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GENERAL LEVEL STRUCTURE OF DUAL RESONANCE MODEL

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A B S T R A C T

The simultaneous diagonalization of the residues of the signaturized and unsignaturized propagators is performed. This is done by means of an orthogonal decomposition of the space of the non-spurious states lying on arbitrary daughter trajectories.

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1. INTRODUCTION

In a recent work ¹⁾ on dual resonance model in operator formalism, it has been shown that it is possible to characterize in a simple way the non-spurious resonance states, and this fact was very useful to investigate the diagonalization of the non-spurious residue operators. For the simultaneous diagonalization of the different residue operators, the results were given for the mother trajectory and the first daughter and only partially for the second daughter. In this article we are presenting a way to obtain simultaneous eigenvectors of the signaturized and unsignaturized residue operators in general cases; we shall see that our method constitutes a considerable improvement over the previous ones ²⁾, as far as the study of the level structure is concerned. For this purpose, we first perform in Section 2 an orthogonal decomposition of the space of the non-spurious states $\tilde{H}_{N,\ell}$ with a given spin ℓ and an eigenvalue N of the operator R . The structure of this space will be studied in some detail. Then, by using these results, we obtain in Section 3 the simultaneous diagonalization of the signature operator $\rho^+ \Omega^+$ and the residue operator β ; we also calculate explicitly the eigenvalues of these operators in general. We are then in a position to discuss, in general, the absence of many levels, due to the signature properties; this will be done in the last section, together with some additional remarks about the sign of the norm of the states and the ghosts states, as well as the abnormal parity states.

2. ANALYSIS OF THE STRUCTURE OF $\tilde{H}_{N,\ell}$

Let us call $H_{N,\ell}$ the space of the states satisfying the two equations

$$R|\psi\rangle = N|\psi\rangle \quad (2.1)$$

$$\vec{J}^2|\psi\rangle = \ell(\ell+1)|\psi\rangle \quad (2.2)$$

where $\vec{J}$ is the angular momentum operator ³⁾. The corresponding space of non-spurious states ¹⁾ at rest, having the mass m_N is ⁵⁾

$$\widetilde{H}_{N,\ell} \equiv \{ |\tilde{\psi}\rangle ; |\tilde{\psi}\rangle = \Omega^+ |\psi\rangle \text{ with } |\psi\rangle \in H_{N,\ell} \}$$

We recall the well-known property of the non-spurious states

$$(A - \alpha_0) |\tilde{\psi}\rangle = 0 \quad \text{i.e.,} \quad \mathcal{P}^+ |\tilde{\psi}\rangle = |\tilde{\psi}\rangle$$

α_0 is the intercept of the mother trajectory and we shall assume in the following that $\alpha_0 \neq \frac{1}{2}$ and $\alpha_0 < 1$.

It turns out to be useful to consider also the states with the same spin and lower masses. For each N' , such that $\ell \leq N' \leq N$, we define a linear subspace of $H_{N',\ell}$

$$H_{N',\ell}^0(m_N) \equiv \{ |\phi\rangle \in H_{N',\ell} \text{ and } L_- |\phi\rangle = 0 \} \quad (2.3)$$

For N' , $\ell \leq N' < N$, we construct a series of linear subspaces as follows

$$H_{N'+j,\ell}^j(m_N) \equiv (L_+)^j H_{N',\ell}^0(m_N) \text{ for } j, 1 \leq j \leq N - N' \quad (2.4)$$

Let us show that $H_{N',\ell}^k$, $k=0, 1, \dots, N'-\ell$ are orthogonal to each other. Let $|\phi_{N'}^k\rangle$ be a vector in $H_{N',\ell}^k(m_N)$. Then, there exists a vector $|\phi_{N',-k}^0\rangle$ in $H_{N',-k,\ell}^0(m_N)$ such that

$$|\phi_{N'}^k\rangle = (L_+)^k |\phi_{N',-k}^0\rangle$$

We now take an arbitrary vector $|\phi_{N'}^{k'}\rangle$ in $H_{N',\ell}^{k'}(m_N)$; then we have ⁶⁾

$$\langle \phi_{N'}^{k'} | \phi_{N'}^k \rangle = \langle \phi_{N'}^{k'} | (L_+)^k \phi_{N',-k}^0 \rangle = \langle (L_-)^k \phi_{N'}^{k'} | \phi_{N',-k}^0 \rangle$$

