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RADIATIVE HYPERON DECAYS AS A TEST OF NON-LEPTONIC

WEAK INTERACTION MODELS

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A B S T R A C T

We discuss decays of the type $B \rightarrow B' + \gamma$ with $|\Delta S| = 1$ as a means of studying the weak mass transition. Only transitions involving neutral hyperons are sensitive to this term. We also discuss the direct emission amplitudes in relation to models of non-leptonic weak interactions.

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1. - INTRODUCTION

With the advent of intense hyperon beams, it may be possible to learn more about the non-leptonic weak interaction by studying rare decay modes. We shall discuss decays of the type

$$B_1 \rightarrow B_2 + \gamma \quad (1)$$

where B_1 and B_2 are $(\frac{1}{2})^+$ octet baryons whose strangeness differs by one unit.

One would a priori expect the amplitude for (1) to contain a "bremsstrahlung" contribution of the type shown in Fig. 1, where H_w is the weak Hamiltonian. Since these diagrams yield terms of $1/k$ and 1 in the amplitude, they might be expected to dominate over the "direct emission" process of order k or higher which is shown in Fig. 2. The decay (1) would then provide a measure of the weak mass transition

$$B_1 \rightarrow B_2 \quad (2)$$

In the problem of interest here k is not an independent variable: in the rest frame of B_1 , $k = (M_1^2 - M_2^2)/2M_1 \simeq M_1 - M_2$. So what we are really doing is expanding the amplitude for the process (1) in powers of the SU_3 breaking parameter $M_1 - M_2$. We then hope that terms of lowest order in this parameter will be dominant.

It turns out that for charged baryon decay (or more generally for transitions between members of the same U spin multiplet) the diagrams of Fig. 1 vanish up to terms proportional to symmetry breaking effects, and therefore cannot be expected to dominate over the "direct emission" process of Fig. 2. This is because, if symmetry breaking corrections to vertices are neglected, the photon sees the weak interaction only as an effective rotation in SU_3 space (or chiral $SU_3 \otimes SU_3$ for the parity violating part) of the semi-strong symmetry breaking interaction. This will be discussed more fully below.

However, for parity conserving transitions between neutral baryons the "bremsstrahlung" contribution (which occurs through magnetic moment couplings) does not in general vanish unless the f/d ratio of the weak mass term is the same as that for the semi-strong mass splittings ($d/f \approx -0.3$). In this case the weak interaction again reduces to an effective rotation of the semi-strong interaction.

By comparing neutral decays where the "bremsstrahlung" term can contribute with charged decays where only direct emission occurs, one may be able to learn something about the d/f ratio of the weak mass term. For example, if the "direct" amplitude is estimated from $\Sigma^+ \rightarrow p \gamma$ and d and f are taken from a pole fit to p wave hyperon decays ($d/f \approx -0.8$), the rate for $\Xi^0 \rightarrow \Sigma^0 \gamma$ can be enhanced by a factor 3-12 by the bremsstrahlung contribution. If the neutral decays are dominated by the bremsstrahlung term, their amplitudes are simply related.

In Section 2 we discuss more fully the vanishing of the bremsstrahlung term for charged decays. In the following Sections we calculate and discuss this contribution for neutral decays. We shall also discuss the "direct" term (Fig. 2) in terms of non-leptonic weak interaction models. We shall derive relations for this term which (a) serve as tests of the general properties of the weak interaction Hamiltonian and, (b) help to separate out the weak mass contribution. Most of these relations have been presented in the literature as consequences of octet dominance of the weak Hamiltonian. However, if the observed $\Delta I = \frac{1}{2}$ rule is due to a dynamical mechanism rather than inherent property of the weak Hamiltonian, octet enhancement may not play a rôle in radiative decay. Therefore we emphasize those relations which can be obtained without the assumption of octet dominance.

2. - CHARGED HYPERON DECAYS

Consider first the parity conserving part of H_w which we shall call H_w^c . In calculating the contribution of Fig. 1, we may neglect terms of order one and higher in k or $M_1 - M_2$ since these are comparable to the contribution of Fig. 2, and may be included by definition in the direct term.

The weak off-shell amplitude responsible for the transition (2) is of the general form

$$h(p_1, p_2) = A_0 + A_1 \not{p}_1 + A_2 \not{p}_2 + A_3 \not{p}_1 \not{p}_2 \quad (3)$$

where the A 's are functions of Lorentz invariants :

$$A_0 = A_0(p_1^2, p_2^2, (p_1 - p_2)^2), \text{ etc.} \quad (4)$$

For comparison with non-leptonic decay models, we are interested in the on-shell amplitude

$$h(p_1, p_2) \rightarrow H(M_1^2, M_2^2, 0) \equiv A_0 + M_1 A_1 + M_2 A_2 + M_1 M_2 A_3 \quad (5)$$

with

$$A_0 = A_0(M_1^2, M_2^2, 0), \text{ etc.}$$

One can show by manipulating the amplitude for Fig. 1 that $h(p_1, p_2)$ may in fact be replaced by Eq. (5). The difference between expressions (3) and (5) gives rise to terms which are either of order k (or $M_1 - M_2$) and may be included in the direct emission contribution, or are of order one but are such that they must be exactly cancelled by a direct emission term in order that gauge invariance be satisfied. This is in conformity with a general theorem due to Low¹⁾ to the effect that only on-shell amplitudes are required for calculating the lowest order bremsstrahlung terms.

