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DUAL APPROACH TO REGGE DIPS

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A B S T R A C T

The analysis of πN charge exchange suggests that some prominent features associated with Regge dips are also properties of each of the leading peripheral resonances, duality holding in an almost local sense. The use of local duality, together with a simple Regge prescription for the dips leads, however, to very strong restrictions for more complicated spin structures. We study as a test case $\pi \Delta$ charge exchange and show how such constraints could still work and produce a nonsense zero. The equivalence between the t channel Regge pole and s channel absorptive description of the dip structure is, however, lost. A different s channel description emerges and leads to a strong prediction for two step decays of higher resonances. A qualitative discussion of a few other reactions is briefly presented.

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I. - INTRODUCTION

The first success of duality was in its application to πN charge exchange (CEX) : resonances in the πN system reproduce indeed some properties characteristic of the high energy behaviour of the scattering amplitudes ¹⁾. In particular it turns out that the contribution of each prominent πN resonance to the A' and B amplitudes vanishes for values of the momentum transfer squared (t), which coincide with the values expected from a high energy analysis, namely $t \simeq -0.2 \text{ GeV}^2$ and $t \simeq -0.5$ to -0.6 GeV^2 respectively. In the Regge language, one associates the zero at $t \simeq -0.6$ with the first nonsense wrong signature zero of the ρ trajectory, expected at least for the t channel spin flip amplitude B . However, duality and the absence of exotic resonances in the $\pi\pi$ system lead to a ρ -f exchange degeneracy pattern which requires a nonsense choosing ρ trajectory and thus implies a zero at this t value ($t \simeq -0.6$) also for the non-flip t channel π nucleon CEX amplitude, A' . Experimentally, however, one finds a zero at $t \simeq -0.2 \text{ GeV}^2$. The standard explanation nowadays is to blame this shift of the nonsense wrong signature zero on a cut effect. Although a successful quantitative calculation of the shift is still lacking, at least qualitatively, it is clear that a cut due to Pomeron ρ interference will affect the non-flip amplitude much more strongly than the flip amplitude ²⁾. Another possibility is of course to appeal to an appropriate dynamical zero in the non-flip amplitude and abandon strong exchange degeneracy. In any case, duality between resonances and leading Regge pole exchanges is, at best, only an approximate property of the strong interactions : to observe leading Regge pole effects in the B amplitude is therefore quite satisfying since, in CEX, the flip amplitude is by far the larger one.

On the other hand, actual dominance of the exchange of a single nonsense choosing ρ Regge pole is thus expected to impose severe constraints on scattering amplitudes for processes where many spin flips can occur : several amplitudes are expected to vanish around $t \simeq -0.6 \text{ GeV}^2$. Can duality cope with such strong constraints ? In

other words, is it possible for resonances to develop zeros at or around the same t value in many of the flip and non-flip amplitudes they contribute to? In the affirmative, this must imply a striking regularity in the behaviour of resonances that should be explored. In this paper we investigate this problem for a particular reaction that we take as a test case. We chose $\pi\Delta$ charge exchange which is a reaction not too different from πN CEX but involves nevertheless high spin flips. Indeed from a purely theoretical point of view, the helicity structure of $\pi\Delta$ scattering is rich enough to test duality in a domain which the πN amplitudes could not cover while at the same time the helicity crossing matrix remains relatively simple thanks to the elastic kinematics.

Zeros of helicity amplitudes have of course immediate implications for the presence of dips in density matrix elements and in differential cross-sections. These dips which, according to duality, can be associated with either the direct channel resonances or the crossed channel Regge poles, can also be seen as the result of a simple diffractive picture. In this approach where the impact parameter is kept fixed, dips can be associated with zeros of the imaginary parts of the amplitudes and these are located according to the dominant s channel helicity flip ³⁾. Such a model ⁴⁾ can then simply be related to a direct channel resonance picture if one assumes that the leading resonances occur in peripheral waves ¹⁾. A priori it is not clear how such a simple s channel picture can be made compatible with the simple t channel picture underlying a Regge pole approach. However, it is well known that no difficulty is met with either description in the case of πN CEX. This will be briefly recalled in Section 2 where the kinematics of πN and $\pi\Delta$ are presented together with a discussion of the helicity crossing matrices. This discussion will illustrate the fact that compatibility between the s and t channel descriptions cannot be trivially generalized to the slightly more complicated case of $\pi\Delta$ charge exchange scattering.

In Section 3, we develop a possible solution to the $\pi\Delta$ problem which would allow a standard Regge analysis to go through. It deviates, however, from a straightforward generalization of the simple s channel diffraction model, although it does lead to a new diffractive picture. We then proceed with a discussion of some experimental consequences of the $\pi\Delta$ solution and end the Section with a confrontation of different models. Other two-body reactions are then briefly discussed in Section 4.

