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ON THE NECESSITY OF EXOTIC MESONS,
PARITY DOUBLETS, OR REGGE CUTS

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A B S T R A C T

It is shown that the assumptions of duality and of Regge pole behaviour imply the necessity of exotic contributions to meson-meson scattering amplitudes, unless there is parity doubling for leading meson trajectories at $t=0$.

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1. INTRODUCTION

In this paper we point out the mutual incompatibility of the following three assumptions:

- i) The imaginary parts of meson-meson scattering amplitudes are saturated by resonances with the quantum numbers of quark-antiquark states. (Saturation by non-exotic resonances.)
- ii) The high energy behaviour of non-diffractive meson-meson scattering amplitudes is given by the exchange of non-exotic simple Regge poles. (Regge pole behaviour.)
- iii) Leading meson trajectories have definite parity at $t=0$. (Absence of parity doubling.)

More precisely, we will show that these assumptions are incompatible when taken together with the assumptions, such as crossing and analyticity, that are usually made for hadronic scattering amplitudes.

We will first prove that these three assumptions i), ii) and iii) are incompatible, and then make some comments on this result. In the proof, we will have to interpret these assumptions very strictly; in particular, we will take assumption i) to imply that there are no exotic resonances anywhere in meson-meson amplitudes, not even in the central partial waves. On the other hand, the proof will not use unitarity, and so will be applicable to the tree approximation to dual models, in which one might have expected that each of the three assumptions could be exactly verified.

2. PROOF

Let us now adopt assumptions i) and ii), and prove that there must be parity doubling; this will show that i), ii) and iii) cannot be simultaneously valid. Let K represent the pseudo-scalar strange meson, and $\bar{K}$ a scalar strange meson; we show in a lemma below that the existence of such a $\bar{K}$ is already implied by the two assumptions which we have made. Call A the scattering ampli-

tude whose s channel is the reaction $K^+ K^- \rightarrow K^0 \bar{K}^0$, and B the scattering amplitude whose s channel is $K^+ K^- \rightarrow K^0 \bar{K}^0$. The contribution to these amplitudes of an s channel state with definite angular momentum J and charge conjugation C obeys the relation

$$A(s, t) = [(-)^J C] B(s, t) . \quad (1)$$

The charge conjugation of a non-strange state formed from quark-antiquark (more generally, from particle-antiparticle) with orbital angular momentum L and spin S is given by

$$C = (-)^{L+S} \quad (2)$$

and hence for $J=0$ the charge conjugation is necessarily positive ^{*)}. Since the imaginary parts of A and B are assumed to be saturated by such states, we can see from Eq. (1) that

$$\text{Im } a_0(s) = \text{Im } b_0(s) \quad (3)$$

where a_0 and b_0 are the s wave projections of A and B , respectively.

The function $a_0(s)$ is defined by

$$a_0(s) = \frac{1}{s} \int dt A(s, t) . \quad (4)$$

^{*)} For positive parity, the rule that non-exotic $J=0$ states have positive charge conjugation is a special case of the rule that non-exotic natural spin-parity states have positive CP; this is the rule which was shown by Schwimmer ¹⁾ to lead to the usual kind of exchange degeneracy. However, the $J=0$ states we will consider have negative parity.

Because its u channel is exotic, the amplitude A will not have a backward peak at large energy ^{*}); thus, barring anomalous contributions from the large angle region, the integral in Eq. (4) will be dominated by the forward peak at large s . Assumption ii) enables us to write, for the forward peak,

$$\text{Im } A(s, t) = \beta(t) s^{\alpha(t)} \quad (5)$$

and by substituting this expression into Eq. (4) we get

$$\text{Im } a_0(s) = \frac{1}{s} \int_{-\infty}^0 dt \beta(t) s^{\alpha(t)}, \quad [s \rightarrow \infty] \quad (6)$$

To estimate the large s behaviour of the integral in Eq. (6), suppose first that neither $\beta(0)$ nor the trajectory slope $\alpha'(0)$ are zero. The function $\beta(t)$ may oscillate; however, for sufficiently large s , the most rapidly varying factor in the integrand is the exponential $s^{\alpha(t)}$. This fact enables us to see that

$$\text{Im } a_0(s) = \frac{\beta(0)}{\alpha'(0) \ln s} s^{\alpha(0)-1} [1 + O(1/\ln s)] \quad (7)$$

More generally, as long as $\alpha(t)$ is not identically a constant it is still true that the large s limit of $\text{Im } a_0(s)$ is given (except for logarithms) by the intercept of the leading trajectory in the t channel of amplitude A . Similarly, the large s limit of $\text{Im } b_0(s)$ is given by the intercept of the leading trajectory in the t channel of amplitude B . We can thus see from Eq. (3) that the intercepts of the leading t channel trajectories in amplitudes A and B must be equal.

