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HIGH ENERGY PRODUCTION OF HADRONS
IN THE COULOMB FIELD OF A NUCLEUS

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A B S T R A C T

We compute the one-photon exchange contribution to coherent production processes at high energy. The Primakoff effect gives a measurement of total photoabsorption cross-section and the integrated cross-section is found to increase with energy like $\log S$. Some discussion of inclusive reactions is given in particular for proton-proton colliding beam experiments.

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1. - INTRODUCTION

The usefulness of performing coherent photoproduction experiments in the Coulomb field of a nucleus has been pointed out twenty years ago by Primakoff ¹⁾ and the π^0 and η^0 lifetimes have been successfully measured by this method ^{2),3)}.

The natural generalization of the so-called Primakoff effect is the computation of the one-photon exchange contribution to hadronic reactions occurring coherently on a target. When the electromagnetic contribution to the cross-section can be isolated it gives a nice way to measure photoproduction cross-sections on unusual targets like π meson, K meson or photons. The essential step in that direction has been done by Halprin, Andersen and Primakoff ⁴⁾ who study the angular distribution in the laboratory system of the hadronic final state. The Weizsäcker-Williams formalism ⁵⁾ or more covariant techniques ⁶⁾ can be used for such calculations.

Recently, Stodolsky ⁷⁾ suggested the possibility to look at the recoil target only considering the reaction as an inclusive one. The hadronic final state is then treated exactly in the same way as in the inelastic lepton scattering experiment when only the final lepton is detected. The only information we retain on it is the effective mass.

The aim of this paper is a systematic approach of the problem using straightforward covariant techniques. The inclusive reaction is studied in a Lorentz invariant way using scalar variables like the squared centre-of-mass energy S , the final state effective mass W and the squared momentum transfer q^2 .

A particular attention has been paid to the production of relatively low W states at high energy $S \gg W^2$. The Primakoff peak at small q^2 is exhibited and the integration over q^2 gives rise to a fixed W^2 cross-section increasing like $\log S$ at high energy. As usual this feature is due to the zero mass of the photon.

The kinematics of these reactions is discussed in the laboratory frame and in the centre-of-mass frame with a special reference to proton-proton storage rings.

II. - ONE-PHOTON EXCHANGE CONTRIBUTION

1) - We consider the process

$$p + A \Rightarrow F + A \quad (I)$$

occurring coherently on a nucleon or a nucleus A . In practice the incident particle p will be a charged massive hadron or a photon. The final hadronic state F with the same electric and baryonic charges as p is not specified except by its effective mass W .

The kinematics is shown on Fig. 1.

We introduce the transferred energy momentum four-vector $q = p'_A - p_A$ and three scalar variables

$$S = -(p + p_A)^2 \quad q^2 = (p'_A - p_A)^2 \quad W^2 = -(p - q)^2$$

The target has a mass M_A and the incident particle a mass M_p .

2) - We are interested in this paper in the one-photon exchange contribution to the process (I) and we compute the diagram drawn on Fig. 2, using the same techniques as for inelastic lepton scattering.

The transition amplitude is the product of two matrix elements of the electromagnetic current

$$T = \frac{e^2}{q^2} \langle F | J_\mu(0) | P \rangle \langle P'_A | J^\mu(0) | P_A \rangle$$

Therefore the differential cross-section is obtained saturating two tensors, each of them being associated to an electromagnetic vertex.

The nucleus tensor can be measured in elastic electron-nucleus scattering whose angular distribution exhibits a Rosenbluth form with two structure functions $E(q^2)$ and $M(q^2)$ describing respectively the electric and magnetic structure of the nucleus

$$\frac{d\sigma(e^-A \Rightarrow e^-A)}{d\sigma_{\text{MOTT}}} = E(q^2) + \frac{q^2}{4M_A^2} M(q^2) \left[1 + 2 \left(1 + \frac{q^2}{4M_A^2} \right) \tan^2 \frac{\theta}{2} \right]$$

θ in the laboratory scattering angle.

