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SIMPLE ASSUMPTIONS FOR 90° SCATTERING AT HIGH ENERGIES

A.S. Goldhaber *)

CERN - Geneva

A B S T R A C T

It is assumed that at very high energies the spin and isospin dependence of 90° scattering must satisfy some simple hypothesis. Two diffractive hypotheses and one statistical hypothesis are considered, and consequences are deduced for the ratio of the elastic cross-section at 90° for identical particles to that for different particles in the same isospin multiplet. The dependence of this ratio on the intrinsic spin s of either scattered particle is deduced. In classical physics (Boltzmann statistics) this ratio would be $2/1$. All three hypotheses yield the classical value for large s (distinguishable particle limit). The results obtained are compared with data on np and pp scattering, and further tests are suggested using nucleus-nucleus scattering.

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*) National Science Foundation Senior Postdoctoral Fellow.

Permanent address: Institute for Theoretical Physics
State University of New York, Stony Brook, N.Y.
11790, U.S.A.

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1. - SOME SIMPLE FORMULAS

It is natural to speculate that at high energies any particular hadronic process will assume some asymptotic form. The questions which arise from this line of thought are :

- 1) what is such an asymptotically high energy ?
- 2) what is the asymptotic form ?

Let us consider the second question for the case of 90° elastic scattering of two particles which are members of the same isospin multiplet. One possible assumption may be called the "diffractive hypothesis", that internal quantum numbers are unchanged by the scattering, which is maximally independent of these quantum numbers. One diffractive amplitude for any two different members of the same isomultiplet is the spin-independent form

$$F_{ab}(\theta) = f(\theta) \mathbb{1} \quad (1)$$

where f at fixed energy is a function only of centre-of-mass (cm) scattering angle θ , and $\mathbb{1}$ is a unit matrix in the intrinsic spin of the two particles. The corresponding amplitude for two identical particles is

$$F_{a'a^2}(\theta) = [f(\theta) + f(\pi - \theta)] P_E + [f(\theta) - f(\pi - \theta)] P_O \quad (2)$$

where $P_E(P_O)$ is the projection operator for even (odd) total spin S , with $\underline{S} = \underline{s}_1 + \underline{s}_2$. This rule is an immediate consequence of isospin invariance and Bose statistics for integer spin particles, but it also applies for half-integer spin particles obeying Fermi statistics. If one takes as exact the assumption of Wu and Yang¹⁾ that, in a range R near 90° , charge exchange is indistinguishable from direct scattering, then $f(\theta)$ is an even function of $\cos\theta$ in R . In this case simple spin sums yield a result for all angles in the range R ,

$$\frac{d\sigma}{d\Omega_{aa}} / \frac{d\sigma}{d\Omega_{ab}} = 2 [1 \pm (2s+1)^{-1}] \quad (3)$$

where $\underline{s}$ is the intrinsic spin of either scattered particle [more explicitly, $\underline{s}_a^2 = \underline{s}_b^2 = s(s+1)\mathbb{I}$, and the $+(-)$ sign holds for integer (half-integer) $\underline{s}$. If Eqs. (1) and (2) hold without the symmetry assumption, then Eq. (3) is still true for $\theta = 90^\circ$.

One can make the amplitude still more symmetrical, while keeping $d\sigma/d\Omega$ independent of initial spin configuration, by changing Eq. (1) to

$$F_{ab}(\theta) = f(\theta) [1 + iE_s] \quad (4)$$

where E_s interchanges the spins of particles 1 and 2. This symmetrization does not affect the result of Eq. (3), but it has the effect of leaving the final spin of $\underline{a}$ equally correlated with the initial spins of $\underline{a}$ and of $\underline{b}$. It must be admitted that this symmetrization is artificial.

