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NEUTRINO AND ANTINEUTRINO TOTAL CROSS-SECTIONS

IN THE QUARK PARTON MODEL

M. Gourdin *)

CERN - Geneva

A B S T R A C T

The information from electroproduction data is used in the quark parton model to obtain predictions for the neutrino and antineutrino total cross-sections.

*) On leave of absence from the Laboratoire de Physique Théorique, 91 Orsay, France.

I. - INTRODUCTION

The old idea of a composite structure of hadrons has recently received a considerable support with the SLAC experiment on inelastic electron-nucleon scattering. Feynman's idea of a parton structure ¹⁾ has been extensively developed by Bjorken and Paschos ^{2),3)}, by Drell, Levy and Yan ⁴⁾ and by Gross and Llewellyn Smith ⁵⁾.

In this model the Lorentz invariant structure functions of inelastic lepton scattering are computed in the infinite momentum frame and the electromagnetic or weak current scatters instantaneously and incoherently off the partons considered as pointlike particles. The partons are nearly free inside the hadrons and final state interaction effects are neglected.

Parton models can be divided into two classes depending on the nature of the partons :

- a) partons are known particles like nucleons, π or K mesons, etc., ^{4),6)} ;
- b) partons are unknown particles like quarks, gluons ^{2),3),5)}.

In this paper we are interested in the quark parton model first introduced by Bjorken and Paschos, and which seems to be extremely natural on the basis of the unitary symmetry $U(3)$.

In Section II, we give the notations of the parton quark model with the application to inelastic electron scattering. Comparison with experiment is made in Section III where some knowledge on the parameters of the model can be obtained. In Section IV, we apply the quark parton model to neutrino and antineutrino scattering with a particular attention to the total cross-sections. A comparison with the available bubble chamber data is made in Section V.

Up to now the quark parton model can give a consistent description of the electron and neutrino data but further neutrino experiments will be of the greatest interest to test all the predictions of the model and to improve or disprove the agreement with experiment.

A complete study of the quark parton model and of its consequence has not been made in this paper where only one type of integrals involving the structure functions has been considered. We refer to the original work of Bjorken and Paschos for completeness.

II. - QUARK PARTON MODEL FOR ELECTROPRODUCTION

1. - The inelastic scattering of unpolarized electrons or muons on an unpolarized target is described in the one-photon exchange approximation by two structure functions when only the final lepton is detected. It is well known for a long time that these two structure functions can be conveniently expressed in terms of the two total cross-sections for the scattering of the polarized electromagnetic current on the same target, σ_L and σ_T where L means longitudinal and T transverse ⁷⁾.

The energy momentum four-vector is called p for the target and q for the virtual photon, and we are interested in the Bjorken limit ⁸⁾ LIM defined by

$$q^2 \Rightarrow +\infty \quad -p \cdot q \Rightarrow +\infty \quad \xi = -\frac{q^2}{2p \cdot q} \text{ fixed}$$

In the physical region $0 \leq \xi \leq 1$ from the kinematics. If the Bjorken limit is non-trivial, the scaling functions are defined by

$$\overline{F}_{L,T}(\xi) = \text{LIM} \frac{-p \cdot q}{\pi} \sigma_{L,T}(q^2, W^2)$$

where W is the effective mass of the undetected hadronic system. In the Californian language

$$\overline{F}_1 = \overline{F}_T \quad \overline{F}_2 = 2\xi(\overline{F}_T + \overline{F}_L)$$

For a spin $\frac{1}{2}$ parton model the Callan-Gross relation ⁹⁾ implies the vanishing of the longitudinal scaling function $F_L(\xi)$ and we have to study only the transverse function $F_T(\xi)$. Experimental data seem to support this feature ¹⁰⁾.

2. - In the parton model, the cross-section is an incoherent sum of contributions of the various types of partons which interact like-point particles ⁴⁾. We then introduce the standard notations :

P_N probability for a configuration with N partons,
 N_j number of partons of type j in this configuration,
 $f_N^j(\xi)$ density of probability of the parton momentum,
 Q_j electric charge of the type j parton,

and we have the normalization constraints

$$\sum_N P_N = 1 \qquad \int_0^1 f_N^j(\xi) d\xi = 1 \qquad (1)$$

For a spin $\frac{1}{2}$ parton model the expression of $F_T(\xi)$ is simply given by ^{2), 11)}

$$2 \overline{F}_T(\xi) = \sum_N P_N \sum_j Q_j^2 N_j(\xi)$$

where $N_j(\xi)$ is a short notation for $N_j f_N^j(\xi)$.

