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ON MOMENTUM LOSS SPECTRA IN PROTON SCATTERING ON DEUTERIUM

R.J. Glauber

Harvard University, Cambridge, Mass.

O. Kofoed-Hansen ^{*)}

CERN -- Geneva

and

B.Margolis

McGill University, Montreal

ABSTRACT

The multiple scattering formalism is used in an investigation of the momentum loss spectra of protons scattered on deuterium at high energies. The results can be used to set limits on the strength of intermediate N^* states in high energy scattering of protons from nuclei as well as to provide information on neutron-proton scattering.

^{*)} On leave from the Danish AEC, Risø, Roskilde

INTRODUCTION

It has been observed by Allaby et al.¹⁾ that the momentum loss spectra for protons scattered on deuterium show interesting and potentially informative structure. Experimentally the momentum spectra of the scattered protons are found to show two distinct peaks lying above a background due to hadron production (see the Figure). The peaks are interpreted as the sum of the nuclear inelastic and elastic scattering involving single scattering (high momentum loss) and double scattering (low momentum loss), respectively.

The experimental data have already been compared¹⁾ with a simplified evaluation of the multiple scattering theory using the closure approximation, which neglects the energy difference between the excited states of the deuteron system. In this approximation the multiple scattering expression may be split, mathematically, into three terms each depending on the single nucleon scattering amplitudes and the deuteron wave function. Thus we write

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{(1)}}{d\Omega} + \frac{d\sigma^{(2)}}{d\Omega} + \frac{d\sigma^{(1,2)}}{d\Omega} \quad (1.a)$$

with

$$\frac{d\sigma^{(1)}}{d\Omega} = \frac{k^2}{(2\pi)^2} \int e^{i\vec{q} \cdot (\vec{b} - \vec{b}')} \langle i | \left[\Gamma_{pp}(\vec{b} - \frac{\vec{z}}{\sqrt{2}}) + \Gamma_{pn}(\vec{b} + \frac{\vec{z}}{\sqrt{2}}) \right] \cdot \left[\Gamma_{pp}^*(\vec{b}' - \frac{\vec{z}}{\sqrt{2}}) + \Gamma_{pn}^*(\vec{b}' + \frac{\vec{z}}{\sqrt{2}}) \right] | i \rangle d^2b d^2b' \quad (1.b)$$

$$\frac{d\sigma^{(2)}}{d\Omega} = \frac{k^2}{(2\pi)^2} \int e^{i\vec{q} \cdot (\vec{b} - \vec{b}')} \langle i | \Gamma_{pp}(\vec{b} - \frac{\vec{z}}{\sqrt{2}}) \Gamma_{pn}(\vec{b} + \frac{\vec{z}}{\sqrt{2}}) \cdot \Gamma_{pp}^*(\vec{b}' - \frac{\vec{z}}{\sqrt{2}}) \Gamma_{pn}^*(\vec{b}' + \frac{\vec{z}}{\sqrt{2}}) | i \rangle d^2b d^2b' \quad (1.c)$$

and

$$\begin{aligned} \frac{d\sigma^{(1,2)}}{d\Omega} = & -\frac{k^2}{(2\pi)^2} \int e^{i\vec{q}(\vec{b}-\vec{b}')} \langle i | [\Gamma_{pp}(\vec{b}-\frac{\vec{s}}{\sqrt{2}}) + \Gamma_{pn}(\vec{b}+\frac{\vec{s}}{\sqrt{2}})] \\ & \cdot \Gamma_{pp}^*(\vec{b}-\frac{\vec{s}}{\sqrt{2}}) \Gamma_{pn}^*(\vec{b}+\frac{\vec{s}}{\sqrt{2}}) | i \rangle d^2b d^2b' + c.c. \end{aligned} \quad (1.d)$$

where the profile function Γ is given by

$$\Gamma(\vec{b}) = \frac{1}{2\pi i k} \int e^{-i\vec{q}'\vec{b}} f(\vec{q}') d^2q' \quad (2)$$

In these expressions k is the momentum of the incoming particle, $\vec{q}$ is the momentum transfer, $\vec{b}$ is the impact parameter, $|i\rangle$ is the deuteron ground state, $\vec{s}$ is the projection of the relative coordinate $\vec{r} = (\vec{r}_n - \vec{r}_p)/\sqrt{2}$ onto the impact parameter plane, and $f(q)$ is the proton-nucleon amplitude with appropriate indices. The terms (1.b)-(1.d) represent respectively contributions from single scattering, from double scattering, and from their interference.