When a summation is made over all the internal variables but W^2 of the final state F , the second tensor on which we wish to extract information is also described by two structure functions depending on q^2 and W^2 . These structure functions are easily interpreted in terms of total cross-sections for the photoproduction process

$$\gamma + p \Rightarrow F \quad (\text{II})$$

with a spacelike polarized photon, $\sigma_T(q^2, W^2)$ and $\sigma_L(q^2, W^2)$ where T means transverse and L longitudinal as viewed in the F rest frame. A straightforward calculation gives the result

$$\begin{aligned} \frac{d^2\sigma}{dq^2 dW^2} (p+A \Rightarrow F+A)_{1\gamma} &= \frac{1}{[3-(M_A+M_p)^2][3-(M_A-M_p)^2]} \frac{2\alpha}{\pi} \frac{1}{q^2 KW} \times \\ &\times \left\{ \left[E(q^2) + \frac{q^2}{4M_A^2} M(q^2) \right] \left[\sigma_T \frac{I_T}{T} + \sigma_L \frac{I_L}{L} \right] + \left(1 + \frac{q^2}{4M_A^2} \right) M(q^2) \frac{KW^2}{4} \left[2\sigma_T - \sigma_L \right] \right\}^{(1)} \end{aligned}$$

where the invariants I_T and I_L are defined by

$$\left| \begin{array}{l} I_T = \frac{1}{4} \left[(p_A + p'_A) \cdot q \right]^2 - \frac{\kappa^2 W^2}{q^2} \left[M_A^2 + \frac{q^2}{4} \right] \\ I_L = \frac{1}{4} \left[(p_A + p'_A) \cdot q \right]^2 \end{array} \right. \quad (2)$$

with

$$\kappa^2 W^2 = (p \cdot q)^2 + q^2 M_p^2$$

As usual α is the fine structure constant $\alpha \simeq 1/137$.

3) - When the final state F is a single particle n , the total cross-sections σ_T and σ_L are reinterpreted in terms of polarized total widths Γ_T and Γ_L for the radiative decay

$$n \Rightarrow p + \gamma \quad (III)$$

The one-photon exchange contribution to the process $p + A \Rightarrow n + A$ becomes

$$\frac{d\sigma}{dq^2} (p+A \Rightarrow n+A)_{1\gamma} = \frac{1}{[S - (M_A + M_p)^2][S - (M_A - M_p)^2]} \frac{8\pi\alpha}{q^2 \kappa^3} \frac{2J_n + 1}{2J_p + 1} \times \\ \times \left\{ \left[E(q^2) + \frac{q^2}{4M_A^2} M(q^2) \right] \left[\Gamma_T I_T + \Gamma_L I_L \right] + \left(1 + \frac{q^2}{4M_A^2} \right) M(q^2) \frac{\kappa^2 W^2}{4} \left[2\Gamma_T - \Gamma_L \right] \right\} \quad (3)$$

where J_n and J_p are the spins of particles n and p . For an incident photon the factor $2J_p + 1$ is replaced by 2.

4) - At $q^2 = 0$ the intermediate photon is real and the longitudinal contribution σ_L (or Γ_L) vanishes. The transverse contribution σ_T (or Γ_T) is then simply related to a photoproduction cross-section or a radiative width

$$\left| \begin{array}{l} \sigma_T \Rightarrow \sigma(\gamma + p \Rightarrow F) \\ \Gamma_T \Rightarrow \frac{1}{2} \Gamma(n \Rightarrow p + \gamma) \end{array} \right. \quad (4)$$

If moreover the incident particle is a photon we must use the Bose statistics in the $q^2 = 0$ limit and a new factor of 2 appears

$$\left| \begin{array}{l} \sigma_T \Rightarrow 2 \sigma(\gamma + \gamma \Rightarrow F) \\ \Gamma_T \Rightarrow \Gamma(n \Rightarrow \gamma + \gamma) \end{array} \right. \quad (5)$$

Concerning the nucleus structure we have the normalization constraints at $q^2 = 0$

$$E(0) = Q_A^2 \quad M(0) = \frac{J_A + 1}{3 J_A} \mu_A^2 \quad (6)$$

where Q_A is the electric charge, μ_A the dipole magnetic moment and J_A the spin of the nucleus.