Therefore, aside from the electromagnetic couplings, the parity conserving part of the lowest order ($1/k$ and 1) bremsstrahlung terms depend only on a single multiplicative parameter, the on-shell weak mass term $H(M_1^2, M_2^2, 0)$. Now one may consider a fictitious model in which the parity conserving weak Hamiltonian H_w^c is related to the SU_3 breaking part of the strong Hamiltonian H_M by a U spin rotation $U = e^{i\alpha U_2}$

$$U H_M U^{-1} \approx H_M + i\alpha [U_2, H_M] = H_M + H_w^c \quad (6)$$

$\alpha \ll 1$ determines the strength of the weak interaction. Since the electromagnetic coupling of hadrons is invariant under this transformation, there will be no strangeness changing radiative decays (nor any strangeness changing decays) in this model, since the weak interaction may be absorbed into the semi-strong interaction and only serves to redefine hypercharge. Nevertheless one may define the matrix element :

$$\bar{u}_2 u_1 H(M_1^2, M_2^2, 0) = \langle B_2 | H_w^c | B_1 \rangle \neq 0 \quad (7)$$

Since the matrix element for the transition (1) vanishes in this model, the coefficient of (7) must vanish. This coefficient is model independent if H_M is neglected (SU_3 vertices for electromagnetic couplings) ; therefore the bremsstrahlung term vanishes up to symmetry breaking terms. In fact the amplitude is proportional to $\mu_1/M_1 - \mu_2/M_2$, where $\mu_{1,2}$ is the anomalous magnetic moment of $B_{1,2}$. For members of a U spin multiplet (Σ^+, p or Ξ^-, Σ^-) $\mu_1 = \mu_2$ in the SU_3 limit.

If we instead consider decays of neutral hyperons (e.g., $\Xi^0 \rightarrow \Sigma^0 \gamma$, $\Lambda^0 \gamma$) we must include $\Sigma - \Lambda$ magnetic transitions (cf. Fig. 3). For example for the decay $\Xi^0 \rightarrow \Sigma^0 \gamma$, two parameters are required to describe the weak mass transitions :

$$\begin{aligned} \Xi^0 &\rightarrow \Sigma^0 \\ \Xi^0 &\rightarrow \Lambda \end{aligned} \quad (8)$$

We may take these to be the f and d parameters of the weak mass matrix. Then in general the results will be non-vanishing since the calculation is model dependent. However, if the weak f/d ratio is the same as that for baryon mass splittings, then

for the purpose of this calculation any model reduces to the fictitious one described above and the amplitude must vanish up to SU_3 breaking corrections. This will be seen explicitly in the next Section.

For the parity violating decays the "bremsstrahlung" term is suppressed by a factor $M_1 - M_2$ relative to the parity conserving transition because of the γ_5 operator. Therefore we need only consider the lowest order term (charge coupling $\sim 1/k$). This also vanishes, as can be understood from the fact that the electromagnetic coupling is invariant under chiral U spin transformations. One may then define a fictitious model in which the parity violating weak interaction is related to the $SU_3 \otimes SU_3$ breaking part of the strong interaction in a manner analogous to Eq. (6). Since the transition (1) vanishes in this model, and since the calculation of the leading terms is model independent, it must vanish identically.

Since the magnetic coupling in parity violating transitions yields a term which is already of order $M_1 - M_2$, we need not consider the parity violating amplitude in the discussion of the weak mass contribution to neutral hyperon decays.

Finally we remark that the above conclusions on the vanishing of the weak mass term are valid for the emission of any number of photons.

3. - NEUTRAL DECAYS

We shall consider the decays

$$\begin{aligned} \Xi^0 &\rightarrow \Sigma^0 \gamma \\ \Xi^0 &\rightarrow \Lambda \gamma \\ \Lambda &\rightarrow n \gamma \end{aligned} \quad (9)$$

The "bremsstrahlung" transition amplitudes are of the general form

$$\mathcal{M}_{B_1 \rightarrow B_2 + \gamma} = e \bar{u}_2 \gamma_\mu k u_1 A_2 \quad (10)$$

$$A_2 = X_2 f + Y_2 d \quad (11)$$

f and d are the weak mass parameters defined by

$$\langle B_i | H_w^c | B_j \rangle = 2 \bar{u}_2 u_1 (-if_{ijk} + d_{ijk} d)$$

where d_{ijk} and f_{ijk} are respectively the symmetric and antisymmetric tensors introduced by Gell-Mann²⁾. The coefficients X and Y depend on masses and magnetic moments. The assignment of SU_3 values for the magnetic moments is not unambiguous.

