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SELECTION RULES FOR REGGE CUTS

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A B S T R A C T

We combine the expectation that diagrams which generate Regge cuts must contain third double spectral functions with the requirements of exchange degeneracy to arrive at a selection rule for Regge cuts. This rule prohibits Regge-Regge cuts from contributing to KN scattering, and we speculate that this prohibition might explain the observation that the K^+p total cross-section is such a remarkably constant function of energy, and that the cross-sections for KN and $\bar{K}N$ charge-exchange scattering rapidly become equal as the energy increases.

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1. - INTRODUCTION

One of the more perplexing problems in the theory of strong interaction scattering amplitudes at high energy is the existence of Regge cuts. The Regge absorption model ¹⁾, if not theoretically justified, has at least had some phenomenological success in explaining cuts due to the exchange of the Pomeron trajectory together with one proper Regge pole, and multiple scattering models have been proposed ²⁾ in order to understand the cuts due to multiple Pomeron exchange. However, there exists at present no well-accepted model for Regge cuts produced by the exchange of more than one proper Regge pole (Regge-Regge cuts).

In this paper, we show that one can at least derive a selection rule for these Regge-Regge cuts by combining the expectations for cuts based on perturbation theory models (necessity of third double spectral functions) with the assumptions of linear trajectories and of the absence of imaginary parts in amplitudes for exotic processes (exchange degeneracy). We consider the cut in the amplitude for the process $a + b \rightarrow c + d$ due to exchange of the two trajectories R_1 and R_2 , as shown in Fig. 1. Letting r_1 and r_2 be any particles lying on the trajectories R_1 and R_2 , we obtain the following rule :

The $(R_1 R_2)$ cut is absent from the amplitude for $a + b \rightarrow c + d$ unless there exists a planar duality diagram (1) connecting the process $a + r_1 \rightarrow c + r_2$ with the process $\bar{c} + r_1 \rightarrow \bar{a} + r_2$.

The condition that such a duality diagram exists is slightly stronger than the condition that neither the channel $(a + r_1)$ nor the channel $(\bar{a} + r_2)$ be exotic.

The "derivation" of this rule (1) will be presented in the second section below ; briefly, the idea is that cuts arise only when there are third double spectral functions, and that in certain cases

the requirement of exchange degeneracy implies the absence of third double spectral functions. We should emphasize that we shall "derive" this rule only in the sense that we shall show how it follows from some (certainly not exactly correct) assumptions that are currently somewhat popular ; essentially nothing about Regge cuts, not even the well-known formula for their position, can be proved from any fundamental starting point. In this same second Section, we discuss some theoretical implications of the selection rule (1), and observe that, while it is compatible with the eikonal prescription for Pomeron-Pomeron and for Pomeron-Regge cuts, it casts some doubt on the validity of the eikonal prescription for Regge-Regge cuts.

In the third Section, we discuss the application of this rule to certain specific processes ; in particular, we point out that it implies the absence of Regge-Regge cuts in KN scattering, which perhaps can explain why the K^+p total cross-section is such a remarkably constant function of energy.

2. - DERIVATION AND DISCUSSION OF THE SELECTION RULE

Let $A(s,t)$ be the amplitude for the process $a + b \rightarrow c + d$, with the definitions

$$\begin{aligned} s &= (p_a + p_b)^2 \\ t &= (p_a - p_c)^2 \end{aligned} \tag{2}$$

as is indicated in Fig. 1. Let M be the particle-Reggeon scattering amplitude which appears in the left half of Fig. 1, and which is redrawn in Fig. 2, and define

$$\begin{aligned} s_1 &= (p_a + p_{R_1})^2 \\ t_1 &= t \\ u_1 &= (p_a - p_{R_2})^2 \end{aligned} \tag{3}$$