5) - The particular case of a spin-zero target has been previously considered in the literature ⁴⁾. The electric function $E(q)$ reduces to the squared electric form factor $[F_A(q^2)]^2$ and the magnetic term is obviously absent. For a spin $\frac{1}{2}$ target, we have the two Dirac form factors $F_1(q^2)$ and $F_2(q^2)$ respectively normalized at $q^2 = 0$ to the electric charge and to the anomalous magnetic moment. The two functions E and M are then given by

$$\left| \begin{aligned} E(q^2) + \frac{q^2}{4M_A^2} M(q^2) &= [\overline{F}_1(q^2)]^2 + \frac{q^2}{4M_A^2} [\overline{F}_2(q^2)]^2 \\ \left(1 + \frac{q^2}{4M_A^2}\right) M(q^2) &= [\overline{F}_1(q^2) + \overline{F}_2(q^2)]^2 \end{aligned} \right.$$

III. - KINEMATICS IN THE LABORATORY FRAME

1) - In the incident nucleus rest frame we introduce the following notations

$$\left| \begin{aligned} P_A &= (\vec{0}, M_A) & P &= (\vec{K}, E) \\ P'_A &= (\vec{K}_A, E_A) & Q &= (\vec{K}_A, E_A - M_A) \end{aligned} \right.$$

the recoil angle is θ_A . The knowledge of E , E_A and θ_A determines completely the scalar variables S , q^2 and W^2

$$\left| \begin{aligned} S &= M_A^2 + M_P^2 + 2M_A E \\ q^2 &= 2M_A (E_A - M_A) \\ W^2 &= M_P^2 - 2(E + M_A)(E_A - M_A) + 2K K_A \cos \theta_A \end{aligned} \right. \quad (7)$$

In particular, the kinetic energy of the recoil nucleus measures only q^2 .

2) - The computation of the invariant I_T in the laboratory system is straightforward and the result is simply

$$\frac{I}{T} = K^2 M_A^2 \frac{q_T^2}{q^2} = K^2 M_A^2 \frac{E_A + M_A}{2M_A} \sin^2 \theta_A \quad (8)$$

where q_T is the transverse part of the photon momentum. The occurrence of the $\sin^2 \theta_A$ term has already been recognized in Ref. 7).

By conservation of the transverse momentum we have the relation

$$k_A \sin \theta_A + k_F \sin \theta_F = 0$$

where the index F refers to the complex state F. Therefore an equivalent expression of I_T looking at the state F is

$$I_T = k_A^2 M_A^2 \frac{k_F^2}{q^2} \sin^2 \theta_F \quad (9)$$

Equation (9) is one of the main results of Ref. 4).

3) - An elementary study of the kinematics leads to the following conclusions :

- a) at fixed incident energy S only final states F with an effective mass W less than a maximal value $W_M(S)$ can be produced

$$W_M(S) = \sqrt{S} - M_A \quad (10)$$

- b) at fixed incident energy S and effective mass $W \leq W_M(S)$ the nucleus recoils inside a cone of axis the incident momentum $\vec{k}$ and the semi-axis $\theta_M(S, W^2)$ is less than $\pi/2$;

- c) at fixed incident energy S and effective mass $W \leq W_M(S)$ in a given recoil direction θ_A inside the allowed cone, the squared momentum transfer q^2 can take two possible values (only one on the cone) and the minimum and maximum of q^2 are reached in the forward direction.

The explicit expression of $\theta_M(S, W^2)$ and $q^2(S, W^2, \theta)$ are given in Appendix 1.