For reasons of simplification we take μ/M to be the quantity which satisfies the SU_3 constraints. Then from the empirical relations :

$$\mu_p(\text{total})/M_p \approx -\frac{3}{2}\mu_n/M_n \approx 3 \text{ GeV}^{-1} \quad (13)$$

we obtain for the other relevant moments :

$$\mu_{\Sigma^0}/M_{\Sigma^0} = -\mu_{\Lambda}/M_{\Lambda} = -\frac{1}{2}\mu_{\Xi^0}/M_{\Xi^0} = 1 \text{ GeV}^{-1} \quad (14)$$

The $\Sigma - \Lambda$ transition moment is determined by SU_3 to be :

$$\mu_{\Sigma\Lambda}/M_{\Sigma\Lambda} = \sqrt{3} \text{ GeV}^{-1} \quad (15)$$

Using Eqs. (13)-(15) we may calculate explicitly the diagrams of Fig. 3 to obtain the X and Y coefficients :

$\Xi^0 \rightarrow \Sigma^0 \gamma$:

$$A_{\Sigma^0} = \frac{3}{2\sqrt{2}} \left\{ f \left[\frac{1}{M_{\Xi} - M_{\Lambda}} - \frac{1}{M_{\Xi} - M_{\Sigma}} \right] - d \left[\frac{1}{M_{\Xi} - M_{\Sigma}} + \frac{1}{3(M_{\Xi} - M_{\Lambda})} \right] \right\} \text{ GeV}^{-1} \quad (16)$$

$$\Xi^0 \rightarrow \Lambda \gamma : \quad A_{\Lambda} = A_{\Sigma^0} / \sqrt{3} \quad (17)$$

$\Lambda \rightarrow n \gamma$:

$$A_n = \frac{\sqrt{3}}{2\sqrt{2}} \left\{ f \left[\frac{1}{M_{\Lambda} - M_N} - \frac{1}{M_{\Sigma} - M_N} \right] + d \left[\frac{1}{M_{\Sigma} - M_N} + \frac{1}{3(M_{\Lambda} - M_N)} \right] \right\} \text{ GeV}^{-1} \quad (18)$$

Since we are neglecting higher order symmetry breaking effects we may make use of the Gell-Mann - Okubo mass formula

$$2M_{\Sigma} + 2M_N = 3M_{\Lambda} + M_{\Xi}$$

to write Eqs. (16)-(18) in the form :

$$A_{\Sigma^0} = \sqrt{3} A_{\Lambda} \approx -\frac{3}{2\sqrt{2}} \left\{ \frac{(M_{\Sigma} - M_{\Lambda})f + \frac{2}{3}(M_{\Xi} - M_N)d}{(M_{\Sigma} - M_{\Lambda})(M_{\Xi} - M_{\Sigma})} \right\} \text{ GeV}^{-1} \approx -(3.25f + 10.6d) \text{ GeV}^{-2} \quad (19)$$

$$A_n \approx -\frac{(M_{\Xi} - M_{\Lambda})(M_{\Xi} - M_{\Sigma})}{(M_{\Lambda} - M_N)(M_{\Sigma} - M_N)} A_{\Lambda} \approx -0.563 A_{\Lambda} \quad (20)$$

We see that the amplitude for $\Xi^0 \rightarrow (\Sigma^0 - \sqrt{3}\Lambda) + \gamma$ vanishes. This combination of Λ and Σ^0 is just the third component of the U spin triplet of which Ξ^0 and n are members. Similarly the weak mass will not contribute to $(\Sigma^0 - \sqrt{3}\Lambda) \rightarrow n + \gamma$, since weak transitions between members of a U spin multiplet are seen by the photon as a rotation of the semi-strong Hamiltonian as discussed above.