Since we easily see that

$$(L_-)^k |\phi_{N'}^{k'}\rangle = 0 \quad \text{if} \quad k' < k$$

it follows that

$$\langle \phi_{N'}^{k'} | \phi_{N'}^k \rangle = 0 \quad \text{if} \quad k' < k \quad (2.5)$$

The opposite situation $k' > k$ can be immediately obtained by taking the Hermitian conjugation. This proves the desired orthogonality of $H_{N',l}^k$, $k=0, 1, 2, \dots, N'-l$.

At this point we want to show that $L_+|\phi\rangle$ for any $|\phi\rangle$ ($\neq 0$) belonging to $H_{N',l}$ ⁷⁾. Suppose this is not the case: let us then define

$$K_{N',l} \equiv \{|\phi\rangle \in H_{N',l} \text{ and } L_+|\phi\rangle = 0\}$$

Then, this linear subspace $K_{N',l}$ is readily seen to be orthogonal to $L_-H_{N'+1,l}$. We choose an orthogonal basis $\{\chi_i\}$ in $K_{N',l}$; therefore, $\langle \chi_i | \chi_i \rangle \neq 0$ for any i .

Let us assume that

$$\langle \chi_i | L_+^n L_-^n \chi_i \rangle = c_n \langle \chi_i | \chi_i \rangle \quad \text{for} \quad n \geq 1 \quad (2.6)$$

We know that this is true for $n=0$ with $c_0=1$. Then, since $L_+|\chi_i\rangle=0$ we have

$$\langle \chi_i | L_+^{n-1} L_+ L_-^n \chi_i \rangle = \langle \chi_i | L_+^{n-1} [L_+, L_-^n] \chi_i \rangle \quad (2.7)$$

Now, let us recall that

$$\begin{aligned} [L_+, L_-^m] &= -2 \{ L L_-^{m-1} + L_- L L_-^{m-2} + \dots + L_-^{m-1} L \} \\ &= -2 L_-^{m-1} \left\{ m L - \sum_{i=1}^{m-1} i \right\} \\ &= -2 m L_-^{m-1} \left\{ L - \frac{m-1}{2} \right\} \end{aligned}$$

By inserting this result into (2.7) we obtain

$$\langle \chi_i | L_+ L_-^m | \chi_i \rangle = -2m \left\{ [\alpha_0 - (N - N')] - \frac{m-1}{2} \right\} \langle \chi_i | L_+^{m-1} L_-^{m-1} | \chi_i \rangle$$

from which we deduce immediately that

$$c_m = -2m \left\{ [\alpha_0 - (N - N')] - \frac{m-1}{2} \right\} c_{m-1}$$

Therefore, c_n is zero for the n values of α_0 given by

$$\alpha_0 = N - N' + \frac{n-1}{2}$$

which are all larger than unity since $N > N'$ and $n \geq 1$.

We have shown that $(L_-)^n | \chi_i \rangle \neq 0$ for any n , which leads to an obvious contradiction since the eigenvalues of R are non-negative integers. We thus conclude that the subspace $K_{N', \ell}$ is empty and hence

$$H_{N', \ell} = L_- H_{N'+1, \ell} \quad (2.8)$$

We now want to derive an analogous equation for the linear subspaces $H_{N', \ell}^k$. Let us consider a vector $|\phi\rangle$ in $H_{N'+1, \ell}^{k+1}(\mathfrak{m}_N)$; for an arbitrary vector $|\chi\rangle$ in $H_{N', \ell}^{k'}(\mathfrak{m}_N)$ we have

$$\langle \chi | L_- \phi \rangle = \langle L_+ \chi | \phi \rangle = 0 \quad \text{if} \quad k' \neq k$$

since $L_+ |\chi\rangle$ belongs to $H_{N'+1, \ell}^{k'+1}$. It follows that $L_- |\phi\rangle$ belongs to $H_{N', \ell}^k$ which means that