IV. - HIGH ENERGY LIMIT

1) - In the high energy limit the effective mass W being fixed $S \gg W^2$ the maximum angle θ_M given by

$$\cos^2 \theta_M(S, W^2) = 2 \frac{(W^2 - M_P^2)(S + M_A^2)}{(S - M_A^2)^2} \quad (11)$$

tends to $\pi/2$ at infinite incident energy. In the forward direction $\theta_A = 0$ the extremal values of q^2 are

$$q_{\min}^2(S, W^2) = M_A^2 \left(\frac{W^2 - M_P^2}{S - M_A^2} \right)^2 \quad (12)$$

$$q_{\max}^2(S, W^2) = \frac{(S - M_A^2)^2}{S} \quad (13)$$

and on the cone $|\theta_A| = \theta_M$ the two solutions for q^2 collapse into the limiting expression

$$q_{\lim}^2(S, W^2) = 2M_A^2 \frac{W^2 - M_P^2}{S + M_A^2 - W^2} \quad (14)$$

2) - The differential cross-sections (1) and (3) in the high energy limit $W^2/S \ll 1$ $q^2/S \ll 1$ take more simple forms

$$\frac{d^2\sigma}{dq^2 dW^2}(p+A \rightarrow F+A) = \frac{\alpha}{4s} \frac{1}{2\pi} \frac{1}{q^2 KW} \left[E(q^2) + \frac{q^2}{4M_A^2} M(q^2) \right] \left[\left(1 - \frac{q_{\min}^2}{q^2} \right) G_T + G_L \right] \quad (15)$$

$$\frac{d\sigma}{dq^2}(p+A \Rightarrow \pi+A) = \frac{2\pi\alpha}{q^2 K^3} \frac{2J_n+1}{2J_p+1} \left[E(q^2) + \frac{q^2}{4M_A^2} M(q^2) \right] \left[\left(1 - \frac{q_{\min}^2}{q^2}\right) \Gamma_{\pi^+} \Gamma_L \right] \quad (16)$$

The factor $1 - q_{\min}^2/q^2$ ensures the vanishing of the transverse contribution in the forward direction and corresponds to the $\sin^2\theta_A$ term of formula (8).

3) - Let us now study the small solution region

$$q_{\min}^2 \leq q^2 \leq q_{\lim}^2 \quad (17)$$

From Eqs. (12) and (14) the values of q^2 remain small in the high energy limit and the nucleus structure occurs only via its value at $q^2 = 0$ which is simply the squared electric charge Q_A^2 .

The transverse contribution to the differential cross-section (15)

$$\frac{d^2\sigma}{dq^2 dW^2} \approx \frac{\alpha}{\pi} \frac{Q_A^2}{W^2 - M_p^2} G_T(0, W^2) \left[\frac{1}{q^2} \left(1 - \frac{q_{\min}^2}{q^2}\right) \right] \quad (18)$$

is a rapidly varying function of q^2 exhibiting a strong narrow peak at $q^2 = 2q_{\min}^2$ as shown on Fig. 3 where the bracket of Eq. (18) is plotted against q^2 in the range (17) and the peak value of the cross-section (18) increases at high energy like S^2

$$\left. \frac{d^2\sigma}{dq^2 dW^2} \right|_{\text{peak}} \approx \frac{\alpha}{4\pi} \frac{Q_A^2}{M_A^2} (S - M_A^2)^2 \frac{G_T(0, W^2)}{(W^2 - M_p^2)^3} \quad (19)$$

This feature is known as the Primakoff effect.

The longitudinal contribution to the different cross-section (15)

$$\frac{d^2\sigma^{(L)}}{dq^2 dW^2} = \frac{\alpha}{\pi} \frac{Q_A^2}{W^2 - M_p^2} \frac{\sigma_L(q^2, W^2)}{q^2}$$

is essentially constant and involves the q^2 derivative of the cross-section σ_L at $q^2 = 0$. It is the only one surviving term in the exact forward direction.