2. - KINEMATICS AND CROSSING MATRICES

We describe the reactions $\pi(p_1) + A(p_2) \rightarrow \pi(p_3) + A(p_4)$. A stands for either the nucleon or the Δ . Following standard notations we introduce

$$\begin{aligned} s &= (p_1 + p_2)^2 & t &= (p_3 - p_1)^2 \\ u &= 2M^2 + 2\mu^2 - s - t \end{aligned} \quad (1)$$

We denote $m_A = M$ and $m_\pi = \mu$ and further define

$$\begin{aligned} \mathcal{S} &= [(s - (M + \mu)^2)(s - (M - \mu)^2)]^{1/2} \\ \mathcal{T} &= [t(t - 4M^2)]^{1/2} \end{aligned} \quad (2)$$

together with

$$\Phi = t(su - (M^2 - \mu^2)^2)$$

The crossing matrix between the s channel and t channel helicity amplitudes reads as follows ⁵⁾:

$$M_{0\lambda_4; 0\lambda_2}^{(s)} = e^{i\pi\lambda_2} \sum_{\lambda'_i} d_{\lambda'_2 \lambda_2}^s(x_2) d_{\lambda'_4 \lambda_4}^s(x_4) M_{00; \lambda'_4 \lambda'_2}^{(t)} \quad (3)$$

The crossing angles χ_2 and χ_4 are defined through the relations

$$\begin{aligned}\cos \chi_2 &= \frac{t(s+M^2-\mu^2)}{s\pi} = -\cos \chi_4 \\ \sin \chi_2 &= \frac{2M\Phi^{1/2}}{s\pi} = -\sin \chi_4\end{aligned}\quad (4)$$

which in the high s (relatively low t) limit give

$$\begin{aligned}\cos \chi &= \cos \chi_2 \simeq \left(\frac{t}{t-4M^2} \right)^{1/2} \\ \sin \chi &= \sin \chi_2 \simeq \frac{2M}{(4M^2-t)^{1/2}}\end{aligned}\quad (5)$$

Deleting the helicity indices for the pion, we introduce s channel and t channel helicity amplitudes which we write respectively as $M_{\mu\lambda}^{(s)}$ for $M_{0\mu/2,0\lambda/2}^{(s)}$ and as $M_{\mu\lambda}^{(t)}$ for $M_{00,\mu/2\lambda/2}^{(t)}$.

In the case of π nucleon scattering there are two independent amplitudes which we choose as

$$M_{11}^{(t)} = -M_{-1-1}^{(t)} \quad \text{and} \quad M_{1-1}^{(t)} = M_{-11}^{(t)}$$

or as

$$M_{11}^{(s)} = M_{-1-1}^{(s)} \quad \text{and} \quad M_{1-1}^{(s)} = -M_{-11}^{(s)}\quad (6)$$

Using (3) one then obtains

$$\begin{aligned}M_{11}^{(s)} &= i \left(\sin \chi M_{11}^{(t)} - \cos \chi M_{1-1}^{(t)} \right) \\ M_{1-1}^{(s)} &= -i \left(\cos \chi M_{11}^{(t)} + \sin \chi M_{1-1}^{(t)} \right)\end{aligned}\quad (7)$$

which, at $t = 0$, reduces to

$$M_{11}^{(s)} = i M_{11}^{(t)} \quad \text{and} \quad M_{1-1}^{(s)} = -i M_{1-1}^{(t)} \quad (8)$$

In addition, the s and t channel flip amplitudes both vanish as $t^{\frac{1}{2}}$. Furthermore, as can be seen from Eqs. (5) and (7), the off diagonal elements of the crossing matrix remain negligible for small values of t . The s and t channel non-flip and flip amplitudes respectively are thus essentially equal to each other for high energy scattering near the forward direction. It is precisely this diagonal character of the helicity crossing matrix which trivially guarantees consistency between the simple (s channel) diffractive picture and a (t channel) Regge pole approach.

Indeed, as is well known, the scattering is dominated by the (s or t channel) flip amplitude. A simple evidence for this is the fact that the differential CEX cross-section turns over near $t = 0$. According to the diffractive model, the imaginary part of the (s channel) flip amplitude is approximated by the Bessel function $J_1(b\sqrt{-t})$ where b is the impact parameter. With $b \simeq 1$ fermi, the second zero of the amplitude occurs near $t \simeq -0.6 \text{ GeV}^2$. The same structure follows of course from a t channel analysis where the zero at $t \simeq -0.6 \text{ GeV}^2$ corresponds to the expected nonsense wrong signature zero. For the (dominant) flip amplitudes, the two descriptions are thus obviously almost equivalent at least for t values of the order $|t| \lesssim 1 \text{ GeV}^2$.

For the much smaller non-flip amplitude, the diffractive point of view takes its imaginary part as proportional to $J_0(b\sqrt{-t})$. With the same impact parameter ($b \simeq 1$ fermi) one now expects a first zero around $t \simeq -0.2 \text{ GeV}^2$ which is indeed observed experimentally. The interpretation of this zero in terms of t channel exchange is not simple any more. One has to invoke a dynamical zero or, as at present favoured, call for a pole cut interference.

Let us now turn to $\pi \Delta$ CEX and see whether this fairly simple two-way description can be extended to this more complicated case. Here we have six independent amplitudes which we choose as

$$\begin{aligned} M_{33}^{(s)} &= M_{-3-3}^{(s)} & M_{3-3}^{(s)} &= M_{-33}^{(s)} & M_{11}^{(s)} &= M_{-1-1}^{(s)} & M_{1-1}^{(s)} &= -M_{-11}^{(s)} \\ M_{31}^{(s)} &= -M_{-3-1}^{(s)} = -M_{13}^{(s)} & & & & & & \\ M_{3-1}^{(s)} &= M_{-31}^{(s)} = M_{-13}^{(s)} & & & & & & \end{aligned} \quad (9)$$

and similarly, six t channel helicity amplitudes which are given by

$$\begin{aligned} M_{33}^{(t)} &= -M_{-3-3}^{(t)} & M_{3-3}^{(t)} &= M_{-33}^{(t)} & M_{11}^{(t)} &= -M_{-1-1}^{(t)} & M_{1-1}^{(t)} &= M_{-11}^{(t)} \\ M_{13}^{(t)} &= M_{31}^{(t)} = M_{-1-3}^{(t)} & & & & & & \\ M_{3-1}^{(t)} &= -M_{-31}^{(t)} = -M_{1-3}^{(t)} & & & & & & \end{aligned} \quad (10)$$