In the evaluation carried out previously, the following assumptions were made : 1) the p-p and p-n scattering amplitudes were assumed to have the same values as functions of q ; 2) the deuteron form factor was approximated by

$$S(q) = \exp(-q^2/4\gamma^2) \quad (3)$$

The single scattering term may easily be found without the latter assumption. The result is

$$\frac{d\sigma^{(1)}}{d\Omega} = |f_{pp}(q)|^2 + |f_{pn}(q)|^2 + (f_{pp}^*(q)f_{pn}(q) + c.c.) S(q)$$

which reduces to

$$\frac{d\sigma^{(1)}}{d\Omega} \approx 2|f(q)|^2(1 + S(q)) \quad (4.a)$$

when we do assume

$$f_{pp}(q) = f_{pn}(q) = f(q)$$

The evaluation of the integrals in Eqs. (1.c)-(1.d) was carried out in Ref. 1) by using the saddle point method of integration. As an illustration, let us consider Eq. (1.c) with Eq. (2) inserted. One may then easily carry out the b , b' and s integrations. The result contains two δ functions in the four q' integration variables. By carrying out the integration over these two δ functions, one finds

$$\begin{aligned} \frac{d\sigma^{(2)}}{d\Omega} = & \left(\frac{1}{2\pi k}\right)^2 \int S(\vec{q}' - \vec{q}'') f^*(\vec{q} - \vec{q}') f(\vec{q} - \vec{q}'') \\ & \cdot f^*(\vec{q}') f(\vec{q}'') d^2 q' d^2 q'' \end{aligned} \quad (5)$$

The individual factors in this integrand are rapidly varying functions of q' and q'' . This suggests the use of the saddle point method which consists of finding the maximum value of the integrand and expanding its logarithm to second order in deviations from the position in $\vec{q}'$ and $\vec{q}''$ yielding the maximum. In this approximation the integral is an integration over four of the six variables in a six-dimensional Gaussian. The result is

$$\frac{d\sigma^{(2)}}{d\Omega} = |f(\frac{q}{2})|^4 \frac{1}{2k^2 \beta} \frac{\gamma^2}{1 + \frac{\gamma^2}{2\beta \gamma^2}} \quad (4.b)$$

Similarly, one finds

$$\begin{aligned} \frac{d\sigma^{(1,2)}}{d\Omega} = & \left\{ \frac{1}{ik\beta} f(q) [f^*(\frac{q}{2})]^2 + c.c. \right\} \frac{4\beta \gamma^2}{1 + \frac{4\beta \gamma^2}{4\beta \gamma^2}} \\ & \cdot \exp \left[-\beta q^2 / (4 + 16\beta \gamma^2) \right] \end{aligned} \quad (4.c)$$

In these relations the quantity β is the logarithmic derivative of the nucleon cross-section evaluated at $q' = q/2$

$$\beta = \left(- \frac{d}{d(q'^2)} \ln |f(q')|^2 \right)_{q' = q/2} \quad (5)$$

The slope parameter for the deuteron form factor was taken ²⁾ to be $\chi^2 = 0.1922 \text{ f}^{-2}$. The p-p cross-sections measured ³⁾ at 19.2 GeV/c together with a constant value for $\text{Re } f(q)/\text{Im } f(q)$ ($= -0.33$) equal to that measured in the forward direction by Bellettini et al. ⁴⁾ were used to give the amplitude $f(q)$. With the above assumptions the cross-section is completely determined without any free parameters.

The experimental results ¹⁾ included summed cross-sections and an estimate of $d\sigma^{(2)}/d\Omega$ reached by estimating separately the area under the double scattering peak. In this preliminary comparison, the agreement of the experimental results with the theoretical expressions (4.a)-(4.c) was quite satisfactory. We therefore propose in the present paper to derive theoretical expressions for the detailed shapes of the momentum spectra in order to make further comparisons with the experimental results.

BASIS FOR MOMENTUM SPECTRA COMPUTATIONS

We note that the main features of the above considerations were :

- 1) the use of experimental data for $|f(q)|$; 2) summation over a complete set of final states with their excitation energy neglected, so that closure may be used; and 3) the use of a Gaussian form factor for the deuteron. We now wish to include the excitation energy of the final deuteron states in our calculations. At the same time, we shall investigate the accuracy of the computation and the sensitivity of the results to particular physical assumptions.

We proceed to write down the different contributions to the momentum loss spectra. The contribution from nuclear elastic and inelastic scattering is in the multiple scattering formalism given by ^{5),6)}:

$$\begin{aligned} \left(\frac{d^2 \sigma}{d\Omega d\Delta} \right)_1 &= \sum_f \left| \frac{ik}{2\pi} \right| e^{i\vec{q} \cdot \vec{b}} \langle f | \Gamma_{pp}(\vec{b} - \frac{\vec{z}}{\sqrt{2}}) + \Gamma_{pn}(\vec{b} + \frac{\vec{z}}{\sqrt{2}}) \\ &\quad - \Gamma_{pp}(\vec{b} - \frac{\vec{z}}{\sqrt{2}}) \Gamma_{pn}(\vec{b} + \frac{\vec{z}}{\sqrt{2}}) | i \rangle d^2 b \|^2 \delta(E_f - E_i) \end{aligned} \quad (7)$$