so that (s_1, t_1, u_1) are the Mandelstam variables for the amplitude M . It can be shown in perturbation theory models ³⁾ that the $(R_1 R_2)$ cut is absent from $A(s, t)$ unless $M(s_1, t_1)$ has a third (i.e., $s_1 u_1$) double spectral function, and, a fortiori, unless $M(s_1, t_1)$ has singularities both in s_1 and in u_1 (a similar restriction comes from the particle-Reggeon amplitude N which appears in the right half of Fig. 1). In the linear trajectory, and hence zero-width, approximation, M will have no singularities in s_1 or u_1 if the amplitudes for $a + r_1 \rightarrow c + r_2$ have no singularities in s_1 or u_1 . If singularities are given by non-exotic resonances, then we expect amplitudes to have no singularities in channels with exotic quantum numbers; to the extent to which this is true, we see that the $(R_1 R_2)$ cut is absent from A if the quantum numbers either of $(a + R_1)$ or of $(\bar{a} + R_2)$ are exotic.

Even if neither channel $(a + R_1)$ nor $(\bar{a} + R_2)$ is exotic, it may be that it is not possible to write a planar $(s_1 u_1)$ duality diagram ⁴⁾ for the process $a + r_1 \rightarrow c + r_2$. In this case we would again expect no cut, because again the amplitude for this process would be expected to have no $(s_1 u_1)$ double spectral function. That is, even though there would be singularities in both the s_1 and the u_1 channels, they would be expected to be in disjoint parts of the amplitude, and hence would not give rise to Regge cuts.

We have thus arrived at the selection rule (1) given in the preceding Section. It has been understood for some time that the idea of exchange degeneracy was in some sense in conflict with the idea of Regge cuts which require third double spectral functions ⁵⁾, but as far as we know it has not previously been noticed that this conflict is restricted to certain special cases, nor has it been suggested that it could be used to systematically exclude cuts from those cases and not from others. Since we have imagined particles to lie on both the trajectories R_1 and R_2 , it might not be appropriate to apply the above discussion to the case in which either trajectory is the Pomeron. In any event, since whenever either R_1 or R_2 has vacuum quantum numbers neither the s_1 nor the u_1 channel will be exotic, we can see that the rule (1) does not exclude cuts involving the Pomeron. The rule (1) will not lead to a conflict between exchange degeneracy and the absorption model.

When the t_1 channel of M (equivalently, the t channel of A) can contain the Pomeron, we are not justified in assuming the absence of singularities in the s_1 or the u_1 channels, even if those channels are exotic. However, we shall suppose that the Pomeron is made up from $(s_1 t_1)$ and $(u_1 t_1)$ terms, and hence does not contribute to the $(s_1 u_1)$ double spectral function. In the absence of any well-established model for the Pomeron, this is clearly a weak point in the argument, and for this reason the selection rule (1) should be more believable in those cases in which the t_1 channel does not have vacuum quantum numbers, so that this problem does not arise. In any case, the rule (1) should not be expected to be exact, since neither the assumption of linear trajectories nor of the absence of singularities in exotic channels can be exactly true. Indeed, it is possible to establish the existence of third double spectral effects, even when there are exotic channels, by explicitly exhibiting a non-zero third double spectral function⁶⁾. However, since there is fairly good evidence for the approximate validity of our assumptions, it is reasonable to expect the third double spectral function effects to be small when either the s_1 or the u_1 channel of M is exotic. Since duality is presumably most accurate for low-mass external particles, we might expect the rule (1) to be more accurate when the "masses" of R_1 and of R_2 are small, that is for momentum transfers not too different from zero.

It is instructive to consider an apparent paradox to which the selection rule (1) gives rise. At least in perturbation-theory models³⁾, one could calculate the diagram in Fig. 1 by summing the contributions of intermediate states in the s_1 channel of the Reggeon-particle amplitudes M and N ; keeping only the lowest single-particle intermediate state leads to the eikonal prescription for Regge cuts. According to the rule (1), if the u_1 channel of M is exotic, then the cut coming from this lowest intermediate state should be cancelled by contributions coming from the higher intermediate states in the same channel. However, it is possible to find cases (e.g., if the s_1 channel of M is $\pi^+ K^- \rightarrow \pi^+ K^-$) in which all of the contributions are positive; how then can there be cancellations?