4) - We now compute a fixed W^2 differential cross-section by integration over q^2 in the small solution region (17). In other terms, we cover all the angular domain for the recoil nucleus summing up all the events with a recoil kinetic energy less than the limiting value on the cone $|\theta_A| = \theta_M$

$$E_A - M_A \equiv T_A \leq M_A \frac{W^2 - M_p^2}{S + M_A^2 - W^2} \quad (20)$$

The result for a multiparticle final state F is

$$\frac{d\sigma^{(1)}}{dW^2} = \frac{\alpha}{\pi} \frac{Q_A^2}{W^2 - M_p^2} \sigma(\delta + p \Rightarrow F) \left[\log \frac{2(S - M_A^2)^2}{(W^2 - M_p^2)(S + M_A^2)} - 1 \right] \quad (21)$$

and for a single particle final state n we get

$$G^{(1)} = g_{n\alpha} \frac{Q_A^2 M_n^3}{(M_n^2 - M_p^2)^3} \frac{2J_n + 1}{2J_p + 1} \Gamma(n \Rightarrow p + \alpha) \left[\log \frac{2(S - M_A^2)^2}{(M_n^2 - M_p^2)(S + M_A^2)} - 1 \right] \quad (22)$$

The index (1) refers to the particular region of integration defined by the inequalities (17).

5) - Let us now consider the complete domain of allowed values of q^2

$$q_{\min}^2 \leq q^2 \leq q_{\max}^2$$

Large values of q^2 are now involved and the electromagnetic structure at both vertices plays a role in the q^2 integration.

As strongly suggested by electron scattering data, we assume the structure functions to decrease rapidly enough at large spacelike q^2 so that integrals of the type

$$\int_0^\infty \left[E(q^2) + \frac{q^2}{4M_A^2} M(q^2) \right] G_{T,L}(q^2, W^2) dq^2$$

will be convergent.

The fixed W^2 differential cross-section has then a behaviour analogous to (21) or (22) in the high energy limit but only the coefficient of the leading $\log S$ term can be computed in a model independent way. The result is

$$\frac{d\sigma}{dW^2}(p+A \Rightarrow F+A)_{18} = \frac{\alpha}{\pi} \frac{Q_A^2}{W^2 - M_p^2} G(\delta + p \Rightarrow F) \left[2 \log \frac{S}{s_0(W)} - a(W^2) \right] \quad (23)$$

$$G(p+A \Rightarrow n+A)_{18} = 8\pi\alpha \frac{Q_A^2 M_n^2}{(M_n^2 - M_p^2)^3} \frac{2J_n + 1}{2J_p + 1} \Gamma(n \Rightarrow p + \delta) \left[2 \log \frac{S}{s_0} - a \right] \quad (24)$$

V. - PROTON-PROTON COLLIDING BEAMS

- 1) - Let us apply the previous considerations to high energy proton-proton collisions observing only a proton in the final state

$$p + p \Rightarrow p + \bar{p}$$

The two incident particles are identical and the one-photon exchange contribution is associated here to two diagrams (Fig. 4). The photon energy momentum four-vector is $q_{13} = p_3 - p_1$ for diagram (a) and $q_{23} = p_3 - p_1$ for diagram (b).

- 2) - We now introduce the centre-of-mass variables

$$p_1 = (\vec{k}_i, E_i) \quad p_2 = (-\vec{k}_i, E_i) \quad p_3 = (\vec{k}_f, E_f)$$

with the centre-of-mass angle θ_{CM} between $\vec{k}_i$ and $\vec{k}_f$. As usual the angular distribution is symmetrical with respect to $\theta_{CM} = 90^\circ$ but in the near forward or backward directions only one diagram of Fig. 4 contributes appreciably, diagram (a) near 0° and diagram (b) near 180° .

Kinematics gives

$$E_i = \frac{\sqrt{s}}{2} \quad E_f = \frac{s + M^2 - W^2}{2\sqrt{s}}$$

The measurement of the final proton energy E_f determines the effective mass W of the unobserved F system. But at high energy a great accuracy in the energy determination is needed to recognize low W production and this accuracy is of order $(W^2 - M^2)/s$.