The equality used in order to define the fifth and sixth independent amplitudes stems from charge conjugation invariance which can be used in the t channel to provide the constraints imposed by parity and time reversal invariance in the s channel. It indeed gives

$$M_{13}^{(s)} = -M_{31}^{(s)} \quad M_{3-1}^{(s)} = M_{-13}^{(s)}$$

The orthogonal crossing matrix, as calculated from Eq. (3), is displayed in Table I. Using Eq. (5) one obtains the matrix elements which do not vanish at $t = 0$. They are given in Table II. Explicit calculations show that for large s these matrix elements are the ones which remain relatively large in the whole near forward region, up to $t = -0.6$, say. In addition we note that near $t \simeq 0$, one finds

$$\eta_{\lambda\mu}^{(s)} \sim (-t)^{\frac{|\lambda-\mu|}{4}} \quad M_{\lambda\mu}^{(t)} \sim (-t)^{\frac{1}{4}} \left(1 - (-1)^{\frac{|\lambda-\mu|}{2}}\right)$$

One immediately sees that the one-to-one correspondence between flip and non-flip amplitudes no longer holds. What matters is the odd or even character of the helicity flip. The s channel non-flip amplitudes draw from the t channel double flip as well as from the t channel non-flip amplitudes. At the same time the s channel single flip and triple flip amplitudes are mainly related (this is an exact statement at $t = 0$) to their t channel single flip and triple flip counterparts.

In addition the absence of exotic exchanges implies an over-all cancellation in the imaginary part of the $I_t = 2$ crossed channel amplitude between direct channel resonances of isospin $\frac{1}{2}$ and $\frac{3}{2}$. The contributions of these resonances must then be added up to build the $I_t = 1$ crossed channel amplitude in $\pi\Delta$ (single) charge exchange scattering. This holds of course for every helicity amplitude. This is a crucial difference between the πN and $\pi\Delta$ problems. Whereas in πN CEX the A' amplitude is small because of the different signs of the N and Δ families in the s channel, we find in $\pi\Delta$ that each of these two families has to obey all constraints by itself and we have to cope with big non-flip amplitudes.

If we now extend to $\pi\Delta$ scattering, the assumption which was verified in πN CEX, namely that a single Regge pole provides a good approximation to the t channel flip amplitudes, we expect again zeros around $t = -0.6$. As implied by the crossing matrices of Tables I and II, these zeros should then be present not only in both s channel single flip amplitudes, but also at least in the s channel triple flip amplitude. This obviously contradicts a simple diffractive picture with fixed impact parameter where, say, the single and triple flip s channel amplitudes are expected to behave as $J_1(b\sqrt{-t})$ and $J_3(b\sqrt{-t})$ respectively. The second zeros of these Bessel functions occur at quite different values of t (in fact, they are in a ratio of the order of three !). Using different impact parameters for different flip amplitudes is of course a possibility : with $b = 1$ and 1.7 fermi for the single and triple flip respectively, they both vanish around $t \simeq -0.6 \text{ GeV}^2$. Such a possibility may be considered. It is completely at odds, however,

with any local dual picture where the same leading resonances contribute to all the different s channel amplitudes in a way prescribed by their respective parities.

Furthermore, the vanishing of the t channel double flip amplitude at the correct t value imposes additional restrictions on any s channel picture : either a subtle cancellation between the s channel non-flip and double flip contributions has to occur or all amplitudes have to vanish around $t = -0.6 \text{ GeV}^2$.

On the other hand, to impose a zero at $t = -0.6 \text{ GeV}^2$ does not necessarily conflict with a simple s channel picture if one argues that for dynamical reasons the s channel single flip amplitude is big in $\pi \Delta$ CEX because of the similarity to πN CEX. Such an argument could, for instance, be made in the quark model where both $\pi N \rightarrow \pi N$ and $\pi N \rightarrow \pi \Delta$ charge exchange reactions involve the same quark spin flip process which through vector dominance is supposed to give a large contribution to the scattering. Extending this picture to $\pi \Delta$ charge exchange obviously results in the dominance of s channel single flip. This necessarily calls for cancellations between direct channel resonances of opposite parity since, as obvious from factorization, each resonance contributes equal amounts (up to a sign) to final states with opposite helicities. For example, if a resonance of a given parity gives a large contribution to, say, $M_{3-1}^{(s)}$, by parity it will give an equally large contribution to $M_{3-1}^{(s)}$ that has to be cancelled by resonance(s) of opposite parity if the s channel single flip dominates. Note, however, that by using only a single isospin in the s channel one cannot arrange a dominance of the single flip over the non-flip amplitude since the latter is composed of contributions of the same sign only. This is a big difference between πN and $\pi \Delta$ CEX.