with Δ the numerical momentum loss $|k| - |k'|$, and where E_f and E_i are the final and initial state energies of the deuteron plus proton. Apart from that the notation is as above and we may again use the saddle point method of integration equivalent to evaluation of $f(q)$ as a Gaussian with locally determined parameters. Thus in the following expressions we shall neglect second order contributions in the following four magnitudes :

$$\epsilon_1 = \ln(|f_{pn}|/|f_{pp}|)_{q'=q/2} \quad (8)$$

$$\epsilon_2 = \ln(\beta_n/\beta_p)_{q'=q/2} \quad (9)$$

$$\epsilon_3 = \left[\left\{ \frac{\partial}{\partial q} \ln \left(\frac{\beta_n + \beta_p}{2} \right) \right\} \sqrt{1/(\beta_n + \beta_p)} \right]_{q'=q/2} \quad (10)$$

where the square root is the e folding distance in q' around $q' = q/2$ involved in the saddle point approximation for the integrals, and

$$\epsilon_4 = \left[\left\{ \frac{\partial}{\partial q} \delta \right\} \sqrt{1/(\beta_n + \beta_p)} \right]_{q'=q/2} \quad (11)$$

with

$$\delta = \alpha_n(q) - \alpha_p(q)$$

where α is the ratio $\text{Re } f / \text{Im } f$. For certain purposes we shall also neglect second order terms in $\delta(q/2)$. However, since usually

$$q > q/2 + [\sqrt{1/(\beta_n + \beta_p)}]_{q'=q/2}$$

we shall consider $\alpha(q) \neq \alpha(q/2)$ in part of the investigations. In this approximation, Eq. (7) reduces to

$$\begin{aligned} \left(\frac{d^2\sigma}{d\Omega d\Delta} \right)_1 &= \sum_f |f_{pp}(q) \langle f | e^{i\vec{q} \cdot \frac{\vec{r}}{2}} | i \rangle + f_{pn}(q) \langle f | e^{-i\vec{q} \cdot \frac{\vec{r}}{2}} | i \rangle \\ &- \frac{1}{ik(\beta_n + \beta_p)} f_{pp}\left(\frac{q}{2}\right) f_{pn}\left(\frac{q}{2}\right) \langle f | e^{-s^2/(\beta_p + \beta_n)} | i \rangle|^2 \end{aligned} \quad (12)$$

$$\cdot \delta(E_f - E_i)$$

where β_n and β_p are the logarithmic derivatives of the np and pp cross-sections [cf., Eq. (5)], and where the $f(q)$'s are the elastic nucleon-nucleon amplitudes.

In addition to the processes which we have considered thus far (i.e., those conserving number and nature of the particles present) there will be hadronic production processes contributing to the background. The differential cross-section for each type of production process (such as N^* production) will contain three terms. Because of the difference between the nucleon-nucleon production amplitude $f^P(q)$ and $f(q)$ second order terms in the corresponding magnitudes ϵ_i of Eqs. (8)-(11) are somewhat larger than for the hadronic elastic amplitudes, however the contribution to "background" from double processes is small and we shall use the same approximation as above. Thus we may write for the production cross-section

$$\begin{aligned} \left(\frac{d^2\sigma}{d\Omega d\Delta} \right)_2 &= \left\{ \sum_f |f_{pp}^{(p)}(q) \langle f | e^{i\vec{q} \cdot \frac{\vec{r}}{2}} | i \rangle \right. \\ &- \frac{1}{ik(\beta_n + \beta_p^{(p)})} f_{pp}^{(p)}\left(\frac{q}{2}\right) f_{pn}^{(p)}\left(\frac{q}{2}\right) \langle f | e^{-s^2/(\beta_p^{(p)} + \beta_n)} | i \rangle|^2 \end{aligned}$$

$$\begin{aligned}
 & + \sum_f |f_{pn}^{(p)}(q) \langle f | e^{-i\vec{q} \cdot \vec{r}} | i \rangle \\
 & - \frac{1}{ik(\beta_p + \beta_n^{(p)})} f_{pp}(\frac{q}{2}) f_{pn}^{(p)}(\frac{q}{2}) \langle f | e^{-s^2/(\beta_p + \beta_n^{(p)})} | i \rangle|^2 \\
 & + \sum_f | - \frac{1}{ik(\beta_p^{(p)} + \beta_n^{(p)})} f_{pp}^{(p)}(\frac{q}{2}) f_{pn}^{(p)}(\frac{q}{2}) \langle f | e^{-s^2/(\beta_p^{(p)} + \beta_n^{(p)})} | i \rangle|^2 \} \quad (13) \\
 & \cdot \delta(E_f - E_i)
 \end{aligned}$$

where index (p) indicates a given production process.

In writing down Eqs. (12) and (13) we have neglected the contributions from, e.g., N^* creation and reversion to a proton in the interval between two collision processes in double scattering. We shall return to this point later.