In Table I, we have listed the A's for different values of f and d. The values ($\mu = m_{\pi^+}$)

$$f = -0.416 \times 10^5 (\mu \text{ sec})^{-1/2} \text{ GeV} \quad d = 0.328 \times 10^5 (\mu \text{ sec})^{-1/2} \text{ GeV} \quad (21)$$

$$d/f = -0.79$$

are obtained from a pole fit to the hyperon p wave non-leptonic decay data ³⁾. The prescription ⁴⁾ which gives a good fit is to use a vector $\bar{B}B\pi$ coupling, or, equivalently, to be guided by current algebra and write

$$B(B_1 \rightarrow B_2 + \pi(q)) = B_p + \lim_{q \rightarrow 0} (q_\mu M_\mu - B_p)$$

$$M_\mu(q) = \frac{1}{f_\pi} \int d^4x e^{iqx} \langle B_2 | T(A_\mu^\pi(x), H_W^c(0)) | B_1 \rangle \quad (22)$$

The latter prescription is independent of the $\bar{B}B\pi$ coupling used. In Eq. (22) B_p is the baryon pole term calculated from the diagrams shown in Fig. 4. The $\bar{B}B\pi$ couplings are obtained from the axial vector couplings via the Goldberger-Treiman relation. For the axial vector couplings we have used the values ⁵⁾

$$F = 0.451 \quad D = 0.777 \quad (23)$$

The values

$$d = 0.063 \times 10^5 (\mu \text{ sec})^{-1/2} \text{ GeV} \quad f = -0.187 \times 10^5 (\mu \text{ sec})^{-1/2} \text{ GeV} \quad (24)$$

$$d/f = -0.34$$

are obtained from a current algebra ⁶⁾ fit to the s wave hyperon non-leptonic decay data ³⁾, viz. :

$$A(B_1 \rightarrow B_2 + \pi(q)) = \lim_{q \rightarrow 0} \frac{i}{f_\pi} \int e^{iqx} \langle B_1 | T(\partial_\mu A_\mu(x), H_W^v(0)) | B_2 \rangle$$

$$= \frac{1}{f_\pi} \langle B_1 | \hat{H}_W^c(q) | B_2 \rangle \quad (25)$$

where H_W^v is the parity violating part of the non-leptonic weak Hamiltonian and $\hat{H}_W^c$ is an isospin partner of H_W^c . The d/f ratio in this case is very close to the mass splitting ratio $d/f = -0.31$ for which the amplitudes (19) and (20) vanish.

4. - DIRECT EMISSION AMPLITUDE

The amplitudes for direct emission may be written

$$\mathcal{M}_{D\mu} = e \bar{u}_2 (a_2 + b_2 \gamma_5) \gamma_\mu k u_1 \quad (26)$$

This term is of order $k \sim M_1 - M_2$, i.e., first order in the symmetry breaking. In the spirit of our expansion we may neglect symmetry breaking corrections to the coefficients a_2, b_2 . Then U spin is a good symmetry for discussing these amplitudes.

For strangeness raising transitions the weak Hamiltonian transforms under U spin like the K^0 , i.e., the $U_3 = 1$ component of a U spin vector. For strangeness lowering transitions it transforms like $\bar{K}^0$, the $U_3 = -1$ component of the same vector. In the usual current current theory of non-leptonic weak interactions, the total Hamiltonian transforms like the first component of a U spin triplet :

$$H_W \sim K^0 + \bar{K}^0$$

This transformation property holds if only charged currents ($U \text{ spin} = \frac{1}{2}$) are used or if neutral currents are included to give octet dominance. In either case $|U| \leq 1$; only the $|U_3| = 1$ component carries strangeness. Since the electromagnetic current is U spin invariant, the effective current which induces radiative transitions is also the first component of a U spin vector :

$$J_\mu = \int d^4x T(H_W(x), j^{e.m.}(x)) = J_\mu^+ + J_\mu^- = J_\mu^{(1)}$$

$$J_\mu^\pm = J_\mu^1 \pm i J_\mu^2$$

The currents $J_\mu^\pm$, responsible for $\Delta S = \pm 1$ transitions satisfy the relation

$$J_\mu^\pm = - e^{i\pi U_2} (J_\mu^\mp) e^{-i\pi U_2} \quad (27)$$

which is analogous to the isospin property of the $I = 1$ current responsible for β decay. Since the CP properties are also the same, the analogy is complete (except for hermiticity) so that arguments similar to those relating to first and second class currents are applicable here :

$$\begin{aligned} \langle B_2(p_2, s_2) | J_\mu | B_1(p_1, s_1) \rangle &= - \langle B_2'(p_2, s_2) | J_\mu | B_1'(p_1, s_1) \rangle \\ &= - \langle B_1'(p_1, -s_1) | J_\mu | B_2'(p_2, -s_2) \rangle \end{aligned} \quad (28)$$

where p_i, s_i are momenta and spin and

$$|B_i'\rangle = e^{i\pi U_2} |B_i\rangle$$

The last expression in Eq. (28) is obtained by applying parity $\times$ time reversal. Using the properties of Dirac spinors and the fact that only the term in $\bar{\psi} \gamma_\mu \psi$ contributes for on-shell photons, one can show that Eq. (28) may be written :