A second form of diffractive amplitude is helicity-independent instead of spin-independent. In this case the amplitude in spin space may be written (for $a \neq b$)

$$F_{ab}(\theta) = f(\theta) e^{i \underline{S} \cdot \hat{n} \theta} \quad (5)$$

where $\hat{n} = (\underline{p} \times \underline{p}') / |\underline{p} \times \underline{p}'|$ is the normal to the scattering plane and $\underline{S}$ is the total spin vector. Now the identical particle amplitude must be

$$F_{aa}(\theta) = [f(\theta) e^{i \underline{S} \cdot \hat{n} \theta} + f(\pi-\theta) e^{i \underline{S} \cdot \hat{n}(\theta-\pi)}] \times P_E + [f(\theta) e^{i \underline{S} \cdot \hat{n} \theta} - f(\pi-\theta) e^{i \underline{S} \cdot \hat{n}(\theta-\pi)}] P_O \quad (6)$$

Here, we use the fact that $\underline{S}$ commutes with P_E and P_0 . For this case, straightforward counting gives

$$\frac{d\sigma}{d\Omega_{aa}} / \frac{d\sigma}{d\Omega_{ab}} = 2 [1 + (2s+1)^{-1}] \quad (7)$$

The second term now has a plus sign both for bosons and fermions. Again the result is valid in the range R if $f(\theta)$ is symmetric in $\cos\theta$ for θ in R .

A second class of assumption may be called "statistical" ²⁾. In this approach each independent partial wave amplitude with distinct isospin, angular momentum, and parity, is written as the sum of a diffractive term and a small statistical amplitude. Near 90° the diffractive amplitudes interfere destructively to give a negligible contribution. The statistical amplitudes vary so rapidly with energy that interference terms wash out and only the sum of squares of the statistical partial wave amplitudes appears in $d\sigma/d\Omega$.

This "strong statistical assumption" is a sufficient, though perhaps not necessary, condition that all spin and isospin amplitudes be nearly equal in magnitude - subject only to the constraints imposed by symmetry. Let us suppose the colliding particles belong to an isospin $\frac{1}{2}$ system of spin $\underline{s}$, e.g., KK or NN scattering. At 90° we may derive constraints on the amplitude for total $I = 0$ or 1 scattering by a variation on the proof of Bohr's theorem ³⁾ on intrinsic parity selection rules in two-body collisions. Taking the initial momentum direction $\underline{p}$ as an axis, make a reflection of this axis, leaving directions in the perpendicular plane invariant. This reflection may be composed of parity inversion and rotation by π about $\underline{p}$. It reverses $\underline{p}$, while leaving the final momentum $\underline{p}'$ unchanged. Ignoring intrinsic parities, which are unchanged by the collision, we see that equality of the effect of this operation before and after collision leads to the condition

$$\Theta = (-1)^{S_{fp} - S_{ip}} \quad (8)$$

where $\mathcal{P}$ is the (conserved) orbital parity, and $S_{ip}(S_{fp})$ is the total spin along p in the initial (final) state. The constraints of spin and statistics may be written, for $I = 1$,

$$\begin{aligned} \text{Even } \mathcal{P} &\Longleftrightarrow \text{Even } S \\ \text{Odd } \mathcal{P} &\Longleftrightarrow \text{Odd } S \end{aligned} \quad (9)$$

and for $I = 0$,

$$\begin{aligned} \text{Even } \mathcal{P} &\Longleftrightarrow \text{Odd } S \\ \text{Odd } \mathcal{P} &\Longleftrightarrow \text{Even } S \end{aligned} \quad (10)$$

The same symmetry operation, followed by rotation about $\underline{p}'$ by π , gives the result that $d\sigma/d\Omega$ is unchanged if S_{fp} and S_{ip} are simultaneously reversed. Time reversal invariance implies that $d\sigma/d\Omega$ is unchanged if (S_f, S_{fp}) and (S_i, S_{ip}) are interchanged. The statistical hypothesis gives the stronger result, equality of all cross-sections not forbidden by Eqs. (8), (9), (10).

The total rate (at 90°) for an identical particle system (pure $I = 1$), e.g., K^+K^+ or pp , is then proportional to

$$\begin{aligned} N(I=1) = & \left[\sum_{\text{even } S=0}^{2[S]} \text{Number of states with } S_{\text{p even}} \right]^2 \\ & + \left[\sum_{\text{even } S=0}^{2[S]} \text{Number with } S_{\text{p odd}} \right]^2 \\ & + 2 \left[\sum_{\text{odd } S} \text{Number with } S_{\text{p odd}} \right] \left[\sum_{\text{odd } S} \text{Number with } S_{\text{p even}} \right] \end{aligned} \quad (11)$$

