neighbourhood are generally expected to be very deep, in agreement with experiment ²⁾ (cf., also Fig. 3 where the sharp edge of the nucleus leads to minima which are somewhat deeper than in experiment). The too high values of the second diffraction maximum is a well-known phenomenon produced by the sharp nuclear edge in our model.

- iii) In order to discuss the position of the maximum in the total cross-section, we consider an even further simplified model for the nucleus: a cylinder of length L and radius R oriented along the incident direction. Then L in Eq. (10) is independent of b and the scattering amplitude $F_{\text{cyl}}(q)$ can be expressed in terms of the (imaginary) amplitude $f_{\text{BD}}(qR)$ produced by a totally black disc of radius R :

$$F_{\text{cyl}}(q) = [1 - e^{i(n-1)kL}] f_{\text{BD}}(qR) . \quad (11)$$

$$f_{\text{BD}}(qR) = i k R^2 J_1(qR) / qR . \quad (12)$$

The total and the integrated elastic cross-sections are found as $(n = n_r + i n_i)$

$$\begin{aligned} \sigma_{\text{tot}} &= 2\pi R^2 [1 - \cos[(n_r - 1)kL] e^{-n_i kL}] \\ \sigma_{\text{el}} &= \pi R^2 |1 - e^{-n kL}|^2 . \end{aligned} \quad (13)$$

The maximum in σ_{tot} coincides with the minimum of $e^{-n_1 kL} \cos[(n_r-1)kL]$, i.e., it occurs for

$$\frac{d(k n_i)}{dE} = - t_9 [(n_r-1)kL] \frac{d[k(n_r-1)]}{dE} . \quad (14)$$

The maximum of σ_{tot} would lie close to the energy where $n_r-1=0$ only if the width $\Gamma(E)$ is independent of energy (which is certainly not the case for πN scattering). Fig. 2 shows that both refractive indices n_K and n_G predict a downward shift in the total cross-section and also reproduce the elastic cross-section. The magnitude of σ_{tot} could have been reproduced with greater precision by choosing the nuclear radius R somewhat larger than the equivalent charge radius ¹²⁾. This would also lead to a proper description of the forward differential cross-sections which now are somewhat low. However, we did not want to introduce any adjustable parameter into the model. Note that the black sphere limit ($n_1 \rightarrow \infty$) with the radius $R=3.2$ fm is smaller than the maximally observed total cross-section. It is remarkable how well even the differential cross-section predicted by the black disc model follows the experimental values for angles up to the first minimum (Fig. 3). The minor refinement of a diffuse edge, semi-transparent nucleus qualitatively explains the filled in minimum and the two high secondary maximum.

Our model neglects correlations between nucleons especially those which lead to the absorption of pions. They seem unimportant. However, part of these processes are indirectly accounted for insofar as a pion is absorbed after first having experienced an inelastic nuclear scattering.

We conclude from our simple and relatively successful model that pion scattering by nuclei in the region of the $(3,3)$ resonance is dominated by strong absorption which is generated by the pion-nucleon resonance amplitude. Within an accuracy of 10% to 30% the results of the π nucleus scattering seem to be insensitive to finer (and more interesting) details like the formation and propagation

of an N^* in nuclear matter. Especially the "shift of the $(3,3)$ resonance" in the total π nucleus cross-section seems to be of rather trivial origin. Although the Kisslinger gradient potential and the Glauber approach are conceptually rather different with respect to the momentum dependence of the optical potential, the numerical results are rather insensitive. The only notable exception is the zero of the real part of the forward scattering amplitude $\text{Re } F(0^0)$. The momentum dependent optical potential predicts a downward shift by an amount Δ compared to the πN resonance energy, while the Glauber model predicts $\text{Re } F(0^0) = 0$ at the πN resonance energy.

FIGURE CAPTIONS

Figure 1 : The complex refractive indices $n_K(E)$, Eq. (6), and $n_G(E)$, Eq. (9), as a function of the pion laboratory energy E . The numbers along the curves give the energy E in MeV.

Figure 2 : The elastic and total cross-sections for $\pi^{12}\text{C}$ scattering as a function of the pion energy E . The experimental values ²⁾ are compared with calculations using the refractive indices $n_K(E)$ (solid line) and $n_G(E)$ (dashed line). The horizontal lines give the total and elastic cross-sections as predicted by a completely black nucleus.

Figure 3 : Differential cross-sections of $\pi^{12}\text{C}$ scattering at the energies $E = 120$ and 180 MeV. The experimental values ²⁾ are compared with calculations using the refractive indices $n_K(E)$ (solid line) and $n_G(E)$ (dashed line). The dash-dot line represents the prediction for a completely black disc, Eq. (12).

Fig. 1

FIG. 2

FIG.3