3. - If the interacting partons are quarks ($j=1,2,3$) and antiquarks ($j=-1,-2,-3$) labelled as shown on Fig. 1 from the $SU(3)$ fundamental representations, the electric charge Q_j is known and we obtain

$$2 \overline{F}_T(\xi) = \sum_N P_N \left\{ \frac{4}{9} [N_1(\xi) + N_{-1}(\xi)] + \frac{1}{9} [N_2(\xi) + N_{-2}(\xi) + N_3(\xi) + N_{-3}(\xi)] \right\} \quad (2)$$

- Figure 1 -

Neutral particles called gluons might also be present for dynamical purpose ¹¹⁾ and we have the inequality

$$N_g = \sum_d N_d \leq N$$

4. - The conservation of the electric charge Q , of the baryonic number B and of the hypercharge Y gives the following constraints

$$N_1 - N_{-1} = B + Q$$

$$N_2 - N_{-2} = B + Y - Q$$

$$N_3 - N_{-3} = B - Y$$

Let us notice that the isotopic spin invariance has not been used but only the I_3 rotational invariance.

5. - In the particular case of nucleons, $B = Y = 1$, these relations become

$$N_1 - N_{-1} = \frac{3}{2} + I_3 \quad N_2 - N_{-2} = \frac{3}{2} - I_3 \quad N_3 - N_{-3} = 0 \quad (3)$$

Charge symmetry allows us to compute the function $N_j(\xi)$ for the neutron in terms of the corresponding one for the proton

$$\begin{aligned} N_1^n(\xi) &= N_2^p(\xi) & N_2^n(\xi) &= N_1^p(\xi) & N_3^n(\xi) &= N_3^p(\xi) \\ N_{-1}^n(\xi) &= N_{-2}^p(\xi) & N_{-2}^n(\xi) &= N_{-1}^p(\xi) & N_{-3}^n(\xi) &= N_{-3}^p(\xi) \end{aligned} \quad (4)$$

From now we will only use the functions $N_j^p(\xi)$ for proton and neutron and the same probability P_N for both nucleons.

6. - We assume the nucleon momentum to be equally distributed in average among the partons so that ¹²⁾

$$\int_0^1 \xi N_d(\xi) d\xi = \frac{N_d}{N} \quad (5)$$

and we introduce the notation

$$\sum_N P_N \frac{N_d}{N} = \langle \frac{N_d}{N} \rangle \quad (6)$$

We want to emphasize that the assumption (5) is rather natural but not necessary in the parton model. It gives the model a strong predictive power and most of the consequences we will derive depend on this assumption.

7. - The scaling functions $F_T(\xi)$ have been experimentally measured for the proton and the neutron ¹⁰⁾. We will use in this paper only the information contained in the two integrals

$$I_{1p} = 2 \int_0^1 \xi \overline{F}_T^p(\xi) d\xi \qquad I_{1n} = 2 \int_0^1 \xi \overline{F}_T^n(\xi) d\xi$$

In the quark parton model with assumption (5), I_{1p} and I_{1n} are given by

$$I_{1p} = \frac{1}{9} (1 - \varepsilon) + \frac{2}{3} \left\langle \frac{1}{N} \right\rangle + \frac{2}{3} \left\langle \frac{N_1^p}{N} \right\rangle \quad (7)$$

$$I_{1n} = \frac{1}{9} (1 - \varepsilon) + \frac{1}{3} \left\langle \frac{1}{N} \right\rangle + \frac{2}{3} \left\langle \frac{N_2^p}{N} \right\rangle \quad (8)$$

The ε parameter measures the proportion of gluons in the sense

$$\left\langle \frac{N_g}{N} \right\rangle = 1 - \varepsilon \quad (9)$$

In obtaining (7) and (8) we have used the constraints (3). Conversely if I_{1p} and I_{1n} are known, the mean values $\langle N_i^p / N \rangle$ depend on the two parameters $\langle 1/N \rangle$ and ε as shown on Table I.