Co-operative effects among resonances do not violate any basic principle. Nevertheless our experience with πN CEX points to the fact that the larger amplitude, $\mathcal{V}_B^{(-)}$ or $A^{(-)}$ is built in a cumulative way from the resonances while the smaller one, $A'^{(-)}$, witnesses important cancellations. Indeed, as well known, each prominent resonance gives a contribution to B which vanishes in the

neighbourhood of $t = -0.6 \text{ GeV}^2$. It is therefore tempting to look for a similar solution in the case of $\pi\Delta$ scattering or, in other words, to apply duality in an almost local sense, thus separating the contributions from each leading resonance.

It is quite possible, however, that such a local dual model which apparently works in π nucleon scattering is of no use in the $\pi\Delta$ case but leads to an awfully complicated picture in terms of t channel exchanges, each prominent resonance spreading its t channel amplitude zeros among quite different t values. Simplicity in the t channel description, on the other hand, would then imply a very complicated s channel description which cannot possibly be checked with the scanty information presently available on the $\pi\Delta$ channel.

It turns out though, as will be discussed in the next Section, that there is a simple solution to the $\pi\Delta$ problem with the property that each leading resonance induces zeros in the t channel helicity amplitudes which cluster around specific values of t , including $t \simeq -0.6 \text{ GeV}^2$.

3. - A SOLUTION TO $\pi\Delta \rightarrow \pi\Delta$ CHARGE EXCHANGE

Except if they have spin $\frac{1}{2}$, resonances can couple to the $\pi\Delta$ channel in two different partial waves. We use this freedom to parametrize the partial widths into helicity $\frac{1}{2}$ and $\frac{3}{2}$ by $\Gamma_p \sin^2 \alpha$ and $\Gamma_p \cos^2 \alpha$ respectively. The contribution of a resonance to a s channel helicity amplitude is then given by

$$\eta_{\lambda\mu}^{(s)} = (-1)^{\frac{\lambda-\mu}{2}} d_{\frac{\mu}{2}\frac{\lambda}{2}}^{\frac{\lambda}{2}}(\theta_s) \frac{\Gamma_p R_{\lambda\mu}}{M_R - \sqrt{s} - i\frac{\Gamma}{2}} \quad (11)$$

where

$$R_{33} = \cos^2 \alpha \quad R_{31} = \cos \alpha \sin \alpha \quad R_{11} = \sin^2 \alpha \quad (12)$$

Parity invariance implies that

$$R_{\mu-\lambda} = \eta (-1)^{J+1/2} R_{\mu\lambda} \quad (13)$$

where J and η denote the spin and intrinsic parity of the resonance respectively.

The zeros of the $M_{\lambda\mu}^{(s)}$ amplitudes depend on the spin J of the resonance through $d_{\mu\lambda}^J(\theta_s)$ but not on its parity. The structure in $M^{(t)}$ will in general depend on the parity as well since it is constructed by sums of $M^{(s)}$ amplitudes and their relative signs may be crucial. The structure of the zeros of s channel amplitudes for various prominent resonances is shown in the Mandelstam plane in Fig. 1. We would like to note several important facts that stem from it.

(A) - In order to get zeros at the same t values one has to increase the spin of the resonance as one increases its mass. This is the well-known effect from πN scattering which leads to the assumption of peripheral resonances as previously discussed.

(B) - An asymptotic relation for the d functions is the following

$$d_{\lambda\mu}^J(\theta) \sim J_{|\lambda-\mu|}((J+1/2)\theta) \quad (14)$$

which holds for high J and low θ such that $J\theta$ is constant. For the low values characteristic of the resonances that we look at, this cannot yet be regarded as a good approximation. Nevertheless, some of

the features of the Bessel function description can already be traced here, namely the order of the appearance of the zeros in the different amplitudes is like that of the various Bessel functions as indicated on Fig. 1.

(C) - The first zeros of the two non-flip amplitudes occur almost at the same place in all resonances (corresponding to the asymptotic description by the same Bessel function J_0). This implies that low zeros will be seen in $M_{11}^{(t)}$, $M_{33}^{(t)}$, $M_{3-1}^{(t)}$. This is so because for very low t values one finds, as already mentioned, that these amplitudes are connected to the non-flip s channel amplitudes.

(D) - The second zero of $M_{33}^{(s)}$ lies quite close to the second zero of $M_{3-1}^{(s)}$. This is true asymptotically because of the closeness of the zeros of J_0 and J_3 and it is interesting to see how nicely it holds for the higher resonances displayed in Fig. 1. We will see in the following that this fact plays an important role in the problem that we discuss.

Now we have all the necessary apparatus in order to construct the t channel amplitudes and to investigate their structure. We are first interested in the location of their zeros. This depends on the spin and parity of the resonance as well as on the angle α of Eq. (12). In Fig. 2, we show this complicated structure for the $\frac{7}{2}^+$ resonance of mass 1950 [the $\Delta(1950)$ is known to decay strongly into $\pi\Delta$] as a function of $\cos^2 \alpha$. The effect that we look for is a concentration of the various zeros around one t value which should be close to -0.6 GeV^2 . We note that there does exist such a point near $\alpha \approx 0$. This curious fact can be explained by one of the properties of the s channel zeros that we discussed above, namely the closeness of the second zeros of $M_{33}^{(s)}$ and of $M_{3-1}^{(s)}$. If $\alpha \approx 0$ then these are the only two amplitudes that have considerable contributions from the resonance in question. Since

$$\frac{d\sigma}{dt} \propto \sum_{\mu\lambda} |M_{\mu\lambda}^{(s)}|^2 = \sum_{\mu\lambda} |M_{\mu\lambda}^{(t)}|^2 \quad (15)$$

there must be a strong dip ³⁾ in $d\sigma/dt$ in the neighbourhood of the two close zeros of $M_{33}^{(s)}$ and $M_{3-3}^{(s)}$ if $\kappa \approx 0$. Because of the equality in (15), it follows that all $M_{\mu\lambda}^{(t)}$ must vanish nearby so that the sum of the absolute values of their squares produces the same effect. We may conclude that for the resonances to show the phenomenon of close zeros they have to prefer a decay into $\pi\Delta$ from a helicity $\frac{3}{2}$ state (implied by $\kappa = 0$).