3) - Let us study the near forward direction restricting to diagram (a) only. The squared momentum transfer q_{13}^2 is given by

$$q_{13}^2 = 2[E_i E_f - K_i K_f \cos \theta_{cm} - M^2]$$

and its minimum value q_{min}^2 is reached in the forward direction $\theta_{cm} = 0$ so that

$$q_{13}^2 = q_{min}^2 + 2K_i K_f (1 - \cos \theta_{cm})$$

From the general equation (18) the fixed W^2 angular distribution at very small θ_{cm} is given by

$$\frac{d^2\sigma}{dW^2 d\Omega_{cm}} \approx \frac{\alpha}{2\pi^2} \frac{\sigma^{\gamma p}(W^2)}{W^2 - M^2} \frac{\theta_{cm}^2}{[\theta_0^2 + \theta_{cm}^2]^2} \quad (25)$$

where $\sigma^{\gamma p}(W^2)$ is the total photoabsorption cross-section on protons. The peak angle θ_0 is defined by

$$\theta_0^2 \equiv \frac{q_{min}^2}{K_i K_f} = \frac{4M^2(W^2 - M^2)^2}{S^2}$$

Formula (25) has the same formal structure as the one obtained in Ref. 4) for the angular distribution of the F state in the laboratory frame ; the coefficients are different because here θ_{cm} is a centre-of-mass angle.

The cross-section (25) has a very narrow peak at $\theta_{cm} = \theta_0$ and its value at the peak is

$$\left. \frac{d^2\sigma}{dW^2 d\Omega_{cm}} \right|_{peak} = \frac{\alpha}{32\pi^2} \frac{S^3}{M^2} \frac{\sigma^{\gamma p}(W^2)}{(W^2 - M^2)^3} \quad (26)$$

The integration from $q_{\min}^2$ to $q_{\lim}^2$ becomes now an integration on θ_{CM} from 0 to $\theta_{\lim}$ defined by

$$\theta_{\lim}^2 = \frac{8M^2(W^2 - M^2)}{S^2}$$

and the result is the cross-section (21).

4) - In order to illustrate the previous considerations we put some numbers considering 10 GeV energy proton-proton colliding beams, $S = 400 \text{ GeV}^2$, and the production of a state F with $W^2 = 3 \text{ GeV}^2$. The electromagnetic effects are expected to be sizeable in the angular range $0 \lesssim \theta_{\text{CM}} \lesssim \theta_{\lim}$ where here $\theta_{\lim} \simeq 10 \text{ mr}$. The peak occurs at $\theta_{\text{CM}} = \theta_0 = 0.5 \text{ mr}$ and the electromagnetic contribution is there

$$\left. \frac{d^2\sigma}{dW^2 d\Omega_{\text{CM}}} \right|_{\text{peak}} \simeq 36,5 \text{ mb/GeV}^2\text{-ster}$$

where a value of $200 \mu\text{b}$ has been used for the total photoabsorption cross-section on proton at $W^2 = 3 \text{ GeV}^2$.

The observed proton energy loss is extremely small, 50 MeV for a total energy of 10 GeV. Integrating now from $\theta_{\text{CM}} = 0$ to $\theta_{\text{CM}} = \theta_{\lim}$ we get the following estimate for the fixed W^2 cross-section

$$\frac{d\sigma^{(1)}}{dW^2} \simeq 1,16 \mu\text{b/GeV}^2$$

Let us notice that even at these very high values of S the numerical contribution of the log in formula (21) is only 6.

VI. - CONCLUDING REMARKS

We have obtained the general expression for the one-photon exchange contribution to the coherent production process on a nucleus $p + A \rightarrow F + A$. At low squared momentum transfer q^2 this contribution is proportional to the squared nucleus electric charge Q_A^2 and to the total photoabsorption cross-section for $\gamma + p \rightarrow F$ one can in principle measure, looking at the recoil nucleus only.

In the laboratory frame the kinetic energy of the recoil nucleus determines q^2 and the electromagnetic contribution exhibits a strong narrow peak at very small values of q^2 usually known as the Primakoff effect. For a fixed effective mass W of the F system the kinetic energy where the peak occurs behaves like S^{-2} at high energy and the cross-section integrated over q^2 increases like $\log S$.