$$\langle B_2(p_2, s_2) | J_\mu | B_1(p_1, s_1) \rangle = \mp \langle B_1'(p_2, s_2) | J_\mu | B_2'(p_1, s_1) \rangle \quad (29)$$

where the upper (lower) sign applies for vector (axial) transitions. Then for $\Delta S = +1$ transitions between U spin $\frac{1}{2}$ states :

$$B_1' = -B_2 \quad B_2' = B_1$$

The axial contribution vanishes (in analogy with the vanishing of the $k_\nu \sigma_{\mu\nu} \gamma_5$ form factor in β decay). Therefore current current theory predicts ⁷⁾ for the decays $\Sigma^+ \rightarrow p \gamma$ and $\Xi^- \rightarrow \Sigma^- \gamma$:

$$a \gg b \quad (30)$$

For the neutral decays we may define

$$\begin{aligned} B_3 &= (-\Sigma^0 + \sqrt{3}\Lambda)/2 \\ B_0 &= (\sqrt{3}\Sigma^0 + \Lambda)/2 \end{aligned} \quad (31)$$

as the U spin vector and scalar, respectively. Then Eq. (28) gives :

$$\begin{aligned} \langle B_3 | J_\mu^+ | \Xi^0 \rangle &= \mp \langle n | J_\mu^+ | B_3 \rangle \\ \langle B_0 | J_\mu^+ | \Xi^0 \rangle &= \pm \langle n | J_\mu^+ | B_0 \rangle \end{aligned} \quad (32)$$

where the upper (lower) sign applies for vector (axial) transitions. In terms of the amplitudes defined in Eq. (26) we obtain

$$a_{\Sigma^0} = (2a_n + a_\Lambda)/\sqrt{3} \quad (33)$$

$$b_{\Sigma^0} = (-2b_n + b_\Lambda)/\sqrt{3} \quad (34)$$

These relations were derived by Lo ⁸⁾ under the assumption of octet dominance, which is in fact unnecessary. However, with this assumption he obtains a further relation for the parity violating amplitudes :

$$b_n = -b_\Lambda = -b_{\Sigma^0}/\sqrt{3} \quad (35)$$

Some authors ⁹⁾, assuming octet dominance, have given further relations for the parity conserving amplitudes ; in particular, they relate Σ^+ and Ξ^- decays to the neutral amplitudes. They constructed SU_3 tensors which they failed to decompose correctly into irreducible representations. From purely SU_3 considerations there are seven independent amplitudes (four 8's, 10, $\overline{10}$ and 27) ; this is reduced to five by the fact that the SU_3 matrices for H_W and the electromagnetic current commute. Charge conjugation relates two of these, leaving four independent amplitudes to describe five observable decays [and not three amplitudes as maintained in Ref. 9]. Therefore Eq. (33), which holds in the usual current current theory without octet dominance, is the only prediction possible. However, there are three self-conjugate amplitudes which have the charge conjugation property required for the parity conserving transitions but cannot contribute to parity violating decay. Thus there is only one independent amplitude in that case.

Note that Eq. (33) is not satisfied by the "bremsstrahlung" amplitudes, Eqs. (19) and (20). This is because SU_3 breaking is important here through the mass differences appearing in denominators.

We stated in Section 2 that the parity violating "weak mass" terms are of the same order in $M_1 - M_2$ as the direct term. For charged decays this amplitude has the form (μ is the anomalous magnetic moment) :

$$b^{u.m.} = \frac{\mu}{M} \frac{h_2^5}{M_1 + M_2}$$

$$\bar{u}_2 \gamma_5 u_1 h_2^5 \equiv \langle B_2 | H_W^V | B_1 \rangle \quad (36)$$

For current current theory h_2^5 vanishes in the SU_3 limit so that the over-all amplitude is of order $(M_1 - M_2)^2$, in conformity with Eq. (30).

However, there are models of non-leptonic weak interactions in which the matrix element (36) does not vanish, for example a bilinear quark coupling model ¹⁰⁾. In this case H_W^C transforms like the first component of a U spin triplet as before, but

$$H_W^V \sim -i(K^0 - \bar{K}^0)$$

transforms like the second component. (Operators with this transformation property are sometimes referred to as "abnormal octets" and those discussed above are called "normal octets".) The effective axial current which induces radiative decay has the U spin property

$$J_\mu^V \sim J_\mu^{(2)} = -i(J_\mu^+ - J_\mu^-) \equiv \tilde{J}_\mu^+ + \tilde{J}_\mu^-$$

$$\tilde{J}_\mu^\pm = e^{i\pi U_2} \tilde{J}_\mu^\mp e^{-i\pi U_2}$$

Then for strangeness raising axial transitions we have instead of Eq. (29) :

$$\langle B_2(p_2, s_2) | J_\mu^V | B_1(p_1, s_1) \rangle = -\langle B_1'(p_1, s_1) | J_\mu^V | B_2'(p_2, s_2) \rangle \quad (37)$$