It is important to note that this is actually the only way in which a concentration of zeros can come about. This is even more than we first expected since the zero is present in all t channel helicity amplitudes instead of in some of them (flip amplitudes vanishing at $t = 0$, say) and they are therefore all a priori amenable to a relatively simple Regge pole description which is suitable only for the flip amplitude in πN charge exchange. Again there is no compelling reason for this specific (and as we see extreme) choice to be made. Nevertheless it is such a remarkable property that it is worth testing its consequences. We indeed find an effect that is not present in the πN scattering: the non-flip amplitudes $M_{11}^{(t)}$ and $M_{33}^{(t)}$ are in general big and acquire a second zero near $t = -0.6$. This is a reflection of the different s channel mechanism in both cases. This different mechanism leads also to different predictions with respect to the exact location of the zeros if a peripheral picture is assumed. In πN one expects a zero at $t \simeq -0.2$ from J_0 and at $t \simeq -0.6$ from J_1 . If one insists in $\pi\Delta$ on the concentration of the $t \simeq -0.6$ zeros in $M^{(t)}$ then it corresponds roughly to the second zero of J_3 . This calls of course for a larger diffraction radius in $\pi\Delta$. The increase in this radius will be given roughly by a factor 1.6. It leads to a shift of the first dip (the first zero of J_0) down to $t \simeq -0.1$.

If the concentration of zeros at $t = -0.6$ is a very interesting property, welcome from the Regge point of view, the relative concentration around $t = -0.1$ may seem puzzling. As shown by Figs. 2 and 3 this may affect only some of the amplitudes and not all of them as the zero at $t = -0.6$ does. It always includes, however, a non-flip and a double flip amplitude and any explanation for this closer zero in terms

of cut effects seems obscure. One should rather consider a so-called dynamical zero attached to the Δ vertex part of the residue function and disconnected from the nonsense choosing zero imposed at the same time.

The newly suggested mechanism can be tested experimentally by observing the decay modes of the prominent resonances in the $\pi\Delta$ channel. A measurement of the $\Delta(1950)$ decay properties would therefore be highly desirable. Until this is available we can compare our predictions only with the lower lying resonances $N(1680)_{\frac{5}{2}}^{-}$ and $N(1690)_{\frac{5}{2}}^{+}$ that were analyzed in $\pi N \rightarrow \pi\Delta$ experiments. Since these are low spin resonances, one cannot yet observe the characteristics of Fig. 2 in detail. Nevertheless, some preference for the value $\alpha \approx 0$ prevails. In Fig. 3, we show the strips in this region for several resonances. We note that regarding the concentration of zeros as a figure of merit, the $N(1680)_{\frac{5}{2}}^{-}$ could be labelled as "fair" whereas the $N(1690)_{\frac{5}{2}}^{+}$ could be regarded as "poor". This is indeed also reflected in the agreement with the analysis of their decay amplitudes. If we denote by ρ_L and ρ_H the partial widths in the lower and higher angular momentum states respectively, then the point $\alpha = 0$ corresponds for the $\frac{5}{2}^{-}$ to $\rho_L/\rho_H \simeq 6$ whereas for the $\frac{5}{2}^{+}$ one would expect 0.66. Kernan and Shepard obtain for both cases very strong dominance of the lower waves ⁶⁾. The decay width of $\frac{5}{2}^{+}$ into $\pi\Delta$ may be about half that of the $\frac{5}{2}^{-}$; we may therefore feel gratified by the fact that the resonance that is more strongly coupled to this channel fits better the description.

As already mentioned, the decay of the $\Delta(1950)_{\frac{7}{2}}^{+}$ would be an ideal test. To this end, one should study the decay chain ⁷⁾

$$\Delta\left(\frac{3}{2}^{+}\right) \rightarrow \Delta\left(\frac{3}{2}^{+}\right) + \pi \quad ; \quad \Delta\left(\frac{3}{2}^{+}\right) \rightarrow N\left(\frac{1}{2}^{+}\right) + \pi$$

If the $\Delta(\frac{3}{2}^{+})$ is produced with helicity $\pm\frac{3}{2}$ then the angular distribution of the nucleon in the $\Delta(\frac{3}{2}^{+})$ centre-of-mass frame should be of the form $I(\theta) = \frac{3}{4}(1 - \cos^2\theta)$ whereas an helicity $\pm\frac{1}{2}$ would yield an angular distribution $I(\theta) = (3\cos^2\theta + 1)/4$. Similar predictions should also hold for high spin $Y(I=1) \rightarrow \pi Y(I=1)$ decays.

Summarizing what we have thus learned from this study of πA scattering, we may say the following.