SPECTRUM CALCULATIONS SIMPLIFIED

The influence of the choice of the complete set of final states on the results has been investigated through the use of two essentially different sets : 1) plane waves, and 2) harmonic oscillator wave functions. Consider for the time being constant q scans. Then as we shall see, since the closure approximation is quite accurate, the areas $A^{(1)}$, $A^{(2)}$ and $A^{(1,2)}$ of the three distributions $d^2\sigma^{(1)}/d\Omega d\Delta$, $d^2\sigma^{(2)}/d\Omega d\Delta$ and $d^2\sigma^{(1,2)}/d\Omega d\Delta$ corresponding to single scattering, double scattering and interference are independent of the choice of final state wave functions. As a particularly simple case, let us consider the set of harmonic oscillator states of which Eq. (3) gives the ground state form factor. Neglecting all differences between $f_{pp}(q)$ and $f_{pn}(q)$ and using the non-relativistic approximation to energy momentum relations, we find in this case

$$\frac{d\sigma^{(1)}}{d\Omega d\Delta} = 4|f(q)|^2 \frac{1}{N!} \left(\frac{q^2}{8\gamma^2}\right)^N e^{-q^2/8\gamma^2} \quad (14)$$

where N runs through all even principal quantum numbers of the harmonic oscillator. The energy interval between the discrete spectrum lines caused by our particular choice of final states is small compared to the experimental energy resolution for the outgoing protons. Its value (~ 30 MeV) is consequently of little importance.

For the large q^2 values of interest to us the spectrum in N given by Eq. (14) may be approximated by a continuous spectrum with the average value $\bar{N} = q^2/8\gamma^2$, the mean square deviation $\sigma_N^2 = q^2/8\gamma^2$, and the third moment around the mean $\mu_3 = (N-\bar{N})^3 = q^2/8\gamma^2$, etc., since the difference between Eq. (14) and a Poisson distribution is small for large q . If we want to transform these results to a distribution in Δ then we find in a rough first approximation $\bar{\Delta} \approx q^2/2M_D + N \Delta W$ with ΔW the energy step between successive harmonic oscillator levels $\Delta W = 2\gamma^2/M$ with M the nucleon mass. Thus, with momentum loss as our variable, we find $\bar{\Delta} \approx q^2/2M$, $\sigma_{\Delta}^2 = (q^2/2M)(\gamma^2/M)$ and $(\Delta - \bar{\Delta})^3 = 2(q^2/2M)(\gamma^2/M)^2$.

In the same approximation we find for the double scattering contribution

$$\frac{d\sigma^{(2)}}{d\Omega d\Delta} = \left(\frac{1}{2k\mu}\right)^2 |f(\frac{q}{2})|^4 \left(\frac{4\mu^2}{1+4\mu^2\gamma^2}\right)^2 \left(\frac{1}{1+4\mu^2\gamma^2}\right)^N \quad (15)$$

where N again runs through all even principal quantum numbers of the harmonic oscillator. Therefore for all values of q we may approximate this geometrical series by a one-sided exponential [i.e., $e^{-\alpha(\Delta - \Delta_0)}$ for $\Delta > \Delta_0$ and 0 for $\Delta < \Delta_0$] having its maximum value at $\Delta_0 = q^2/4M$, i.e., the elastic scattering momentum loss. Such a distribution is completely characterized by the area of the peak and the coefficient α in the exponential $\exp(-\alpha\Delta)$. Expanding α into a

power series in βy^2 one finds $\alpha = 2\beta M(1 - 2\beta y^2 + (16/3)(\beta y^2)^2 - \dots)$. The central moments of such a distribution are easily calculated but since there is no reason to approximate the distribution further we shall omit such expressions. We note, however, that the $N=0$ contribution gives a measure of the elastic scattering on the deuteron as a whole. It then follows from Eq. (15) that elastic scattering constitutes the fraction $4\beta y^2(2 + 4\beta y^2)/(1 + 4\beta y^2)^2$ of the area $d\sigma^{(2)}/d\Omega$. For the values of β of interest to us this is roughly speaking 30%. This estimate is crude because of the crudeness of the wave function for the deuterium ground state.

Finally, we find the momentum loss spectrum associated with the interference term

$$\frac{d\sigma^{(1,2)}}{d\Omega d\Omega} = \left\{ \frac{1}{ik\beta} f(q) \left[f\left(\frac{q}{2}\right) \right]^2 + c.c. \right\} \frac{4\beta y^2}{1 + 4\beta y^2} e^{-\frac{q^2}{4 + 16\beta y^2}} \cdot \frac{1}{\left(\frac{N}{2}\right)!} \left(\frac{q^2}{8\beta y^2} \frac{1}{2 + 8\beta y^2} \right)^{\frac{N}{2}} e^{-\frac{q^2}{8\beta y^2} \frac{1}{2 + 8\beta y^2}} \quad (16)$$

where N runs through all even principal quantum numbers of the harmonic oscillator. The three last factors in Eq. (16) constitute a properly normalized Poisson distribution. The moments of N are for all values of q

$$\overline{N/2} = \sigma_{N/2}^2 = \mu_{3,N/2} = \left(\frac{q^2}{8\beta y^2} \frac{1}{2 + 8\beta y^2} \right)$$