	$\frac{3}{2} I_{1p}$	$\frac{3}{2} I_{1n}$	$\frac{1}{6}(1-\varepsilon)$	$\langle 1/N \rangle$
$\langle N_1^p/N \rangle$	1	0	-1	1
$\langle N_{-1}^p/N \rangle$	1	0	-1	-1
$\langle N_2^p/N \rangle$	0	1	-1	$\frac{1}{2}$
$\langle N_{-2}^p/N \rangle$	0	1	-1	$-\frac{1}{2}$
$\langle N_3^p/N \rangle = \langle N_{-3}^p/N \rangle$	-1	-1	5	0

- TABLE I -

III. - BOUNDS ON ε AND $\langle 1/N \rangle$ FROM ELECTROPRODUCTION DATA

1. - By definition of the quantities ε and $\langle 1/N \rangle$ in the quark parton model we have the restrictions

$$0 \leq \langle \frac{1}{N} \rangle \leq \frac{1}{3} (1-\varepsilon) \leq \frac{1}{3} \quad (10)$$

More information on ε and $\langle 1/N \rangle$ can be obtained if we have some knowledge of the integrals I_{1p} and I_{1n} by using the property for any number of antiquarks to be non-negative but smaller or equal to the total number of antiquarks

$$0 \leq \langle \frac{N_d}{N} \rangle \leq \frac{1}{2} (1-\varepsilon) - \frac{3}{2} \langle \frac{1}{N} \rangle \quad \text{for } d=-1, -2, -3$$

2. - Let us first consider the strange antiquarks ($j = -3$).
From Table I, we have the inequality

$$\frac{4}{9} (1 - \varepsilon) + \frac{3}{2} \langle \frac{1}{N} \rangle \leq \frac{3}{2} (\overline{I}_{1p} + \overline{I}_{1n}) \leq \frac{5}{6} (1 - \varepsilon) \quad (11)$$

The relations (10) and (11) are compatible if and only if

$$\overline{I}_{1p} + \overline{I}_{1n} \leq \frac{5}{9} \quad (12)$$

The upper bound (12) which is a non-trivial constraint of the quark parton model seems to be largely satisfied experimentally (see below).
We deduce from (11) an upper bound for ε

$$\varepsilon \leq 1 - \frac{9}{5} (\overline{I}_{1p} + \overline{I}_{1n}) \quad (13)$$

3. - To make more precise the discussion, we have to use numerical values for $\overline{I}_{1p}$ and $\overline{I}_{1n}$. From the SLAC data ¹⁰⁾, we have

$$\overline{I}_{1p} = 0.14 + x_p \quad \overline{I}_{1n} = 0.10 + x_n$$

where x_p and x_n correspond to the non-measured parts of the integrals from $\xi = 0$ to $\xi = 1/12$. Assuming the integrands to remain constant and of order 0.3 in that region, we obtain

$$x_p \approx x_n \approx 0.025$$

For numerical applications we will take

$$\bar{I}_{1p} = 0.16 \qquad \bar{I}_{1n} = 0.12 \qquad (14)$$

and reasonable error estimates give $\pm 20\%$ for I_{1p} and I_{1n} .

4. - The domain of allowed values of ϵ and $\langle 1/N \rangle$ is limited in the first quadrant by the straight lines corresponding to the conditions $N_{-1}^p = 0$, $N_{-2}^p = 0$ and $N_{-3}^p = 0$. This domain is represented on Fig. 2, using the experimental numbers (14) without errors.

- Figure 2 -

Small modifications in the experimental data will change this domain. For instance, if it turns out that experimentally $I_{1n} < \frac{1}{9}$ then the value $\epsilon = 0$ is excluded. Clearly the necessity of gluons has not yet been proved in the quark parton model considering only the quantities I_{1p} and I_{1n} ¹³⁾.