(1) - If local duality is a valid property of the prominent amplitudes, in the sense that, similarly to what happens for $B^{(-)}$ in πN , each resonance gives a zero where the amplitude, as a whole, vanishes, then the only way to achieve consistency between a simple Regge picture and a simple peripheral s channel picture is to have the dominant resonances couple much more strongly to helicity $\pm \frac{3}{2}$ than to helicity $\pm \frac{1}{2}$. The impact parameter needed to get a zero at $t = -0.6 \text{ GeV}^2$ is 1.6 times as large as in πN CEX and while all t channel amplitudes do vanish near $t = -0.6$, some of them have an additional zero around $t \simeq -0.1 \text{ GeV}^2$.

(2) - Any alternative simple s channel picture with local duality necessarily conflicts with a nonsense choosing Regge model. Suppose for example that $M_{11}^{(s)}$ and $M_{1-1}^{(s)}$ are the dominant s channel amplitudes. They necessarily vanish at different t values and lead to large t channel amplitudes of all kinds with zeros scattered around, as can be seen in Fig. 2.

(3) - If one gives up the local duality assumption it is of course fairly easy to reconcile the s and t channel pictures. As an extreme example we may consider the case where for each resonance, of either parity but still of the same isospin (to insure the absence of exotic exchanges), only $M_{11}^{(s)}$ and $M_{1-1}^{(s)}$ are non-zero. By averaging over a couple of resonances of opposite parity, one could be left with only $M_{11}^{(s)}$ (which is positive definite and cannot be averaged out to zero) as a large amplitude in the s channel. With a standard peripheral picture it would vanish say around $t = -0.2$ but changing the impact parameter one can get the zeros at -0.1 and -0.6 . By crossing, the t channel non-flip and double flip amplitudes would then have to vanish at these same t values. This may be an acceptable Regge picture but it is, to say the least, not a very simple one. It is worth noting that even without local duality, at least one of the s channel non-flip amplitudes always has to be a dominant component of the scattering.

(4) - One could finally have a weak cut solution similar to the one met in π nucleon scattering, with approximately good Regge behaviour for the odd flip amplitudes. We would consider the Regge pole description as already accurate for the t channel amplitudes which vanish at $t = 0$, the effect of the cuts being of some importance only for the even flip amplitudes. Unless we look at case (4) as a mere rephrasing of case (1), we would expect in general big non-flip amplitudes and, as a result, no dip in the differential cross-section.

4. - DIRECT CHANNEL VERSUS CROSSED CHANNEL DESCRIPTIONS IN OTHER INELASTIC TWO-BODY REACTIONS

It would be tempting to extend our study to inelastic high spin processes such as $\pi N \rightarrow \omega N$, $\pi N \rightarrow \omega \Delta$, etc. However, it is not possible to push forward our present analysis in any meaningful way since we know essentially nothing about the couplings of individual resonances to, say, a vector meson-decuplet final state. We will thus restrict ourselves mostly to qualitative remarks about some simple correlations between direct channel and crossed channel zeros which are required by local duality, as previously defined.

A remarkable correlation between direct channel resonant masses and the intercept of the leading crossed channel trajectory follows in a locally dual approach from the mass dependence of the momentum transfer at which a resonance contribution to a scattering amplitude vanishes. In πN charge exchange, as repeatedly emphasized, prominent direct channel peripheral resonances vanish around $t = -0.6$ which is the value at which the crossed channel trajectory goes through zero. In $\pi N \rightarrow K \Lambda$ or $\pi N \rightarrow K \Sigma$, the same peripheral resonances must give a vanishing contribution for a much smaller value of t because of heavier external masses. Therefore, the crossed channel trajectory has to pass through zero sooner, i.e., lies below the ρ trajectory. This prediction is of course verified since in the latter

case the leading trajectory is the $K^* - K^{**}$ trajectory ! In KN charge exchange on the other hand, not only the external masses but also the resonance masses themselves are heavier than in the πN case. The two effects essentially cancel each other out so that the resonance contributions vanish again around $t \simeq -0.6$.

The case of $\pi N \rightarrow \eta N$ is more interesting since the direct channel resonances are identical to many of those relevant for πN scattering. $\rho - A_2$ exchange degeneracy forces the imaginary part of the amplitude to vanish when the leading (A_2) trajectory goes through zero, i.e., around $t = -0.6$. With local duality, this should also occur as a co-operative effect between direct channel resonances. A zero in the angular distribution of the πN and ηN processes corresponds, however, to quite different t values which in first approximation are in the ratios

$$\frac{t_{\pi N}}{t_{\eta N}} = \frac{(s_R - (M + \mu)^2)(s_R - (M - \mu)^2)}{(s_R - (M + m_\eta)^2)(s_R - (M - m_\eta)^2)}$$

where s_R is the mass squared of any resonance. This ratio gets as big as 1.7 for example, in the case of the $N^*(1990)$ whose contribution to πN scattering agrees perfectly with local duality. The leading crossed channel trajectories cannot be dual to the same leading resonances in both reactions. In order to shift the dip in ηN scattering to a larger t value than the one expected from the leading resonances met in πN scattering, we have to call for important contributions from resonances with lower spin. As a matter of fact, it is known that (daughter) resonances such as the $S_{11}(1535)$ contribute a lot to η production as opposed to the more peripheral resonances whose contributions are of the order of only a few percent. Further experimental information will eventually allow a more precise test of local duality in the ηN channel but at least qualitatively the proper conditions seem to be met.

Let us remark that a similar type of solution is of course not possible for $\pi\Delta$ scattering since at least some of the leading peripheral resonances are known to strongly couple to this channel.