^{*}) We will use only the result that the backward amplitude disappears relative to the forward amplitude at least as rapidly as some power of the energy.

Since the t channel of amplitude A is natural spin-parity, while that of amplitude B is unnatural spin-parity, we have in this example parity doubling at $t=0$. More specifically, suppose that the leading t channel trajectory in amplitude A were the ρ , and that in amplitude B the Buddha, we would then have

$$\alpha_{\rho}(0) = \alpha_B(0). \quad (8)$$

Ordinary exchange degeneracy tells us that there will not be a unique leading trajectory in these amplitudes; the ρ and the A_2 have the same trajectory function. However, the relative signs of the contributions of the ρ and the A_2 to the amplitude A must be such that their imaginary parts cancel in the exotic u channel, and hence add in the s channel. Thus $\beta(t)$ in Eq. (5) can be interpreted as the contribution of the exchange degenerate ρ - A_2 pair, and the argument leading to Eq. (8) remains valid.

It is possible, under the same assumptions, to establish the need for parity doubling for other trajectories as well. By an extension of the above argument, it can be seen that the intercepts of non-leading trajectories contributing to A and to B must also coincide. As shown in the lemma below, we can find a K which does not decouple from the state (πK) , which means that the π trajectory will not decouple from amplitude B , and so we would conclude that there would be an even signature, even parity trajectory degenerate with the π at $t=0$. Analogous results can be obtained for $I=0$ exchange. Also, by comparing the reactions $K^+ K^- \rightarrow \pi^+ S^-$ and $K^+ K^- \rightarrow S^+ \pi^-$ (with S an $I=1$ scalar), one can see the necessity for parity doubling for strange meson trajectories.

Lemma: We now show that the existence of the K follows from assumptions i) and ii). Consider the reaction $\pi^+ K^- \rightarrow \pi^+ K^-$. By the same sort of argument as that given above, the imaginary part of the s wave will be given, at high energy, by an integral over the forward peak, and so cannot vanish. By assumption i) there must therefore be s wave resonances. Any such resonance is a suitable K .

This completes the proof of the incompatibility of assumptions i), ii) and iii), and explains why nobody has succeeded in constructing a model incorporating all of them.

3. REMARKS

i) It has been shown by Rosner ²⁾ that exotic mesons should be expected to couple to baryon-antibaryon states. Studies of duality diagrams ³⁾ have led to the conclusion that exotic mesons should not couple to states of two non-exotic mesons, which is what has been assumed here. However, duality diagrams do not take the quark spin into account.

ii) If we allow J_t plane singularities more complicated than simple poles, such as Regge cuts or colliding Regge poles, we can no longer establish a relation between the high energy behaviour of the forward peak and that of the s wave. For example, if the amplitude $A(s, t)$ had a zero at a value of t which approached $t=0$ as $s \rightarrow \infty$, then the integral in Eq. (4) could be zero for arbitrarily large s . In an absorption model, this possibility corresponds simply to complete absorption of the s wave.

Unitarized dual models which have Regge cuts clearly do not satisfy our assumptions; however, one might have expected that the "Born approximation" to these models would. On the other hand, it has been suggested that Regge cuts might be incorporated into the Born approximation of dual models in order to remove MacDowell parity doubling, and that these cuts might appear also in boson channels ⁴⁾. It would be interesting to understand the relationship between this suggestion and the result presented here, especially in view of the fact that we have neither considered amplitudes involving fermions nor had to invoke the fermion nature of quarks.

iii) Consideration of the quark spin had led to the construction of dual models that do exhibit parity doubling for meson trajectories ⁵⁾.

Parity doubling at $t=0$ would also be expected if leading meson trajectories were conspirators, whether or not the parity doublets had parallel trajectories. While this possibility might be realized in the duality limit, it seems not to be a good approximation to the real world.

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