The amplitudes for Σ^+ and Ξ^- decay do not in general vanish, and the amplitudes for neutral decays satisfy the same sum rule as the parity conserving amplitudes :

$$b_{\Sigma^0} = (2b_n + b_\Lambda)/\sqrt{3} \quad (38)$$

Since the "weak-mass" amplitudes (36) do not depend sensitively on the mass differences they will also approximately satisfy Eq. (38) in this model. Although there is no reason to expect these amplitudes to dominate over other contributions, we list them for completeness. Again for reasons of simplification we take $h^5/(M_1+M_2)$ to be the quantity which satisfies SU_3 constraints :

$$h^5/(M_1+M_2) \equiv 2(d_{721}d^5 - if_{721}f^5) \quad (39)$$

where d^5 and f^5 are dimensionless constants. The radiative decay amplitudes are given by

$$b_p^{w.m.} = -2(d^5 - f^5) \text{ GeV}^{-1} \quad (40)$$

$$b_\Lambda^{w.m.} = -b_n^{w.m.} = -\sqrt{3} b_{\Sigma^0}^{w.m.} = \sqrt{6} f^5 \text{ GeV}^{-1} \quad (41)$$

$$b_{\Sigma^-}^{w.m.} = 0 \quad (42)$$

The vanishing of the amplitude for $\Xi^- \rightarrow \Sigma^- \gamma$ is due to the vanishing of the anomalous magnetic moments.

All of the relations derived in this Section are of course only expected to be valid to the extent that SU_3 is a good symmetry, i.e., to about 10%.

5. - DECAY PARAMETERS

The total transition amplitude is given by

$$\mathcal{M}_\mu = \mathcal{M}_{B_\mu} + \mathcal{M}_{D_\mu} = \bar{u}_2 (a'_2 + b_2 \gamma_5) \gamma_\mu \not{k} u_1$$

$$a'_2 = A + a \quad (43)$$

The partial width is related to the amplitudes by :

$$\Gamma = 4\alpha k^3 (|a'|^2 + |b|^2) = \alpha R^2 (M_1^2 - M_2^2)^3 / 2 M_1^3 \quad (44)$$

where we have defined the squared amplitude

$$R = (|a'|^2 + |b|^2)^{1/2} \quad (45)$$

If the weak mass d/f ratio is close to -0.3 , the semi-strong ratio, the bremsstrahlung terms vanish and for neutral decays the amplitudes are expected to satisfy the triangle relation [cf. Eqs. (33), (34) and (38)]

$$|2R_n - R_\Lambda| < \sqrt{3} R_{\Sigma^0} < |2R_n + R_\Lambda| \quad (46)$$

The relation (46) is expected to hold to within 10% for the usual charged current current theory and for any model in which the non-leptonic weak interaction belongs to an octet with well-defined charge conjugation properties. Furthermore all radiative decays are expected to be roughly of the same order of magnitude in this case.

On the other hand, if the d/f ratio is very different from -0.3 so that the weak mass terms dominate the neutral decays, they are expected to be enhanced relative to the charged decays and to satisfy approximately the relations [cf. Eqs. (19) and (20)] :

$$R_{\Sigma^0}^2 \approx 3R_\Lambda^2 \approx 5.3 R_n^2 \quad (47)$$

However, for, say, the f and d parameters obtained from a pole fit to p wave hyperon decays, this is not likely to be the case. From the branching ratio for $\Sigma^+ \rightarrow p \gamma$:

$$\Gamma_{\Sigma^+ \rightarrow p \gamma} / \Gamma_{\Sigma^+ \rightarrow \text{all}} = (1.16 \pm 0.17) 10^{-3} \quad (48)$$

one obtains for the direct emission term

$$R_p = 0.78 \text{ GeV}^{-1} \times 10^5 (\mu\text{sec})^{1/2} \quad (49)$$

which is not negligible in comparison with the values listed in Table I. The f and d values given there are obtained from soft pion theorems and contain the small factor

$$f_\pi = .138 M_N$$

Furthermore, the p wave ratio $d/f \approx -0.8$ is not so different from -0.3 so that a partial cancellation occurs. Nevertheless, the rate for $\Xi^0 \rightarrow \Sigma^0 \gamma$, which gets the largest contribution, may be enhanced with respect to the direct term if we use the amplitude (49) to estimate the latter. To illustrate, we have listed in Table II the rate and branching ratio for $\Xi^0 \rightarrow \Sigma^0 \gamma$ for several d/f values, assuming $R_{\Sigma^0} \approx R_p$ and (a) no interference between the weak mass and direct emission terms, (b) maximum