The electromagnetic contribution can be measured if and only if it can be isolated from the one due to strong interactions. This will certainly be the case at very high energy and proton acceleration could be very convenient machines to study electromagnetic interactions. A careful study of the signal to noise ratio must be done in each specific case and we do not make any attempt of this type here referring to previous studies ^{4),7),8)}.

The most delicate problem is certainly the detection of the nucleus recoil either in the laboratory frame where the interesting kinetic energies are very small or in the centre-of-mass frame where the emission angle and the energy loss are both very small. Nevertheless new techniques are rapidly progressing and we hope that in a near future these calculations would be of interest. Estimates are given in Appendix 2.

ACKNOWLEDGEMENTS

We thank Professor L. Van Hove for an interesting discussion.

APPENDIX 1

KINEMATICS

The laboratory scattering angle θ_A is computed from Eqs. (7) to be

$$\cos \theta_A = \frac{W^2 - M_p^2 + q^2 \left(1 + \frac{E}{M_A}\right)}{2k k_A}$$

We then deduce a second order equation in q^2 for fixed S , W^2 and θ . The existence of real roots defines a cone of axis $\vec{k}$ and semi-angle θ_M with

$$\cos^2 \theta_M (S, W^2) = \frac{(W^2 - M_p^2) [2(S + M_A^2) - (W^2 + M_p^2)]}{[S - (M_A + M_p)^2] [S - (M_A - M_p)^2]}$$

The two solutions in q^2 at fixed S , W^2 and θ are

$$\frac{q^2}{2M_A^2} = \frac{\cos^2 \theta_A [S - (M_A + M_p)^2] [S - (M_A - M_p)^2] \left[1 \pm \sqrt{1 - \frac{\cos^2 \theta_M}{\cos^2 \theta_A}}\right] - (W^2 - M_p^2)(S + M_A^2 - M_p^2)}{(S + M_A^2 - M_p^2)^2 - \cos^2 \theta_A [S - (M_A + M_p)^2] [S - (M_A - M_p)^2]}$$

and for $\theta_A = \theta_M$ the limiting function is simply

$$q_{lim}^2 (S, W^2) = 2M_A^2 \frac{W^2 - M_p^2}{S + M_A^2 - W^2}$$

APPENDIX 2

ESTIMATES

The quantitative importance of the one-photon exchange contribution is evaluated for three particular reactions

We make the following choice for the incident laboratory energies

$$E_\gamma = 16 \text{ GeV}$$

$$E_K = 8 \text{ GeV}$$

$$E_\pi = 50 \text{ GeV}$$

Results are given in the Table with reasonable estimates of the radiative widths

$$\Gamma(X^0 \Rightarrow 2\gamma) \approx 5 \text{ keV}$$

$$\Gamma(K^{*\pm} \Rightarrow K^\pm \gamma) = 72 \text{ keV}$$

$$\Gamma(\rho^\pm \Rightarrow \pi^\pm \gamma) = 0,11 \text{ MeV}$$

	$T_{\min}$ (MeV)	$T_{\lim}$ (MeV)	$d\sigma/dq^2$ peak ($\mu\text{b}/\text{GeV}^2$)	$d\sigma/dq^2$ lim ($\mu\text{b}/\text{GeV}^2$)	$\sigma^{(1)}$ (μb)
R_1	0.5	30	0.093	0.0059	0.0013
R_2	0.26	26	18.2	0.7	0.28
R_3	0.016	5.6	450	5.26	0.29

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FIGURE CAPTIONS

Figure_1 General process $p + A \Rightarrow F + A$.

Figure_2 One-photon exchange diagram.

Figure_3 Plot of $\frac{1}{q^2}(1 - \frac{q_{\min}^2}{q^2})$ versus q^2 .

Figure_4 One-photon exchange diagrams for $p + p \Rightarrow p + F$.

FIG.1

FIG.2

FIG.3

FIG.4