IV. - QUARK PARTON MODEL FOR NEUTRINO REACTIONS

1. - In the local V-A theory of weak interactions or in a theory with intermediate bosons only one unit of angular momentum is exchanged between the leptons and the hadrons. Therefore the inelastic scattering of neutrinos or antineutrinos on an unpolarized target is described by three structure functions ^{14), 15)} when only the final lepton is detected. These structure functions can be written in terms of the three total cross-sections for the scattering of the polarized weak current on the same target, σ_L and $\sigma_T^{(\pm)}$ where $\pm$ means helicity ± 1 .

In a spin $\frac{1}{2}$ parton model, the longitudinal scaling function vanishes and the two transverse scaling functions are defined as previously by

$$F^{(\pm)}(\xi) = \lim_{\substack{q^2 \rightarrow 0 \\ W^2 \rightarrow 0}} \frac{-p \cdot q}{q^2} \sigma_T^{(\pm)}(q^2, W^2)$$

It is convenient to separate the contributions coming from the $\Delta S = 0$ transitions and the $|\Delta S| = 1$ transitions ¹⁶⁾

$$F(\xi) = \cos^2 \theta G(\xi) + \sin^2 \theta H(\xi)$$

where θ is the Cabibbo angle.

2. - We call s the square of the total energy in the neutrino (antineutrino) target c.m. system. We refer to the Bjorken high-energy limit as s, q^2, W^2 going to infinity so that their ratios two by two remain fixed. We assume that there exists, in this high energy limit, a region where the local Fermi theory is still applicable and where the scaling takes place. With a vanishing longitudinal scaling function, the differential cross-sections can be written as

$$\left| \begin{array}{l} \frac{d^2 G^{\nu}}{d\xi d\rho} \Rightarrow \frac{G^2 S}{2\pi} \xi \left[\rho^2 F^{(+)\nu}(\xi) + \bar{F}^{(-)\nu}(\xi) \right] \\ \frac{d^2 G^{\bar{\nu}}}{d\xi d\rho} \Rightarrow \frac{G^2 S}{2\pi} \xi \left[\rho^2 \bar{F}^{(-)\bar{\nu}}(\xi) + F^{(+)\bar{\nu}}(\xi) \right] \end{array} \right. \quad (15)$$

where ρ is the ratio of the lepton energies in the laboratory frame.

In order to find the total cross-sections we have to integrate the differential cross-sections in the region $0 \leq \xi \leq 1$; $0 \leq \rho \leq 1$. Now comes the strong assumption that the result is not too different from the one obtained by using the scaling form (15) in all the domain. We then deduce a linear rising of the total cross-section with S

$$G_{TOT}(S) \Rightarrow \frac{G^2 S}{2\pi} A \quad (16)$$

where

$$A = \int_0^1 \xi A(\xi) d\xi$$

For neutrino induced reactions from (15)

$$A^{\nu}(\xi) = \frac{1}{3} F^{(+)\nu}(\xi) + \bar{F}^{(-)\nu}(\xi) \quad (17)$$

and for antineutrino induced reactions from (15)

$$A^{\bar{\nu}}(\xi) = \frac{1}{3} \bar{F}^{(-)\bar{\nu}}(\xi) + F^{(+)\bar{\nu}}(\xi) \quad (18)$$

The quantities $A^{\nu, \bar{\nu}}$ and $A^{\bar{\nu}, \nu}$ will also be splitted into two parts

$$A^{\nu, \bar{\nu}} = \cos^2 \theta B^{\nu, \bar{\nu}} + \sin^2 \theta C^{\nu, \bar{\nu}}$$

3. - The determination of the scaling functions in the quark parton model has been explicitly given in Refs. 3) and 10), assuming pointlike quarks and chiral symmetry ($g_A = g_V$ for quarks)

$$\begin{aligned} G^{(+)\nu}(\xi) &= 2 \sum_N P_N N_{-1}(\xi) & H^{(+)\nu}(\xi) &= 2 \sum_N P_N N_{-1}(\xi) \\ G^{(-)\nu}(\xi) &= 2 \sum_N P_N N_2(\xi) & H^{(-)\nu}(\xi) &= 2 \sum_N P_N N_3(\xi) \\ G^{(+)\bar{\nu}}(\xi) &= 2 \sum_N P_N N_{-2}(\xi) & H^{(+)\bar{\nu}}(\xi) &= 2 \sum_N P_N N_{-3}(\xi) \\ G^{(-)\bar{\nu}}(\xi) &= 2 \sum_N P_N N_1(\xi) & H^{(-)\bar{\nu}}(\xi) &= 2 \sum_N P_N N_1(\xi) \end{aligned} \quad (19)$$