The total rate for different particles, e.g., K^+K^0 or $\pi^+\pi^0$, is proportional to $4^{-1}[N(I=1) + N(I=0)]$, with

$$N(I=0) = \left[\sum_{\text{odd } s} \text{Number with } s_p \text{ even} \right]^2 + \left[\sum_{\text{odd } s} \text{Number with } s_p \text{ odd} \right]^2 + 2 \left[\sum_{\text{even } s} \text{Number } s_p \text{ odd} \right] \left[\sum_{\text{even } s} \text{Number } s_p \text{ even} \right] \quad (12)$$

These sums are easily computed, and one may quickly confirm the result

$$N(I=0) \leq N(I=1) \quad (13)$$

where the equality holds for all half-integer s . As a consequence, the 90° cross-section ratio is

$$\frac{d\sigma}{d\Omega_{aa}} / \frac{d\sigma}{d\Omega_{ab}} = 2 \quad (14)$$

for all half-integer s . Direct calculation gives for integer s (bosons)

$$\frac{d\sigma}{d\Omega_{aa}} / \frac{d\sigma}{d\Omega_{ab}} = 2 \left[1 + \frac{1}{(2s+1)[s^2 + (s+1)^2]} \right] \quad (15)$$

Like the previous results, this converges to the value 2 in the limit of large s , but much more quickly. The limit 2 is easily understood from classical considerations. As s diverges, the identical a particles become distinguishable because of their different spin orientation. However, the counters in an experiment without

polarization measurements do not distinguish between two a particles of different spin orientation, while they do distinguish between particles of type a and type b because of the difference in charge. If the intrinsic rates are equal, twice as many a particles will be counted in 90° aa scattering as in 90° ab scattering, hence the ratio 2 in this distinguishable particle "classical" limit.

2. - COMMENTS

The only available data relevant to these considerations are np/pp cross-section ratios up to 7 GeV ⁴⁾. The trend of these data favours an asymptotic ratio of unity, but with an uncertainty of nearly a factor 2. The predictions of the spin-independent, helicity-independent and statistical assumptions are respectively 1, 3 and 2, so that helicity independence ⁵⁾ is least acceptable.

If diffractive scattering is dominant, why should the amplitude be a unit matrix in spin, and not helicity? The appeal of the latter lies in the fact that helicity is a relativistic invariant for a particle of infinite energy. Vector-meson exchange produces such an helicity-conserving amplitude. On the other hand, scalar-meson exchange is purely helicity-reversing (!) for fixed angle ($\neq 0$) scattering of spin $\frac{1}{2}$ particles. This fact indicates that helicity-independence is by no means uniquely favoured. Spin-independence, however, implies that the cm frame is singled out from others. For example, the colliding particles may come to a stop before boiling off again at 90° to the incident direction. The difficulty with this notion is that it seems contrary to the usual picture of diffraction, since some explicit dynamical process appears necessary: could unitarity effects alone produce a spin and isospin-independent effective hard-core potential? In fact, one must require even more; the forward-backward symmetry of large-angle scattering, predicted by Wu and Yang ¹⁾ and observed in the np data ⁴⁾, implies for the effective central potential in this diffractive model that it vanishes

for odd angular momentum states, or, if not, that odd and even parity amplitudes are 90° out of phase ! Such an exchange symmetry is surprising.

The advantage of the statistical model is that symmetry about 90° follows at once from the non-interference of even and odd orbital parity amplitudes. The statistical model has many other predictions. In particular, the same 90° symmetry should be present in πp elastic scattering, not at all an evident consequence of the diffraction picture : symmetry in np scattering means equal-direct and charge-exchange amplitudes, while symmetry in πp means equal-direct and baryon-exchange amplitudes. Also, the statistical model predicts definite, non-unit, ratios for elastic scattering in the different charge states of the πN system, as well as for charge-exchange scattering. The diffraction approach would give unit ratio for the elastic scattering, and ambiguous predictions for charge-exchange, perhaps also unit ratio. Either or both of these models could be ruled out by new data on πN scattering ⁶⁾.

The main disadvantage of the statistical approach is an aesthetic one. It seems hard to believe that partial-wave amplitudes would have such rapid energy-dependence that all interference would disappear. Such a phenomenon may not violate smoothness or analyticity requirements, but it does not seem a natural consequence of such requirements.