Transforming to momentum loss, one has correspondingly $\overline{\Delta} = (q^2/4M) \times (1 + 1/(1 + 4\beta y^2))$, $\sigma_{\Delta}^2 = q^2 y^2 (M^2(1 + 4\beta y^2))$, and $\mu_{3,\Delta} = 8q^2 y^4 / (M^3(1 + 4\beta y^2))$. Allowing $\alpha(q)$ to be different from $\alpha(q/2)$ we may rewrite the factor R in brackets $\{ \}$ to be

$$R = - \frac{2}{k\beta} \left[\sqrt{\frac{d\sigma}{d\Omega_H}} \right]_{q'=q} \left[\frac{d\sigma}{d\Omega_H} \right]_{q'=q/2} \cdot \frac{1 + \alpha^2(q) - (\alpha(q) - \alpha(q/2))^2}{\sqrt{1 + \alpha^2(q)} (1 + \alpha^2(q/2))} \quad (17)$$

which shows that the interference term is destructive for constant α but may become constructive in exceptional cases.

It is evident from Eqs. (14)-(16) that the resulting spectra are very similar to those observed experimentally. The displacements of the two peaks are close to those observed and for not too different values of $\alpha(q)$ and $\alpha(q/2)$ the destructive interference aids in separating the peaks.

VARIATION OF FINAL STATE WAVE FUNCTIONS

Although the use of the particular set of harmonic oscillator wave functions we have chosen as final states seems natural, it is worth verifying that the cross-sections predicted do not depend materially on that choice. For this reason we have repeated the preceding calculations using plane waves as our complete set of final states. Let us first recall that the three areas $d\sigma^{(1)}/d\Omega$, $d\sigma^{(2)}/d\Omega$, and $d\sigma^{(1,2)}/d\Omega$ are independent of our assumption of any particular set of final states. Considering area, mean momentum, mean square deviation, and third central moment as 0th, 1st, 2nd and 3rd order magnitudes and expanding in powers of $\beta\gamma^2 \simeq 0.075$ we see from the results in Table 1 that deviations first occur in third order terms only. Thus possible effects of final state interactions are at most of the order of a few per cent, where the spectra are intense. For single scattering and interference we compare the moments of the distributions, for double scattering we compare the coefficient in the exponential. The results for single scattering and interference are given in the $q^2/\gamma^2 \gg 1$ limit of interest to us. In the plane wave case the double scattering term deviates slightly from a one-sided exponential. The sharp edge is rounded and smeared out over a few MeV. Considering the experimental resolution, this can be completely forgotten.

The foregoing analytical expressions are useful as guides, but in order to carry out a more detailed comparison with experiment we must use more exact expressions for the kinematics than the non-relativistic limit, $E = p^2/2M$, used thus far. We must be prepared to deal numerically

with the observed p-p amplitudes, and finally we must also investigate the consequences of the use of more accurate deuteron wave functions by including in particular the 6% d state admixture in the ground state. For these reasons we have used numerical methods in order to proceed further.

We note from Eqs. (12) and (13) that we shall be dealing with matrix elements of spin independent magnitudes. Consider now inelastic scattering and sum over final state spin orientations. The result is then equal to an initial state matrix element of a spin independent magnitude. It is easy to verify that the average over initial spins of such a matrix element does not contain terms corresponding to interference between s and d states. For elastic scattering, we must average over the square of an initial state matrix element. In this case interference terms are important. Thus we should only expect relatively small changes due to d state admixture because we are dealing with dominantly inelastic scattering.

KINEMATICS

The experimentally observed spectra ¹⁾ represent constant angle, θ , scans. An appropriate approximation to the kinematics is obtained as follows. Let $\Delta = |k| - |k'|$ be the difference between the numerical values of incoming and outgoing proton momenta. We then expand in terms of Δ/k and M/k with M the proton mass. Then

$$E_p - E_{p'} \approx \left(1 - \frac{1}{2} \frac{M^2}{k^2}\right) \Delta + \dots \equiv a\Delta + \dots \quad (18)$$

with E_p and $E_{p'}$ the incoming and outgoing proton energies. For the deuteron energy difference we have

$$E_D - E_{D'} = M_D - \sqrt{M_D^2 + Q^2} \quad (19)$$

where $M_D \simeq 2 \cdot M$ and Q is the three-dimensional momentum transfer to the deuteron system of invariant mass M_D^* . M_D^* is given by