We have only six types of quarks and antiquarks and therefore only six independent scaling functions. Two trivial relations between neutrino and antineutrino functions follow :

$$H^{(+)\nu}(\xi) = G^{(+)\nu}(\xi) \quad H^{(-)\bar{\nu}}(\xi) = G^{(-)\bar{\nu}}(\xi) \quad (20)$$

and the electroproduction scaling function $F_T^e(\xi)$ can be written as a linear combination of the neutrino and antineutrino ones

$$2 F_T^e(\xi) = \frac{2}{9} [G^{(+)\nu}(\xi) + G^{(-)\bar{\nu}}(\xi)] + \frac{1}{18} [G^{(-)\nu}(\xi) + G^{(+)\bar{\nu}}(\xi) + H^{(-)\nu}(\xi) + H^{(+)\bar{\nu}}(\xi)] \quad (21)$$

4. - We now study the functions $B^{\nu, \bar{\nu}}(\xi)$ and $C^{\nu, \bar{\nu}}(\xi)$ involved in the neutrino and antineutrino total cross-sections and we restrict ourselves to the realistic case of a nucleon target.

From charge symmetry, the neutron distributions $N_j^n(\xi)$ are simply related to the proton distributions $N_j^p(\xi)$ and we have only six independent scaling functions with a nucleon target. This property has two consequences :

- a) two relations between the B's and C's from (20),
- b) two relations between the B's and C's and the electroproduction functions F_T^{ep} and F_T^{en} from (21).

These relations are valid between scaling functions in the quark parton model ¹⁷⁾. They are here written in the integrated form for simplicity

$$8(C^{up} - C^{un}) = 3B^{\bar{u}n} + B^{un} - 3B^{\bar{u}p} - B^{up} \quad (22)$$

$$8(C^{\bar{u}p} - C^{\bar{u}n}) = 3B^{un} + B^{\bar{u}n} - 3B^{up} - B^{\bar{u}p} \quad (23)$$

$$36(I_{1p} - I_{1n}) = 7[B^{up} + B^{un} + B^{\bar{u}p} + B^{\bar{u}n}] + 2[C^{up} + C^{un} + C^{\bar{u}p} + C^{\bar{u}n}] \quad (24)$$

$$4(I_{1p} - I_{1n}) = B^{un} + B^{\bar{u}n} - B^{up} - B^{\bar{u}p} \quad (25)$$

Equations (22), (23), (24) and (25) are linear combinations of the results obtained by Llewellyn Smith in Ref. 11). Using relations (22) and (23) between the B's and C's, we obtain an equivalent form of (25) for $|\Delta S| = 1$ transitions

$$2(I_{1p} - I_{1n}) = C^{up} + C^{\bar{u}p} - C^{un} - C^{\bar{u}n} \quad (26)$$

Combining now (25) and (26) in a single form,

$$(A^{un} + A^{\bar{u}n}) - (A^{up} + A^{\bar{u}p}) = (4 - 6S_{1n}^2 \theta)(I_{1p} - I_{1n}) \quad (27)$$

5. - We now use the assumption (5) to parametrize the total cross-sections $A^{\nu p}$, $A^{\nu n}$, $A^{\bar{\nu} p}$, $A^{\bar{\nu} n}$ in terms of the electroproduction integrals I_{1p} and I_{1n} and of the two parameters $\langle 1/N \rangle$ and ϵ defined in Section II.

This has been done in Table II where the results of Table I have been combined with the inequalities (17), (18) and (19).

