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NO-GO THEOREMS FOR DUAL MODELS (ADDENDUM)

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A B S T R A C T

Previous results concerning the difficulty of multiplicative modification of an N particle dual amplitude are extended.

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The difficulty of finding a simple multiplicative modification of the N particle dual resonance amplitudes was pointed out in a previous paper ¹⁾ (hereafter referred to as I). The main reason for seeking such modifications is of course to try to write down a dual amplitude for positive intercept trajectories without imaginary mass poles. It was shown in I that a dual amplitude for N identical particles of the form

$$A_N = F_N(p_1 \dots p_N) B_N \quad (1)$$

where B_N is the generalised beta function and F_N is a scalar function of the external momenta, cannot be consistent with factorisation if we insist that the external particle occurs as an internal state in the three particle channel. To prove this assertion it was sufficient to consider A_6 and its factorisation at the poles $\alpha_{123} = 0$ and $\alpha_{234} = 0$; the requirement of factorisation at these two poles then overconstrains any non-trivial function $F_6(p_1 \dots p_6)$, unless there are also vector and axial-vector particles degenerate at the pion mass. The conclusion of I was therefore that one cannot write a physically reasonable many pion amplitude of the form, Eq. (1), with one cyclic symmetric term.

The purpose of this addendum is to consider the possibility of generalising the form of the amplitude to

$$A'_N = \sum_i F_N^i(p_1 \dots p_N) B_N^i \quad (2)$$

where the B_N^i are not separately invariant under the cyclic permutation $p_i \rightarrow p_{i+1}$, but their sum A_N is.

In particular it is interesting to consider the case where the sum in Eq. (2) has two terms. To be specific, Brower ²⁾ has considered the amplitude for N pions (N even)

$$A'_N = \sum \frac{1}{2} \text{Tr}(\tau_1 \dots \tau_N) (B_N^+ + B_N^-) \quad (3)$$

where the sum is over all inequivalent cyclic permutations and where in $B_N^\pm$ the internal trajectories have intercepts given by

$$\alpha_{ij}(0) = \pm \frac{1}{2} c (-1)^i (1 + (-1)^{i+j}) + \alpha_\pi(0) \quad (4)$$

respectively. Thus $B_N^\pm \rightarrow B_N^\mp$ under a cyclic permutation $p_i \rightarrow p_{i+1}$. The odd G parity channels have trajectory α_π , while the even G parity channels alternate between $\alpha_\rho = \alpha_\pi + c$ and $\alpha_\sigma = \alpha_\pi - c$.

The amplitude of Eq. (3) has the remarkable property ²⁾ that the Adler zeros are incorporated

$$A'_N \rightarrow 0 \quad \text{for} \quad p_i \rightarrow 0. \quad (5)$$

Because of this property, it is therefore worth investigating whether this candidate for a many pion amplitude can be made more physical by removal of the imaginary mass $\alpha_\rho = 0$ pole. As a preliminary step note that, defining the following function

$$\begin{aligned} f(123i) &= \frac{1}{2} \text{Tr}(\tau_1 \tau_2 \tau_3 \tau_i) (B_4^+(\alpha_{12}, \alpha_{23}) + B_4^-(\alpha_{12}, \alpha_{23})) \\ &+ \frac{1}{2} \text{Tr}(\tau_1 \tau_3 \tau_2 \tau_i) (B_4^+(\alpha_{13}, \alpha_{32}) + B_4^-(\alpha_{13}, \alpha_{32})) \\ &+ \frac{1}{2} \text{Tr}(\tau_2 \tau_1 \tau_3 \tau_i) (B_4^+(\alpha_{21}, \alpha_{13}) + B_4^-(\alpha_{21}, \alpha_{13})) \end{aligned}$$

the factorisation of A'_6 at the pion pole in α_{123}^π is given ⁽⁶⁾ by

$$A'_6 \sim \frac{1}{\alpha_{123}^\pi} \left(\sum_{i=1}^3 f(123i) f(456i) + \text{isoscalar part} \right) \quad (7)$$

for $\alpha_{123}^\pi \sim 0$. Now consider a modification

$$A_N'' = \sum_{\substack{\text{inequ.} \\ \text{perms}}} \frac{1}{2} \text{Tr}(\tau_1 \dots \tau_N) \left\{ R_N^+(p_1 \dots p_N) B_N^+ + R_N^-(p_1 \dots p_N) B_N^- \right\} \quad (8)$$

where the $R_N^\pm$ are Lorentz scalar functions formed from the external momenta and must have the same properties under cyclic permutations as $B_N^\pm$, in order that A_N'' be cyclic symmetric.

To remove the imaginary mass pole at $\alpha_p = 0$ in the four pion amplitude we must choose $R_4^+(p_1 p_2 p_3 p_4) = \alpha_p(s_{12})$. Since this now fixes A_4'' we must now require that A_6'' factorise at internal pion poles into a product of A_4'' factors (bootstrap condition). Exactly as in I we try to enforce such factorisation at $\alpha_{123}^\pi = 0$ and $\alpha_{234}^\pi = 0$, leading to the Taylor series expansions [(by considering a single term of the residue in Eq. (7))]

$$\begin{aligned} R_6^+(p_1 \dots p_6) &= \alpha_p(s_{12}) \alpha_p(s_{56}) + \alpha_{123}^\pi \tilde{R}_6^+ \\ &= \alpha_p(s_{34}) \alpha_p(s_{56}) + \alpha_{234}^\pi \hat{\tilde{R}}_6^+ \end{aligned} \quad (9)$$

Putting $\alpha_{123} = \alpha_{234} = 0$ in Eq. (9) leads to a contradiction since we may put also $s_{12} \neq s_{34}$. The function R_6^+ is thus over-constrained and no non-trivial R_6^+ exists. Thus no simple multiplicative modification of Eq. (3) can be found which has no imaginary mass pole at $\alpha_p = 0$.

From the results of I and of this addendum the conclusion is that the generalised beta function integrand must be modified in a non-trivial fashion in order that the imaginary mass states on positive intercept trajectories be absent, and the factorisation properties preserved.

REFERENCES

- 1) A.P. Balachandran, L.N. Chang and P.H. Frampton, No-Go Theorems for Dual Models, Nuovo Cimento A (to be published), hereafter referred to as I.
- 2) R.C. Brower, CERN preprint TH. 1268 (1970).