$$M_D^* = 2\sqrt{M^2 + p^2/2} \quad (20)$$

with p given directly in the plane wave case. For the case of the harmonic oscillator states, on the other hand, we have $p^2 = 2MN \Delta W$ for the final state with principal quantum number N if ΔW is the energy separation between successive states. Since Q in Eq. (19) is determined from k , Δ and θ through momentum conservation, we may use Eqs. (18)-(20) and energy conservation to find Δ for given k , θ and M_D^* . A sufficiently accurate expansion is

$$\begin{aligned} \Delta \simeq & \frac{M_D^{*2} - M_D^2 + 2k^2(1 - \cos\theta)}{[2aM_D + 2k(1 - \cos\theta)]} \\ & + \frac{(1 - a^2)(M_D^{*2} - M_D^2 + 2k^2(1 - \cos\theta))^2}{[2aM_D + 2k(1 - \cos\theta)]^3} + \dots \end{aligned} \quad (21)$$

Having thus obtained Δ from k , θ and M_D^* we find either

$$Q_{\perp} = (k - \Delta) \sin\theta \quad (22.a)$$

or

$$-t = Q^2 - (E_p - E_p')^2 \simeq Q^2 - a^2 \Delta^2 \quad (22.b)$$

which permit us to interpolate, using numerical tables of $f(Q_{\perp})$ or $f(\sqrt{-t})$. We shall use both procedures since a possible difference in results through the interpretation of q in Eqs. (12) and (13) as Q or $\sqrt{-t}$ indicates a measure of the precision of the multiple scattering approximation.

THE pp INPUT DATA

The elastic pp data were taken from Ref. 3) and the inelastic pp data were taken from Ref. 7). Since the spectrum associated with elastic scattering is a δ function in Δ no further approximation is needed. For the inelastic data (needed only for an estimate of the "background"), we make the approximation

$$\left(\frac{d^2\sigma}{d\Omega d\Delta}\right)_{pp,inel} = A(q) [p'_{el} - p'] = A(q) \cdot \Delta' \quad (23)$$

for $p' < p'_{el}$, i.e., with p'_{el} the position of the elastic peak. $A(q)$ was determined as an average over the spectrum in the $0 < \Delta' < 1$ GeV/c interval. The simplifying assumption Eq. (23) yields an $A^2 \Delta'^3/6$ spectrum for double inelastic scattering. The experimental pp data were fitted by hand-drawn curves and tabulated from these curves. They were then numerically differentiated by the use of seven-point numerical differentiation to yield β . Following this, the data were taken through double four-point mid-value interpolation (out to mid-point and back to the original point), until repetition gave changes in β well below 0.5 %. This procedure acts as a filter for value stagger around a smooth curve. The maximum change in β resulting from this smoothing procedure was $\sim 1\%$ for β and $\sim 5\%$ for $\beta^{(p)}$ over the interval in q of interest to us $0.30 \lesssim q \lesssim 0.70$ GeV/c. Near the knee in the elastic pp cross-section the changes were much larger. Here also the uncertainty in the data is more significant but fortunately we only need values of $f(q)$ for $q > 0.70$ GeV/c, not of β . The tables of $f(q)$ and $\beta(q)$ were obtained from the finally smoothed set of data.

RESULTS

It is of course not possible to split the experimental data so as to obtain a simple measure of the interference "peak". Instead we can talk about the area, position, width, etc., of the single and double

scattering peaks (each including their share of the interference) and of the height of the minimum between the peaks. One main result of the work of Allaby et al.¹⁾ is the areas d_1 and d_2 of the single and double scattering peaks. Table 2 compares the experimental and theoretical numbers. Statistical analysis shows that no significance can be attributed to possible differences between theory and experiment when in addition to the statistical errors included in Table 2, one takes into account the possible 6% systematic errors in the cross-section values.

The computed spectra are compared with the experimental data in the Fig. The experimental resolution has been folded into the theoretical spectra. The results presented both in Table 2 and in the Fig. are based on 6% d state and $\alpha(q) = -0.33$. No significant effect of the choice $\alpha(q) = -0.22$ will result (see below). The agreement between the theoretical and experimental spectra is also quite good in view of the uncertainties in the data and considering the approximations made. In addition to the errors in cross-sections there is also uncertainty in the absolute momentum determination and jitter in this determination during experimental runs and systematic variations from run to run of ~ 0.03 GeV/c.

INVESTIGATION OF SPECTRUM SENSITIVITY TO PARAMETERS

We have investigated, with the computational tools at hand, the influence of various physical assumptions on the areas d_1 and d_2 , the peak positions Δ_1 and Δ_2 the widths of the peaks σ_1 and σ_2 and the height of the minimum m between the peaks. We find the following results.

The validity of the multiple scattering approximation to within a few per cent is indicated by comparison of the $\sqrt{-t}$ vs. $Q_{\perp}$ approach. For details, see Table 3. For $\theta_L = 70$ mrad the value of m is seen to be somewhat sensitive to this point (18%).

