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RELATIVE IMPORTANCE OF DUAL LOOPS

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A B S T R A C T

We suggest that the size of a dual loop term is determined by the number of quark lines, rather than the power of g . This could explain the smallness of the triple-Pomeron vertex.

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In this letter, it is suggested that it is useful in estimating the magnitudes of terms in a dual loop expansion to count the number (m) of different quark lines occurring in the corresponding quark diagram ; this is an alternative to the more usual expansion (useful for considerations of perturbative unitarity ¹⁾) in terms of the number of hadron loops, or equivalently of powers of the dual coupling constant (g) .

Expansion of the hadronic scattering amplitude into dual loop terms implies the important requirement that the renormalization contributions are, in some sense, small so that the Born term remains a good approximation to the final solution. Let us now give a definition of what we mean by saying that the renormalization corrections are small. Suppose that there are N external hadrons and $m = (m_{\text{minimum}} + m')$ quark lines where m_{minimum} is the number of quarks (or antiquarks) contained in the external legs. Then, we should expand the amplitude as

$$A = \sum_{m'=0}^{\infty} A_{m'} \quad (1)$$

each term of which may be expanded into the number (L) of hadron loops

$$A_{m'} = g^{N-2} \sum_{L=m'}^{\infty} g^{2L} A_{m',L} \quad (2)$$

All terms with $m' = 1$ are renormalization effects and the fundamental assumption of the dual loop work is that the series, Eq. (1), is very rapidly convergent [much more convergent, for example, than an expansion in powers of g , such as Eq. (2), is expected to be]. As a zeroth order approximation, we should take only $m' = 0$, that is only terms with the minimum possible number of quark lines.

Let us now consider in turn some of the consequences.

i) For the orientable primitive graphs ²⁾ of the dual resonance theory (Fig. 1), we see that only the planar tadpole (Fig. 1b) has an additional quark line ($\delta m = +1$). Both the non-planar loop (Fig. 1a) and the non-planar tadpole (Fig. 1c) have no extra quark line ($\delta m = 0$). Similarly the powers of the dual coupling constant are clear ; we may then summarize the properties of these graphs :

non-planar loop	$\delta m = 0$	g^2
planar tadpole	$\delta m = 1$	g^2
non-planar tadpole	$\delta m = 0$	g^4

To zeroth order, in Eq. (1), we include all loop contributions without planar tadpoles. Thus we include in zeroth order the Pomeron $\mathbb{P}$ (Fig. 1a) and its iterations (Fig. 1a with any number of Fig. 1c), which are dual to multi-Regge exchange cuts. Also included are the Pomeron-Regge cuts (Fig. 1c).

ii) The assumed small higher orders are renormalization effects. For example, the planar tadpole gives rise to mass renormalizations as indicated in Fig. 2b. It is crucial to the success of the dual loop programme that such renormalizations be small, in order that the Born term remain a good approximation.

iii) Similarly, renormalization corrections apply to vertices which are forbidden at the $m' = 0$ level. For example, the coupling of an exotic meson ($qq\bar{q}\bar{q}$) to two non-exotic mesons ($q\bar{q}$) may proceed through the renormalization graph ($m' = 1$) of Fig. 2b. Clearly, it is again crucial that such corrections be small. For example, if the vertex we have chosen to illustrate were appreciable, we would no longer easily understand the experimental verification of exchange degeneracy predictions derived from the absence of exotic states in, for example, $\pi^+\pi^+ \rightarrow \pi^+\pi^+$.

iv) Now, we turn to the three-point vertices involving the Pomeron. First, we look at the triple-Pomeron vertex, which is indicated in Fig. 3a, both as a quark diagram and in terms of integration contours on a Riemann surface. Note that four quark lines (or integration boundaries) are here needed to couple three ($q\bar{q}$) systems so that the $\mathbb{P}\mathbb{P}\mathbb{P}$ vertex has $\delta_m = 1$.