	I_{1p}	I_{1n}	$\frac{1}{3} \langle 1/N \rangle$	$-(\frac{4}{9} - 2\sin^2 \theta)(1 - \epsilon)$
$A^{\nu p}$	$1 - 3 \sin^2 \theta$	$3(1 - 2 \sin^2 \theta)$	$1 - 3 \sin^2 \theta$	1
$A^{\nu n}$	$3(1 - 2 \sin^2 \theta)$	$1 - 3 \sin^2 \theta$	$5 - 6 \sin^2 \theta$	1
$A^{\bar{\nu} p}$	$1 - 3 \sin^2 \theta$	$3(1 - 2 \sin^2 \theta)$	$-1 + 3 \sin^2 \theta$	1
$A^{\bar{\nu} n}$	$3(1 - 2 \sin^2 \theta)$	$1 - 3 \sin^2 \theta$	$-5 + 6 \sin^2 \theta$	1

- TABLE II -

Of course, the general relation (27) is immediately checked on these explicit expressions.

As a consequence of charge symmetry, relations (3) imply an equality

$$\frac{A^{\nu n} - A^{\bar{\nu} n}}{5 - 6 \sin^2 \theta} = \frac{A^{\nu p} - A^{\bar{\nu} p}}{1 - 3 \sin^2 \theta} = \frac{2}{3} \langle \frac{1}{N} \rangle \quad (28)$$

which is independent of the numerical value of I_{1p} and I_{1n} , e.g., relates neutrino and antineutrino cross-sections only.

The bounds found in Section III for $\langle 1/N \rangle$ and ϵ can be used to obtain estimates for the neutrino and antineutrino total cross-sections on nucleons and all the various linear combinations one can imagine.

6. - Let us begin studying the $\langle 1/N \rangle$ dependence after eliminating ε by subtraction. Equations (28) give a double determination of $\langle 1/N \rangle$ and from Table II we deduce other interesting combinations

$$A^{\nu n} - A^{\nu p} = (2 - 3\delta \sin^2 \theta)(I_{1p} - I_{1n}) + \left(\frac{4}{3} - \delta \sin^2 \theta\right) \langle \frac{1}{N} \rangle \quad (29)$$

$$A^{\bar{\nu} p} - A^{\bar{\nu} n} = -(2 - 3\delta \sin^2 \theta)(I_{1p} - I_{1n}) + \left(\frac{4}{3} - \delta \sin^2 \theta\right) \langle \frac{1}{N} \rangle \quad (30)$$

$$A^{\nu p} - A^{\bar{\nu} n} = -(2 - 3\delta \sin^2 \theta)(I_{1p} - I_{1n}) + (2 - 3\delta \sin^2 \theta) \langle \frac{1}{N} \rangle \quad (31)$$

$$A^{\nu n} - A^{\bar{\nu} p} = (2 - 3\delta \sin^2 \theta)(I_{1p} - I_{1n}) + (2 - 3\delta \sin^2 \theta) \langle \frac{1}{N} \rangle \quad (32)$$

The difference between (29) and (30) or between (32) and (31) is the relation (27) valid on more general grounds. The sum of (29) and (30) or the sum of (31) and (32) are various linear combinations of the relations (28). The comparison of ν and $\bar{\nu}$ scattering is also a very simple measure of $\langle 1/N \rangle$

$$(A^{\nu p} + A^{\nu n}) - (A^{\bar{\nu} p} + A^{\bar{\nu} n}) = (4 - 6\delta \sin^2 \theta) \langle \frac{1}{N} \rangle \quad (33)$$

The positive character of $\langle 1/N \rangle$ and the experimental observation $I_{1p} > I_{1n}$ imply the inequalities

$$A^{\nu n} > A^{\nu p} > A^{\bar{\nu} p} \quad A^{\nu n} > A^{\bar{\nu} n} \quad (34)$$

If, moreover, it turns out that experimentally $\langle 1/N \rangle \geq \frac{3}{2}(I_{1p} - I_{1n})$ we have a complete hierarchy of the total cross-sections written as

$$A^{\nu n} > A^{\nu p} > A^{\bar{\nu} p} > A^{\bar{\nu} n}$$

7. - Let us briefly study some direct consequences of the bounds (13) for ε

$$\frac{9}{5} (\bar{I}_p + \bar{I}_n) \leq 1 - \varepsilon \leq 1$$

by eliminating $\langle 1/N \rangle$ between total cross-sections in Table II. For instance, typical relations are

$$A^{pp} + A^{\bar{p}p} = 2(1 - 3\delta_{in}^2\theta)\bar{I}_{1p} + 6(1 - 2\delta_{in}^2\theta)\bar{I}_{1n} - \left(\frac{8}{9} - 4\delta_{in}^2\theta\right)(1 - \varepsilon) \quad (35)$$

$$A^{pn} + A^{\bar{p}n} = 6(1 - 2\delta_{in}^2\theta)\bar{I}_{1p} + 2(1 - 3\delta_{in}^2\theta)\bar{I}_{1n} - \left(\frac{8}{9} - 4\delta_{in}^2\theta\right)(1 - \varepsilon) \quad (36)$$