The resolution of this paradox lies in the fact that the diagram in Fig. 1 can contain, in the t channel, not only the Regge cut in which we are interested but also Regge poles coming from the t_1 channel poles of M , and that the rule (1) is supposed to apply to the cut only. That is, when one sums all the s_1 channel contributions in cases in which the u_1 channel is exotic, the cut should cancel, but one may be left with a non-zero renormalization of the pole residue in the amplitude A .

From the above discussion, it is clear that Regge-Regge cuts calculated from the eikonal model will not always obey the rule (1). In the eikonal model, one customarily approximates M by the contribution of only the lowest single particle states in the s_1 channel. Since this procedure can violate crossing symmetry - one may get a different answer by calculating in the s or in the u channel - it was suggested ⁷⁾ that one should use instead a crossing symmetric eikonal prescription obtained by simply averaging the answers that one would obtain from the ordinary eikonal prescription in the s and in the u channels. Either of these two procedures will tend to give non-zero cuts when the u_1 channel is exotic but the s_1 is not. The conflict between the eikonal prescription and the rule (1) is analogous to the conflict between the interference model and duality: in the crossing symmetric eikonal model, one adds the leading contributions from the s_1 and the u_1 channels, while according to what we have said here, one should instead take either all the contributions in the s_1 channel, or all the contributions in the u_1 channel, and should get the same answer for the cut. We can do the calculation in whichever channel is more convenient; when one of the channels is exotic, then the calculation of the cut becomes trivial when done in that channel, since there are no contributions to add.

It is easy to see that the cuts that have been found in non-planar dual loops ⁸⁾ obey the rule (1), since these cuts arise in loops which are formed by the iteration, through t channel unitarity, of $(s_1 u_1)$ planar diagrams. In this paper we do not consider the specific dynamics of any particular cut model, and so what we are doing may be thought of as an exploration of the consequences of the quantum number content of the non-planar dual loop cuts.

3. - APPLICATIONS

As a first application of the rule (1), consider the $(\rho\rho)$ cut in the amplitude for the scattering of pions from any target ; that is, let $a = c = \pi$, $R_1 = R_2 = \rho$. There can be no cut when either the s_1 or the u_1 channel of M has isospin two ; there is then a unique solution for the amplitude M , which has the property that there is no $I = 1$ in the t_1 channel. Thus the $(\rho\rho)$ cut does not have $I = 1$ in the t channel for the scattering of pions from any target.

Actually, this same result can be obtained in another way. Another feature of cuts that we learn from perturbation theory models is that the signature of a Regge cut is the product of the signatures of the Regge poles from which it formed⁹⁾ ; this means that the $(\rho\rho)$ cut has even signature, and so does not couple to a $(\pi\pi)$ state with $I = 1$. This last argument depends on the identity of the two pions ; if we had considered an example in which $a = \pi$, $c = A_2$, $R_1 = R_2 = \rho$, then a knowledge of the signature would not tell us the isospin, and so rule (1) and the signature rule would be independent. On the other hand, if we consider the (A_1A_2) cut in the scattering of pions from any target (i.e., take $a = c = \pi$, $R_1 = A_1$, $R_2 = A_2$), we find that the signature rule tells us that the cut has odd signature, while rule (1) insists that the cut cannot have $I = 1$ in the t channel. The two rules together imply that the cut vanishes completely in this case.