constructive interference, and (c) maximum destructive interference. We have used the f value of Eq. (21). The enhancement relative to the case

$$d/f \approx 0.3 \quad A_{\Sigma^0} = 0$$

varies from a factor 3 to 50 ; this will in general be less for the decays $\Sigma^0 \rightarrow \Lambda \gamma$ and $\Lambda \rightarrow n \gamma$. For comparison the rate for $\Xi^- \rightarrow \Sigma^- \gamma$ should also be measured ; assuming $R_{\Sigma^-} \approx R_p$, the expected rate is :

$$\Gamma_{\Xi^- \rightarrow \Sigma^- \gamma} = 2.10 \times 10^6 \text{ sec}^{-1} = 3.49 \times 10^{-4} \Gamma_{\Xi^- \rightarrow \text{all}} \quad (50)$$

The tests can of course be sharpened if polarization measurements are feasible, allowing a separation of the amplitudes a' and b . For an initial hyperon of polarization $\vec{P}_1$, the angular distribution in the centre-of-mass is of the form :

$$\begin{aligned} M &\propto 1 + \lambda \vec{P}_1 \cdot \vec{k} / k \\ \lambda &= 2 \operatorname{Re}(a' b^*) / (|a'|^2 + |b|^2) \end{aligned} \quad (51)$$

For an unpolarized initial hyperon, the final state baryon will be longitudinally polarized with polarization :

$$P_L = +\lambda \quad (52)$$

To resolve the ambiguity $a' \leftrightarrow b^*$, it is necessary to look at the final state polarization from an initially polarized hyperon decay. If the initial polarization is $\vec{P}_1$, the final state baryon acquires a transverse polarization. In the direction

$$\hat{n} = \hat{P}_1 - (\hat{P}_1 \cdot \hat{k}) \hat{k}$$

the transverse polarization is

$$\begin{aligned} P_T &= 2\rho P_1 [1 - (\hat{k} \cdot \hat{P}_1)^2] / (1 + \lambda \vec{k} \cdot \vec{P}_1 / k) \\ \rho &= (|a'|^2 - |b|^2) / (|a'|^2 + |b|^2) \end{aligned} \quad (53)$$

If the absorptive parts of the amplitudes are important, there will also be a component of polarization in the direction

$$\hat{n}' = \hat{k} \times \hat{P}_1$$

with magnitude

$$\begin{aligned} P_T' &= -\eta P_1 [1 - (\hat{k} \cdot \hat{P}_1)^2] / (1 + \lambda \vec{k} \cdot \vec{P}_1 / k) \\ \eta &= 2 \operatorname{Im}(a' b^*) / (|a'|^2 + |b|^2) \end{aligned} \quad (54)$$

The longitudinal polarization in this case becomes :

$$P_L = [\lambda + (1 - 4\rho E_2/M_2) \vec{k} \cdot \vec{P}_1 / k] / (1 + \lambda \vec{k} \cdot \vec{P}_1 / k) \quad (55)$$

There are three amplitudes, say, a_n , a_Λ and A_{Σ^0} , which determine completely the parity conserving part of the three observable neutral decays [cf. Eqs. (19), (20) and (33)]. If the parameters λ , ρ and η can be determined, it will be possible to separate out the different contributions.

Of course, Eq. (33) may not be satisfied. However, if the parity violating amplitudes satisfy :

$$\sqrt{3} b_{\Sigma^0} = b_\Lambda \pm 2 b_n \quad (56)$$

this would give one confidence in the analogous relation for the parity conserving direct emission terms. Since only the relative sign between a' and b can be determined, a choice must still be made between Eq. (34) (current current theory) and Eq. (38) (quark pseudoscalar density) for the parity violating amplitudes. This can be resolved by studying $\Sigma^+ \rightarrow \gamma p$ and $\Xi^- \rightarrow \Sigma^- \gamma$. If $a' \gg b$ for these decays, one can reasonably assume that Eq. (34) will also be satisfied. If $a \sim b$ for Σ^+ and Ξ^- decay, but Eq. (56) is nevertheless satisfied, this would be evidence in favour of the quark density model (or any model with the same charge conjugation properties). Then Eq. (38) may be assumed.

6. - DISCUSSION

The idea that the f/d ratio for the weak mass should be the same as that for the semi-strong baryon mass splitting has been suggested in conjunction with various theoretical proposals : the Coleman-Glashow tadpole mechanism ¹¹⁾, the Dashen-Frautschi bootstrap perturbation theory ¹²⁾, the domination of both the weak and semi-strong masses by a bilinear quark term ⁹⁾. This idea is supported empirically by a current algebra analysis of s wave hyperon decays. On the other hand a pole analysis of p wave decay amplitudes favours a different weak f/d ratio.