In fact, we may regard the $\mathbb{P}\mathbb{P}\mathbb{P}$ vertex as a renormalization correction to the $\mathbb{P}\mathbb{P}\mathbb{R}$ vertex ($\delta_m = 0$) shown in Fig. 4b, since it is obtained from it by adding a planar tadpole ⁺). The observed smallness ⁺⁺) of the strength $G(\mathbb{P}\mathbb{P}\mathbb{P})$ thus coincides with the fact that for dual-loop Pomerons this vertex is only a renormalization effect ⁺⁺⁺), expected to be very small as are the corresponding effects discussed above and depicted in Fig. 2.

v) Finally, we turn to the $\mathbb{P}\mathbb{P}\mathbb{R}$ vertex, which is also of considerable interest experimentally. This vertex is shown in Fig. 3b, and has $\delta_m = 0$. Consequently, there is no reason for believing that its strength $G(\mathbb{P}\mathbb{P}\mathbb{R})$ should be very small. On the contrary, this vertex has to be of normal strength, that is not very different from $G(\mathbb{P}\mathbb{R}\mathbb{R})$ and $G(\mathbb{R}\mathbb{R}\mathbb{R})$, since it can be reached by duality transformations from the latter two vertices (Fig. 3c).

⁺) This is noted independently by Brower and Waltz, CERN Preprint TH.1335 (1971).

⁺⁺) We refer to the rapid fall-off in energy (like $1/s$) of the background in missing-mass experiments ³).

⁺⁺⁺) The smallness has alternatively been discussed in terms of the nearness of the Pomeron intercept to one, by Abarbanel et al. ⁴).

By duality and factorization at the intermediate state poles of the graphs in Fig. 3c, we find that

$$G(RRR) \quad G(\overline{P}P R) \sim G(\overline{P}RR)^2$$

Note that we cannot similarly constrain $G(\overline{P}PP)$ because duality transformations preserve the number of quark lines.

Normal strength for $G(\overline{P}P R)$ is thus predicted, and this seems to have some support from the energy dependence of resonance production in $p + p \rightarrow p + N^*(1400, 1520, 1690)$ [see Fig. 4 on p. 859 of Anderson et al., Phys.Rev.Letters 16, 855 (1966)]. Correspondingly, one should expect to see evidence of double Pomeron-exchange at (very) high energy, for example by observing an energy independent cross-section for production of isoscalar $(\pi^+\pi^-)$ systems in the centre of mass of $p + p \rightarrow p + p + (\pi^+\pi^-)$.

REFERENCES

- 1) J. Scherk, Orsay Preprint, LPTHE 71/17 (1971).
- 2) D.J. Gross, A. Neveu, J. Scherk and J.H. Schwarz, Phys.Letters 31B, 592 (1970).
- 3) J.M. Wang and L.L. Wang, BNL Preprint (1971) ;
P.D. Ting and H.J. Yesian, Phys.Letters 35B, 321 (1971) ;
G. Pancheri-Srivastava and Y. Srivastava, Northeastern University Preprint (1971) ;
R. Rajamaram, Trieste Preprint IC/71/25 (1971) ;
R.M. Edelstein, V. Rittenberg and H.R. Rubinstein, Phys.Letters 35B, 408 (1971).
- 4) H.D.I. Abarbanel, G.F. Chew, M.L. Goldberger and L.M. Saunders, Phys.Rev.Letters 26, 937 (1971).

FIGURE CAPTIONS

Figure 1 : Orientable primitive graphs.

(a) Non-planar loop. (b) Planar tadpole. (c) Non-planar tadpole.

Figure 2 : Renormalization diagrams.

(a) Mass renormalization. (b) Vertex renormalization.

Figure 3 : Pomeron couplings.

(a) $\mathbb{P}\mathbb{P}\mathbb{P}$ vertex. (b) $\mathbb{P}\mathbb{P}R$ vertex. (c) Duality transformation for $\mathbb{P}\mathbb{P}R$ vertex.

a)

b)

c)

FIG. 1

a)

b)

FIG. 2

a)

b)

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c)

FIG.3