The sensitivity to a possible difference between $|f_{pp}|$ and $|f_{pn}|$ is maximal in d_2 because d_2 depends directly on $|f_{pn}|^2$.

However, since in this case we are dealing with the $q' = q/2$ value, a possible difference must show up in d_1 long before it appears in d_2 as one goes to larger and larger scattering angles. Similarly, possible differences between the production amplitudes $|f_{pp}^{(p)}|$ and $|f_{pn}^{(p)}|$ will show up in the production background as a consequence of single collisions. There is a trend both in Table 2 and the Fig. indicating that towards $\theta_L > 55$ mrad both $|f_{pn}|$ and $|f_{pn}^{(p)}|$ become smaller than the corresponding pp amplitudes by as much as 20% for $\theta_L = 65$ mrad.

By varying $\alpha(q)$ and $\alpha(q/2)$ independently [see Eq. (17)], one finds effects which might result from phase variations. One also finds that the double scattering peak is quite insensitive to the interference term. This result is important because it means that the sensitivity of the interference term to different physical assumptions will manifest itself in the single scattering peak and in the height m of the minimum, but not in the double scattering peak. In other words, this result permits us to connect the mathematical division of the cross-section in three terms to the physical division in two peaks in a very simple way. As regards variations of $\alpha(q)$ the conclusion is that if $\text{Im}f(q)$ goes through a zero then there will be a region of scattering angle θ_L where $\alpha(q)$ and $\alpha(q/2)$ become sufficiently different so that the single scattering peak might become slightly broader but up to 40% higher and m up to 100% higher than the result for small constant α values. The deviation between the theoretical and experimental spectra at 50 mrad and the smaller difference at 45 and 55 mrad can be explained in this way and would, if these results are confirmed in future experiments, indicate a zero in $\text{Im}f(q)$ at a q value slightly before the appearance of the knee in the pp cross-section.

The sensitivity to the deuterium ground state wave function was investigated by varying γ^2 and by examining the analytical expressions derived above. It follows from this and from the fact that double scattering depends on shorter distance wave function details than single scattering that possible effects should manifest themselves first in single scattering but there only in the width and higher moments of the distribution. The effect of the 6% d state was examined separately. All these sensitivity investigations are summarized in Table 3.

As a final result, we note that the double scattering area d_2 shows very little sensitivity to the above assumptions. However, double scattering will be sensitive to the regenerative amplitude resulting from first producing nucleon isobars on one nucleon and then regenerating the proton on the other nucleon. It is found, from Table 2, that the average deviation between theoretical and experimental values for d_2 is $d_{2,\text{exp}} - d_{2,\text{th}} = 4 \pm 8\%$ using the statistical errors only. However, possible systematic errors and the small theoretical uncertainties listed in Table 3 suggest that a safer estimate is $4 \pm 20\%$ for the possible contributions from regeneration. Undoubtedly this number can be improved when the recent data obtained by Allaby et al.⁸⁾ at 24 GeV/c become available for analysis.

CONCLUSIONS

We first conclude that the results of high energy proton scattering on deuterium can be quite well understood through the theory of multiple scattering. The theoretical results show at most small deviations from experiment. Given the available data, there is no conclusive evidence for any sizeable difference between the p-p and p-n scattering amplitudes nor is there compelling evidence for a variation of $\text{Re} f / \text{Im} f$ with q . However, there are indications both of the p-n elastic and inelastic amplitudes becoming smaller than the corresponding p-p amplitudes at $q^2 > 1.1 (\text{GeV}/c)^2$ and there is some indication of a zero in $\text{Im} f$ at $q^2 \simeq 0.9 (\text{GeV}/c)^2$. Furthermore we may conclude that at the energies and momentum transfers under consideration scattered protons resulting from double scattering involving intermediate N^* etc., production is limited to $4 \pm 20\%$ of the intensity of double scattering events.

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Table 1

Characteristics of distributions.
Index HO indicates harmonic oscillator results.
Index PW indicates plane wave results.