The difference between (35) and (36) is the relation (27) and for the sum of (35) and (36), we obtain the inequality

$$2(4 - 9\delta_{in}^2\theta)(\bar{I}_{1p} + \bar{I}_{1n}) - \left(\frac{16}{9} - 8\delta_{in}^2\theta\right) \leq A^{pp} + A^{pn} + A^{\bar{p}p} + A^{\bar{p}n} \leq \frac{6}{5}(4 - 3\delta_{in}^2\theta)(\bar{I}_{1p} + \bar{I}_{1n}) \quad (37)$$

V. - COMPARISON WITH EXPERIMENT

1. - An experiment made at CERN in a propane bubble chamber gives the result ^{18), 19)}

$$\bar{A}^{p \text{ prop.}} = (0.52 \pm 0.13) \text{ per nucleon}$$

The same quantity can be computed in the quark parton model and the theoretical expression is ²⁰⁾

$$\bar{A}^{\text{prop}} = \left(\frac{20}{14} - \frac{93}{22} \sin^2 \theta\right) I_{1p} + \left(\frac{24}{14} - \frac{105}{22} \sin^2 \theta\right) I_{1n} + \left(\frac{20}{33} - \frac{31}{22} \sin^2 \theta\right) \langle \frac{1}{N} \rangle - \left(\frac{4}{3} - 2 \sin^2 \theta\right) (1 - \epsilon)$$

Using the numerical values (14) for I_{1p} and I_{1n} and the bounds found in Section III for $\langle 1/N \rangle$ and ϵ we have the prediction

$$0.15 \leq \bar{A}^{\text{prop}} \leq 0.45$$

The experimental result is consistent with the theoretical calculation up to one standard deviation and it looks very close to the upper bound.

2. - Let us now try to extract more information on $\langle 1/N \rangle$ and from the neutrino data. In the $\langle 1/N \rangle, \epsilon$ plane, a fixed value of $\bar{A}^{\text{prop}}$ is represented by a straight line. As shown on Fig. 3, the straight line corresponding to the lower limit 0.39 crosses the domain D of allowed values of $\langle 1/N \rangle$ and ϵ . Neglecting the experimental errors in electroproduction data as previously, we can obtain lower bounds for $\langle 1/N \rangle$ and ϵ . The result is

- Figure 3 -

These estimates are very speculative but a quark parton model without gluons cannot accommodate the neutrino data.

3. - For the ratio $R = (A^{\nu n}/A^{\nu p})$ the experimental situation seems to be more confusing. The results quoted in Ref. 18) are

$$\left| \begin{array}{ll} R = 1.8 \pm 0.3 & \text{for the propane experiment,} \\ R = 1.1 \pm 0.3 & \text{for the freon experiment.} \end{array} \right.$$

The theoretical predictions are obtained using the expression for $A^{\nu p}$ and $A^{\nu n}$ given in Table II and the electroproduction data (19) taken, as previously, without errors. We obtain

$$1.25 (1.73) \leq R \leq 1.90$$

The value in parentheses corresponds to the dashed domain Δ of Fig. 3, limited by the one standard deviation lower limit of the neutrino propane data.

Again the theoretical estimates are consistent with the propane data or with the average value computed from propane and freon experiments 19)

$$R = 1.45 \pm 0.21$$

VI. - CONCLUDING REMARKS

The quark parton model is consistent with the available data for electroproduction and neutrino production but most of its predictions have not yet been tested. In fact the model has been built in order to have the scaling in the Bjorken sense and the magnitude of the cross-section via the pointlike assumption.