For processes involving higher spin particles, one expects that the higher the spin flip, the more peripheral the pertinent resonance contributions will be, and, therefore, one can first try to enforce local duality for the maximum s channel flip amplitude assuming it to be large ⁸⁾. As seen in Section 2, this is certainly possible for πN and $\pi\Delta$ charge exchange (although not compelling in the $\pi\Delta$ case).

From a resonance point of view two effects have then to be taken into account to determine the location of a zero : for any given spin value J , the bigger the flip the further its zeros move out. At the same time, in order to increase the maximum flip we need to increase the external masses so as to reach the proper spin value ; as mentioned before, this will push the zero towards lower $|t|$ values and the two effects thus tend to compensate each other. One finds indeed, as illustrated in Table III, that for the pseudoscalar decuplet channel the compensation works satisfactorily at least for the peripheral medium energy resonances which are expected to saturate the FESR. It leads us to expect at least for the maximum s channel flip amplitude a zero structure compatible with a simple nonsense choosing Regge pole. From a simple diffractive point of view, however, this requires quite large variations in the impact parameters for the various reactions.

For vector meson production processes $\pi N \rightarrow \rho$ (or ω) N (or Δ) our analysis is blurred by the proliferation of helicity amplitudes on the one hand and, on the other hand, by the experimental evidence for important contributions from non-leading exchanges (such as π or B). This last fact is quite obvious from the density matrix elements : for $\pi^+ p \rightarrow \rho^0$ (or ω^0) Δ^{++} , for example, at 3.7 GeV/c, ρ_{00} which is built up only from unnatural parity exchanges in the t channel is at least as large as $\rho_{11} + \rho_{1-1}$ which isolates

natural parity exchanges (ρ or ω) in the crossed channel. It is satisfying, however, that this latter quantity behaves in a way consistent with a simple t channel picture : in $\pi^+ p \rightarrow \rho^0 \Delta^{++}$ where A_2 is the leading natural parity exchange, no zero is seen nor expected at $\alpha_{A_2}(t) = 0$ in the density matrix elements since they involve the real and imaginary parts of the various helicity amplitudes. In $\pi^+ p \rightarrow \omega \Delta^{++}$ it is the ρ trajectory which is exchanged and the turnover of the cross-section near the forward direction indicates that flip amplitudes do dominate the scattering and we therefore expect a significant structure in $\rho_{11} + \rho_{1-1}$ at the nonsense wrong signature point $\alpha_\rho(t) = 0$. Such a structure is indeed suggested by the data ⁹⁾. The presence in all vector meson production processes of very large contributions from unnatural parity exchanges prevents us, however, from making any reliable predictions about the presence or absence of a dip in the differential cross-sections.

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	$+i M_{\frac{3}{2} \frac{3}{2}}^{(s)}$	$i\sqrt{2} M_{\frac{3}{2} \frac{1}{2}}^{(s)}$	$+i\sqrt{2} M_{\frac{3}{2} \frac{1}{2}}^{(s)}$	$i M_{\frac{3}{2} \frac{3}{2}}^{(s)}$	$-i M_{\frac{1}{2} \frac{1}{2}}^{(s)}$	$-i M_{\frac{1}{2} \frac{1}{2}}^{(s)}$
$M_{\frac{3}{2} \frac{3}{2}}^{(t)}$	$-\frac{1}{4} \sin^3 \chi$	$\frac{\sqrt{3}}{2\sqrt{2}} \sin^2 \chi \cos \chi$	$-\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{1}{2} \sin^2 \chi)$	$\cos \chi (1 - \frac{1}{4} \sin^2 \chi)$	$\frac{3}{4} \sin^3 \chi$	$-\frac{3}{4} \sin^2 \chi \cos \chi$
$\sqrt{2} M_{\frac{3}{2} \frac{1}{2}}^{(t)}$	$\frac{\sqrt{3}}{2\sqrt{2}} \sin^2 \chi \cos \chi$	$-\sin \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\cos \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{1}{2} \sin^2 \chi)$	$-\frac{3\sqrt{3}}{2\sqrt{2}} \sin^2 \chi \cos \chi$	$\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{3}{2} \sin^2 \chi)$
$\sqrt{2} M_{\frac{3}{2} \frac{1}{2}}^{(t)}$	$-\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{1}{2} \sin^2 \chi)$	$\cos \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\sin \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\frac{\sqrt{3}}{2} \sin^2 \chi \cos \chi$	$\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\frac{3\sqrt{3}}{2\sqrt{2}} \sin^2 \chi \cos \chi$
$M_{\frac{3}{2} \frac{3}{2}}^{(t)}$	$\cos \chi (1 - \frac{1}{4} \sin^2 \chi)$	$\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{1}{2} \sin^2 \chi)$	$\frac{\sqrt{3}}{2} \sin^2 \chi \cos \chi$	$\frac{1}{4} \sin^3 \chi$	$\frac{3}{4} \sin^2 \chi \cos \chi$	$\frac{3}{4} \sin^3 \chi$
$M_{\frac{1}{2} \frac{1}{2}}^{(t)}$	$\frac{3}{4} \sin^3 \chi$	$-\frac{3\sqrt{3}}{2\sqrt{2}} \sin^2 \chi \cos \chi$	$\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\frac{3}{4} \sin^2 \chi \cos \chi$	$-\sin \chi (\frac{9}{4} \sin^2 \chi - 2)$	$-\cos \chi (1 - \frac{9}{4} \sin^2 \chi)$
$M_{\frac{1}{2} \frac{1}{2}}^{(t)}$	$-\frac{3}{4} \sin^2 \chi \cos \chi$	$\frac{\sqrt{3}}{2} \sin \chi (1 - \frac{3}{2} \sin^2 \chi)$	$\frac{3\sqrt{3}}{2\sqrt{2}} \sin^2 \chi \cos \chi$	$\frac{3}{4} \sin^3 \chi$	$-\cos \chi (1 - \frac{9}{4} \sin^2 \chi)$	$\sin \chi (\frac{9}{4} \sin^2 \chi - 2)$