An alternative approach to experimental study of these questions is to examine $\underline{s}$ dependence of cross-section ratios, as indicated in Eqs. (3), (7), (14), (15). Unfortunately, the only identical elementary isobar scattering reaction open to practical experiments is nucleon-nucleon scattering. This is especially frustrating, since scalar particle scattering would be very illuminating : in the diffraction picture, with forward-backward symmetry of $f(\theta)$ in R , the ratio of, e.g., K^+K^+ to K^+K^0 scattering would remain constant at the value 4 throughout R . On the other hand, the statistical model, like all others, must give the ratio 4 at 90° , but the ratio should go to 2 for $|\theta - \pi/2| \approx 1/L$, where L is a typical orbital angular momentum in the collision.

While this fascinating experiment is impossible in the foreseeable future, there is an intriguing alternative for study of s dependence, as well as other questions. This is to use light $I = \frac{1}{2}$ nuclei of spin $-\frac{1}{2}$ (He^3 , H^3) and spin $-\frac{3}{2}$ (Li^7 , Be^7). Comparing elastic collisions of these nuclei at hundreds of MeV with NN scattering at tens of GeV, could provide a new empirical test of the hypothesis that nucleons behave like composite systems. In particular, if the ratio of 90° high-energy pp to np scattering is the same as the ratio of He^3 - He^3 to H^3 - He^3 scattering, it will be natural to look at the corresponding ratio for Li^7 and Be^7 . The result could permit a choice of one or none of the simple hypotheses discussed earlier. For $s = \frac{3}{2}$, the three different guesses yield : $\frac{3}{2}$, $\frac{5}{2}$, 2.

Of course in all of these considerations, the question 1) mentioned at the beginning remains open : what is an asymptotically high energy ? On this point, the only practical criterion would seem to be a stabilization of some feature such as a cross-section ratio, so that it becomes independent of energy and therefore presumably asymptotic. Thus, experiments at only one energy, no matter how high, cannot be proven relevant to the question raised here.

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REFERENCES

- 1) T.T. Wu and C.N. Yang - Phys.Rev. 137, B708 (1965).

Wu and Yang make three predictions relevant to the considerations in this paper. First, ratios of 90° cross-sections have a much weaker energy dependence than the cross-sections themselves, so that ratios of logarithms of cross-sections approach unity at infinite energy. Secondly, scattering at large angles does not depend on the sign of longitudinal momentum transfer in the cm frame.- cross-sections are symmetric about 90° . Thirdly, they point out that the statistical model (also discussed in this paper) satisfies their first two predictions, but also yields specific, finite, values for the asymptotic ratios of cross-sections related by isospin rotations.

- 2) This is discussed in the Appendix of Ref. 1). Fuller discussions of the physical conditions which may lead to statistical scattering have been made by Chernavsky. See :
D.S. Chernavsky - Rapporteur Talk, Proceedings of the XVth International Conference on High Energy Physics, Kiev (1970).
- 3) A. Bohr - Nuclear Phys. 10, 486 (1959).
- 4) M.L. Perl, J. Cox, M.J. Longo and M.N. Kreisler - Phys.Rev. D1, 1857 (1970),

This paper reviews previous theoretical work in the course of presenting new experimental data.

- 5) For the two diffractive hypotheses, one may consider a weaker assumption, proposed by Perl et al. [Ref. 4] : the amplitude may depend on spin (or helicity), but the spin (or helicity) may still be conserved, for the scattering of two different members of the same isomultiplet. If this idea is applied to NN scattering,

there is no new restriction on the pp/np ratio from the spin-conservation hypothesis. However, the helicity-conservation hypothesis implies the ratio must lie between 2 and 4. This statement is in disagreement with the assertion of Perl et al. that helicity-conservation implies a ratio of exactly 4. It appears that these authors omitted certain helicity-exchange amplitudes in deriving their results.

- 6) Recently data have become available on 90° scattering by protons of 5 GeV/c $K^\pm$, $\pi^\pm$ and $\bar{p}$. No simple universal behaviour of the 90° cross-sections is manifest in these data, which seem to be dominated by the tails of forward and backward peaks, and (in the case of pions) perhaps πp resonances as well. This certainly leaves open to doubt the asymptotic character of NN scattering at 7 GeV beam energy, but even np scattering changes significantly between 5 and 7 GeV. More (very difficult) experiments, at still higher energies, are required to determine the degree of relevance of the considerations in the present paper. The 5 GeV/c results mentioned above were reported by Baglin et al., to appear in Phys. Letters.