	$\bar{\Delta}$	σ_{Δ}^2	$\mu_{3,\Delta}$
$\frac{d^2 \sigma^{(1)}}{d\Omega d\Delta} \frac{\overline{\Delta}}{HO}$	$\frac{q^2}{2M}$	$\frac{q^2}{2M} \frac{Y^2}{M}$	$2 \frac{q^2}{2M} \left(\frac{Y^2}{M} \right)^2$
$\frac{d^2 \sigma^{(1)}}{d\Omega d\Delta} \frac{M_{PW}}{PW}$	$\frac{q^2}{2M}$	$\frac{q^2}{2M} \frac{Y^2}{M}$	$3 \frac{q^2}{2M} \left(\frac{Y^2}{M} \right)^2$
$\frac{d^2 \sigma^{(1,2)}}{d\Omega d\Delta} \frac{\overline{\Delta}}{HO}$	$\frac{q^2}{4M} \left(1 + \frac{1}{1 + 4\beta Y^2} \right)$	$\frac{q^2 Y^2}{M^2} \frac{1}{1 + 4\beta Y^2}$	$8 \frac{q^2 Y^4}{M^2} \frac{1}{1 + 4\beta Y^2}$
$\frac{d^2 \sigma^{(1,2)}}{d\Omega d\Delta} \frac{M_{PW}}{PW}$	$\frac{q^2}{4M} \left(1 + \frac{1}{1 + 4\beta Y^2} \right) [1 + (\beta Y^2)^2 - \dots]$	$\frac{q^2 Y^2}{M^2} \frac{1}{1 + 4\beta Y^2} [1 - 2\beta Y^2 + \dots]$	$8 \frac{q^2 Y^4}{M^2} \frac{1}{1 + 4\beta Y^2} \left[\frac{3}{4} - \dots \right]$
Coefficient in exponential (α)			
$\frac{d^2 \sigma^{(2)}}{d\Omega d\Delta} \frac{\overline{\Delta}}{HO}$	$2M\beta \left[1 - 2\beta Y^2 + \frac{16}{3} (\beta Y^2)^2 - \dots \right]$		
$\frac{d^2 \sigma^{(2)}}{d\Omega d\Delta} \frac{M_{PW}}{PW}$	$2M\beta \left[1 - 2\beta Y^2 + \frac{12}{3} (\beta Y^2)^2 - \dots \right]$		

Table 2

Proton-deuteron scattering cross-section (lab. system) at 19.2 GeV/c

Θ_L mrad	$(d\sigma/d\Omega) \text{ cm}^2/\text{sr}$						
	d_{1+2}		d_1		d_2		*)
	Exp.	Theor.	Exp.	Theor.	Exp.	Theor.	
40	1.31 ± 0.06	1.12		1.04		0.08	E-25
45	3.77 ± 0.24	3.53	3.27 ± 0.25	2.98	0.50 ± 0.13	0.55	E-26
50	1.18 ± 0.08	1.06	0.81 ± 0.10	0.75	0.36 ± 0.09	0.31	E-26
55	3.63 ± 0.48	3.60	1.80 ± 0.33	2.05	1.83 ± 0.27	1.56	E-27
60	1.57 ± 1.26	1.63	0.80 ± 0.15	0.94	0.76 ± 0.10	0.69	E-27
65	8.40 ± 1.34	9.88	5.07 ± 0.87	7.03	3.32 ± 0.49	2.85	E-28
70			4.40 ± 0.76	5.52			E-28

*) The expression E-25 denotes a multiplicative factor 10^{-25} , etc.; d_{1+2} is the total scattering cross-section, d_1 and d_2 being the cross-sections associated with lower and upper peaks (see the Figure), respectively. Experimental data from Allaby et al.¹⁾ under headings Exp., Theoretical data under headings Theor.

Table 3

Variation of $d^2\sigma/d\Omega d\Delta$ characteristics with input assumptions
for $45 \text{ mrad} \leq \theta \leq 65 \text{ mrad}$ at 19.2 GeV/c

Item		d_{1+2}	d_1	Δ_1	σ_1	m	d_2	Δ_2	σ_2
		pct.	pct.	GeV/c	pct.	pct.	pct.	GeV/c	pct.
$\sqrt{-t}$ vs. q_1	Max	1	4	0.02	2	-18	-8	0.00	< 1
	Av	< 1	2	0.01	2	-10	-4	0.00	< 1
plane wave vs. harm. osc.	Max	0	0	0.02	3		0	0.00	< 1
	Av	0	0	0.01	2		0	0.00	< 1
-10 pct. on γ^2	Max	-3	2	0.00	-4	-10	-10	-0.01	-4
	Av	-2	2	0.00	-4	-9	-9	0.00	-4
6 pct. D-state	Max	1	2	0.00	13	26	-3	0.00	< 1
	Av	0	1	0.00	8	15	-2	0.00	< 1
$ f_{pp} ^2$ 10 pct. diff from $ f_{pn} ^2$	Max	7	5	0.00	0		10	0.00	0
	Av	6	5	0.00	0		10	0.00	0
$\alpha(q/2) = +5.0$ $\alpha(q) = -0.5$	Max	48	57	-0.02	15	107	4	0.00	-14
	Av	32	46	-0.01	12	99	3	0.00	-2

The standard case against which this is normalized is the $\alpha = 0$, q_1 , harmonic oscillator, zero D-state, $\gamma^2 = 0.1922 \text{ f}^{-2}$ and $f_{pp} = f_{pn}$ case.

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FIGURE CAPTION

Momentum spectra of 19.2 GeV/c protons scattered on deuterium through angles between 40 and 65 mrad. Experimental data from Ref. 1), approximate magnitude of statistical error inserted in circle. Curves are the present theoretical results, with experimental resolution folded in.

p' (GeV/c) $\rightarrow$