As another application of rule (1), consider kaon scattering ; let $a = c = K$, and let R_1 and R_2 be any vector or tensor meson trajectories. It can be seen that there is no (s_1u_1) duality diagram, and hence, by rule (1), no cut, unless either R_1 or R_2 is the ϕ or the f' trajectory. Because of the low intercept of these trajectories, we can expect that Regge-Regge cuts will be relatively unimportant in kaon reactions. Furthermore, if a and b are nucleons there will be no cut if either R_1 or R_2 is the ϕ or f' , and so by applying rule (1) to both sides of Fig. 1 in the case of KN scattering, we see that there are no Regge-Regge cuts whatsoever

formed from leading trajectories that can contribute to this process. There will presumably be cuts corresponding to the exchange of more than two trajectories, but these are expected to disappear quite rapidly with increasing energy, and so we would expect a suppression of Regge-Regge cut effects in KN (also in $\bar{K}N$) scattering.

It is easy to generalize the above example to show that there are no Regge-Regge cuts in meson-baryon scattering whenever either the s or the u channel of the amplitude A is exotic. However, this property is not true in general, and in particular does not hold either for meson-meson or for baryon-baryon scattering. Just as Regge cuts, formed from two non-exotic poles, can introduce exotic quantum numbers in the t channel, so too they can introduce them in the s and the u channel, so long as the s_1 and the u_1 channels on each side of the diagram are non-exotic.

The obvious place to look for experimental evidence about the properties of Regge-Regge cuts is in reactions, such as forward $\pi^- p \rightarrow K^+ \Sigma^-$ scattering, in which no ordinary trajectory can be exchanged. Unfortunately, the s_1 and u_1 channels of M turn out to be non-exotic in these cases, so rule (1) gives us no information. However, the predicted suppression of Regge-Regge cuts in KN scattering does offer us the possibility of confronting rule (1) with experiment.

Consider first the energy dependence of total cross-sections of exotic channels. If it is true that the Pomeron contribution to total cross-sections is constant with energy, and that the proper Regge poles together with Pomeron-Regge cuts contribute nothing to total cross-sections for exotic channels - although both of these assumptions are certainly open to question - then the energy dependence of these total cross-sections would presumably arise from the presence of Regge-Regge cuts. This would lead us to expect that the total cross-section

for Kp scattering should be a more nearly constant function of energy than is the total cross-section for pp scattering, for which Regge-Regge cuts are not excluded by rule (1), and this expectation is in agreement with the data ¹⁰⁾.

A second possible confrontation of rule (1) with experiment occurs in the comparison of the differential cross-sections for line-reversed reactions. For the purpose of the argument, let us assume that the breaking of the duality prediction of equality of the cross-sections for certain line-reversed reactions is due to the presence of Regge-Regge cuts, although this assumption is by no means the only one possible ¹¹⁾. We would then expect the discrepancy between the differential cross-sections for the reactions $K^-p \rightarrow \bar{K}^0 n$ and $K^+n \rightarrow K^0 p$ to be smaller, and to disappear more rapidly with energy, than the discrepancy between the differential cross-sections for, say, the reactions $K^-p \rightarrow \pi^- \Sigma^+$ and $\pi^+ p \rightarrow K^+ \Sigma^+$, since Regge-Regge cuts are not forbidden by rule (1) for this second pair of reactions, while for the first pair the leading (non-Pomeron) cuts would be triple Regge cuts. The experimental evidence, although not entirely unambiguous, seems to indicate that this is what happens ¹²⁾. Indeed, it has been pointed out ¹³⁾ that the discrepancy between KN and $\bar{K}N$ charge exchange disappears with increasing energy more rapidly than would be expected if this discrepancy were due to Regge-Regge cuts, which is in agreement with the prediction from rule (1).

In order to make these comparisons with experiment, we have had to assume that the Regge singularities which lie to the right of the Regge-Regge cuts do not contribute to the effects in question. If this assumption is not correct, then the comparison with experiment would have to be more subtle ; however, the derivation of the rule (1) is completely independent of this extra assumption.

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FIGURE CAPTIONS

Figure_1 The $(R_1 R_2)$ cut in the amplitude $A(a + b \rightarrow c + d)$.

Figure_2 The particle-Reggeon amplitude M , drawn in two equivalent ways.

FIG. 1

FIG. 2