We propose here a further test in the study of radiative hyperon decays. The test is negative in the sense that if the above idea is correct the weak mass will not contribute to radiative decays. Then the parity conserving amplitudes are expected to satisfy the relations

$$\sqrt{3} a'_{\Sigma^0} \approx 2 a'_n + a'_\Lambda \quad (57)$$

On the other hand, if the weak f/d ratio is very different from the semi-strong ratio, the amplitudes for neutral decay may be dominated by the weak mass contribution, in which case they will satisfy the relations :

$$a'_\lambda \approx -0.56 a'_n \approx a'_{\Sigma^0} / \sqrt{3}$$

and are expected to be enhanced relative to the charged decay amplitudes. In the intermediate case where the weak mass term is present but not dominant (as expected if the f and d parameters are those suggested by a pole fit to p wave hyperon decay), the direct emission and weak mass amplitudes can in principle be extracted separately from the experimental study of radiative decay, as discussed in the previous Section.

Whether or not the weak mass term is present, the parity violating amplitudes may be studied to distinguish between a current current (or "normal octet") theory which predicts

$$\begin{aligned} b_p &\approx b_{\Sigma^-} \approx 0 \\ \sqrt{3} b_{\Sigma^0} &\approx -2 b_n + b_\lambda \end{aligned} \quad (58)$$

and a bilinear quark density (or "abnormal octet") theory which predicts

$$\sqrt{3} b_{\Sigma^0} \approx 2 b_n + b_\lambda \quad (59)$$

Furthermore, if Eqs. (58) hold, the additional relations, predicted by octet dominance

$$b_n \approx -b_\lambda \approx -b_{\Sigma^0} / \sqrt{3} \quad (60)$$

may be tested to give some indication as to whether the observed $\Delta I = \frac{1}{2}$ rule is of dynamic origin or is inherent to the weak interaction. Finally, if Eq. (59) holds, and in particular if

$$b_{\Sigma^-} \approx 0 \quad b_\lambda \approx -b_n \approx -\sqrt{3} b_{\Sigma^0} \quad (61)$$

it would be reasonable to assume that the parity violating "weak mass" dominates these amplitudes. The amplitude for $\Sigma^+ \rightarrow p \gamma$ could then be compared with the neutral decay amplitudes to determine the f/d ratio of this term. However, there is no a priori reason for this term to dominate and it is not otherwise distinguishable from the direct term.

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f	d	d/f	A_{Σ^0}	A_{Λ}	A_n
-0.187	0.063	-0.34	-0.061	-0.035	0.020
-0.187	0	0	0.608	0.351	-0.198
0	0.063	∞	-0.669	-0.386	0.218
-0.416	0.328	-0.79	-2.13	-1.23	0.693
-0.416	0	0	1.35	0.780	-0.439
0	0.328	∞	-3.47	-2.01	1.13

TABLE I : Bremsstrahlung amplitudes as a function of the weak mass parameters. The weak mass parameters f and d are expressed in units of $10^5 \times (\mu \text{ sec})^{-\frac{1}{2}} \times \text{GeV}$, the amplitudes A in units of $10^5 \times (\mu \text{ sec})^{-\frac{1}{2}} \text{GeV}^{-1}$.

d/f	Interference	$R_{\Sigma^0}^2$	$\Gamma_{\Xi^0 \rightarrow \Sigma^0 \gamma} (\text{sec}^{-1})$	Branching Ratio
-0.31	-	0.61	1.91×10^6	6.12×10^{-4}
-0.79	-	5.23	1.64×10^7	5.25×10^{-3}
-0.79	Constructive	8.59	2.69×10^7	8.62×10^{-3}
-0.79	Destructive	1.87	5.87×10^6	1.88×10^{-3}
+0.79	-	23.8	7.45×10^7	2.39×10^{-2}
+0.79	Constructive	31.3	9.81×10^7	3.14×10^{-2}
+0.79	Destructive	16.3	5.10×10^7	1.63×10^{-2}

TABLE II : $\Xi^0 \rightarrow \Sigma^0 \gamma$ partial widths for $|f| = 0.416 \times 10^5 (\mu \text{ sec})^{-\frac{1}{2}} \text{GeV}$ and $a_{\Sigma^0}^2 + b_{\Sigma^0}^2 = R_p^2$. $R_{\Sigma^0}^2$ is in units of $10^{10} (\mu \text{ sec})^{-1} \text{GeV}^{-2}$.

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FIGURE CAPTIONS

Figure_1 Bremsstrahlung diagrams for charged hyperon decay.

Figure_2 Direct emission diagram for radiative hyperon decay.

Figure_3 Bremsstrahlung diagrams for $\Xi^0 \rightarrow \Sigma^0 \gamma$.

Figure_4 Pole diagrams for non-leptonic hyperon decay.

FIG. 1

FIG. 2

FIG. 3

FIG. 4