TABLE I : The crossing matrix

	$i M_{\frac{3}{2} \frac{3}{2}}^{(s)}$	$i\sqrt{2} M_{\frac{3}{2} -\frac{1}{2}}^{(s)}$	$-i M_{\frac{1}{2} \frac{1}{2}}^{(s)}$
$M_{\frac{3}{2} \frac{3}{2}}^{(t)}$	$-\frac{1}{4}$	$-\frac{\sqrt{3}}{2\sqrt{2}}$	$\frac{3}{4}$
$\sqrt{2} M_{\frac{3}{2} -\frac{1}{2}}^{(t)}$	$-\frac{\sqrt{3}}{2\sqrt{2}}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2\sqrt{2}}$
$M_{\frac{1}{2} \frac{1}{2}}^{(t)}$	$\frac{3}{4}$	$-\frac{\sqrt{3}}{2\sqrt{2}}$	$-\frac{1}{4}$

	$i\sqrt{2} M_{\frac{3}{2} \frac{1}{2}}^{(s)}$	$i M_{\frac{3}{2} -\frac{3}{2}}^{(s)}$	$-i M_{\frac{1}{2} -\frac{1}{2}}^{(s)}$
$\sqrt{2} M_{\frac{3}{2} \frac{1}{2}}^{(t)}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2\sqrt{2}}$	$-\frac{\sqrt{3}}{2\sqrt{2}}$
$M_{\frac{3}{2} -\frac{3}{2}}^{(t)}$	$\frac{\sqrt{3}}{2\sqrt{2}}$	$\frac{1}{4}$	$\frac{3}{4}$
$M_{\frac{1}{2} -\frac{1}{2}}^{(t)}$	$\frac{-\sqrt{3}}{2\sqrt{2}}$	$\frac{3}{4}$	$\frac{1}{4}$

TABLE II : Crossing matrix at $t = 0$

J	M	$\pi N \rightarrow \pi N$ $\Delta h = 1$	$\pi N \rightarrow \pi \Delta$ $\Delta h = 2$	$\pi \Delta \rightarrow \pi \Delta$ $\Delta h = 3$
$\frac{3}{2}$	$N^*(1520)$	-0.55	-0.41	-
$\frac{5}{2}$	$N^*(1680)$	-0.45	-0.49	-0.42
$\frac{7}{2}$	$N^*(2000)$	-0.51	-0.69	-0.77
J	M	$\bar{K}N \rightarrow \bar{K}N$ $\Delta h = 1$	$\bar{K}N \rightarrow \bar{K}\Delta$ $\Delta h = 2$	$\bar{K}\Delta \rightarrow \bar{K}\Delta$ $\Delta h = 3$
$\frac{3}{2}$	$\Lambda(1700)$	-0.49	-	-
$\frac{5}{2}$	$\Lambda(1815)$	-0.41	-	-
$\frac{5}{2}$	$\Sigma(1910)$	-0.53	-0.57	-0.43
$\frac{7}{2}$	$\Lambda(2100)$	-0.47	-	-
$\frac{7}{2}$	$\Sigma(2030)$	-0.41	-0.52	-0.52

TABLE III : Location in t of the first non-trivial zeros of resonance contributions to the s channel amplitudes with maximal helicity flip.

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FIGURE CAPTIONS

- Figure_1 Mandelstam plot exhibiting the zeros of the s channel helicity amplitudes. Resonances with spins below $\frac{5}{2}$ were not included. For comparison, we show the zeros of the relevant Bessel functions, namely those that correspond to the asymptotic limit, Eq. (14). A $\frac{9}{2}$ resonance, at the location shown in the Figure, lies on the $N_{\frac{7}{2}}^{7+}$ trajectory and is exchange degenerate with the $\Delta_{\frac{7}{2}}^{7+} - \Delta_{\frac{7}{2}}^{11+}$ trajectory. It does therefore show a similar structure. n denotes the helicity flip.
- Figure_2 The zeros of the t channel helicity amplitudes are plotted in a plane of t vs. $\cos^2 \alpha$. The case studied here is the contribution of $\Delta_{\frac{7}{2}}^{7+}(1950)$ to $\pi\Delta$ scattering. The concentrations of zeros for $\cos \alpha \approx 1$ is the striking result of this exercise.
- Figure_3 Segments of plots similar to Fig. 2 (and using the notation defined there) are shown for several resonances. The $\frac{5}{2}$ ones have relatively too low spins, nevertheless the $\frac{5}{2}^-$ has characteristics similar to the $\frac{7}{2}^+$ of Fig. 2. The $\frac{7}{2}^-$ is a classic case of a resonance of a too high mass (or low spin), as can also be seen in Fig. 1, and is expected therefore to couple weakly to the $\pi\Delta$ channel.

- Figure 1 -

- Figure 2 -

- Figure 3 -