The simplest configuration of three quarks is excluded by experiment and we need relatively complicated structures to accommodate the data. In particular, the ad hoc introduction of gluons which are not interacting with the currents is certainly required by the neutrino experiment but the exact role of these gluons is not very well understood from a dynamical point of view.

The neutrino data in their present form are systematically larger than the theoretical prediction from the electroproduction data. But we must keep in mind that the neutron data are only preliminary and in our estimates of upper bounds for the neutrino cross-sections, we have not included the electroproduction errors.

Some predictions for the neutrino and antineutrino cross-sections are given in the Appendix.

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APPENDIX

Bounds for total cross-sections

$$\begin{array}{|l} 0.12 (0.28) \leq A^{\nu p} \leq 0.33 \\ 0.19 (0.50) \leq A^{\nu n} \leq 0.60 \\ 0.12 (0.21) \leq A^{\bar{\nu} p} \leq 0.28 \\ 0.10 (0.11) \leq A^{\bar{\nu} n} \leq (0.22) 0.36 \end{array}$$

$$\begin{array}{|l} 0.32 (0.78) \leq A^{\nu p} + A^{\nu n} \leq 0.92 \\ 0.27 (0.32) \leq A^{\bar{\nu} p} + A^{\bar{\nu} n} \leq (0.50) 0.64 \end{array}$$

$$0.38 \leq \frac{A^{\bar{\nu} p} + A^{\bar{\nu} n}}{A^{\nu p} + A^{\nu n}} \leq (0.64) 1$$

$$\begin{array}{|l} 0.64 (1.13) \leq A^{\nu p} + A^{\nu n} + A^{\bar{\nu} p} + A^{\bar{\nu} n} \leq 1.28 \\ 0 (0.28) \leq A^{\nu p} + A^{\nu n} - A^{\bar{\nu} p} - A^{\bar{\nu} n} \leq 0.57 \end{array}$$

The bounds are computed in the domain D and the numbers in parentheses correspond to the restricted region Δ .

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- 12) In the simplest parton models, $f_N^j(\xi)$ is assumed to be the same
for all types of partons and satisfies the condition (5).
Our assumption is less restrictive and the independence with
respect to j is only for the first moment of the distribu-
tion.
- 13) The conclusion is different if we consider particular $U(3)$ con-
figurations of quarks and antiquarks for the nucleon belonging
to an octuplet $[2,1,0]$ of $U(3)$.
1 - Configuration I : Three quarks in an octuplet $[2,1,0]$ and
the rest of quarks and antiquarks in a singlet $[0,0,0]$

$$\langle \frac{1}{N} \rangle = 3 (\bar{I}_p - \bar{I}_n) \approx 0.12$$

$$E = 1 - \frac{9}{2} \bar{I}_1 \approx 0.46$$

2 - Configuration II : A quark-antiquark pair in an octuplet $[1, 0, -1]$ and the rest of quarks and antiquarks in a singlet $[1, 1, 1]$

$$\langle \frac{1}{N} \rangle = 3(I_p - \bar{I}_n) \approx 0.12 \quad \mathcal{E} = 1 - \frac{9}{2} \bar{I}_n - 3(I_p - \bar{I}_n) \approx 0.34$$

3 - Configuration III : Three antiquarks in an octuplet $[0, -1, -2]$ and the rest of quarks and antiquarks in a singlet $[2, 2, 2]$

$$\langle \frac{1}{N} \rangle = -3(I_p - \bar{I}_n) \approx -0.12 \quad \mathcal{E} = 1 - \frac{9}{2} \bar{I}_n \approx 0.46$$

Clearly a pure configuration III is excluded by experiment $\langle 1/N \rangle$ being computed negative. The effect of mixing of configurations is not absolutely clear to us because the argument of incoherence cannot be used in this specific case. See some discussion in : C.H. Llewellyn Smith - SLCA Pub. 817 (1970).

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- 17) In the integrated form they can be obtained from current algebra in the gluon model. See Refs. 5) and 12).
- 18) I. Budagov et al. - Phys.Letters 30B, 364 (1969).
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- 20) For C^3H^8 we have taken the proportion 26/44 protons and 18/44 neutrons assuming that the probability of obtaining an event is the same on a bound proton or a bound neutron.