$$L_- H_{N+1, \ell}^{k+1} (m_N) = H_{N, \ell}^k (m_N)$$

Owing to Eq. (2.8), however, we immediately obtain

$$H_{N, \ell}^k (m_N) = L_- H_{N+1, \ell}^{k+1} (m_N) \quad (2.9)$$

In passing we record here some immediate consequences of our results obtained above:

$$\begin{aligned} (L_+)^j (L_-)^i H_{N, \ell}^k &= H_{N, \ell}^k \quad \text{for } j, 1 \leq j \leq k \\ &= 0 \quad \text{for } j > k \end{aligned} \quad (2.10)$$

We now proceed to a proof of the orthogonality of the non-spurious subspaces $\tilde{H}_{N, \ell}^k$, defined as $\tilde{H}_{N, \ell}^k \equiv \Omega^+ H_{N, \ell}^k$.

Let us consider two vectors $|\tilde{\psi}^k\rangle$ and $|\tilde{\psi}^{k'}\rangle$ ($k' \neq k$) belonging to $\tilde{H}_{N, \ell}^k$ and $\tilde{H}_{N, \ell}^{k'}$ respectively. We then have

$$\begin{aligned} \langle \tilde{\psi}^{k'} | \tilde{\psi}^k \rangle &= \langle \psi^{k'} | \Omega \Omega^+ \psi^k \rangle \\ &= \langle \psi^{k'} | e^{L_+} e^{L_-} \psi^k \rangle \\ &= \langle \psi^{k'} | \sum_{j=0}^{\infty} \frac{1}{(j!)} (L_+)^j (L_-)^j \psi^k \rangle \end{aligned} \quad (2.11)$$

The last expression in the above equation vanishes because of (2.10). This completes our proof.

To summarize we have obtained in a natural way an orthogonal decomposition of the non-spurious space $\tilde{H}_{N, \ell}$ into the linear subspaces $\tilde{H}_{N, \ell}^k$, which are obtained from $H_{N-k, \ell}^0(m_N)$ as follows:

$$\begin{aligned}
 \tilde{H}_{N,l}^k &= \Omega^+ H_{N,l}^k \\
 &= \sum_{j=0}^k \frac{(-1)^{N-j}}{j!} (L_-)^j H_{N,l}^k \\
 &= \sum_{j=0}^k \frac{(-1)^{N-j}}{j!} (L_-)^j (L_+)^k H_{N-k,l}^0(m_N)
 \end{aligned} \tag{2.12}$$

It is evident that we have the relation

$$\dim \tilde{H}_{N,l}^k = \dim H_{N-k,l}^0(m_N)$$

and from (2.9) it follows that

$$\dim H_{N-k,l}^0(m_N) = \dim H_{N-k,l} - \dim H_{N-k-1,l}$$

Therefore, we conclude that

$$\dim \tilde{H}_{N,l}^k = \dim H_{N-k,l} - \dim H_{N-k-1,l} \tag{2.13}$$

By similar arguments to what we have just given, for instance in (2.11), we can easily show that if we choose an orthogonal basis $\{|\phi_{N-k,i}^0\rangle\}$ in $H_{N-k,l}^0$ then $\{|\phi_{N,i}^k\rangle\}$ form an orthogonal basis in $H_{N,l}^k$ where

$$|\phi_{N,i}^k\rangle \equiv (L_+)^k |\phi_{N-k,i}^0\rangle$$

Also, $\{|\tilde{\phi}_{N,i}^k\rangle\}$ form an orthogonal basis in $\tilde{H}_{N,l}^k$, where

$$|\tilde{\phi}_{N,i}^k\rangle \equiv \Omega^+ |\phi_{N,i}^k\rangle$$

The above results show the importance of the subspaces $H_{N',l}^0(m_N)$ which play, in some sense, the role of "generating subspaces". On the contrary $H_{N',l}^k(m_N)$ ($k \neq 0$) may be called "generated subspaces".

3. DIAGONALIZATION OF THE SIGNATURE OPERATOR AND OF THE NON-SPURIOUS RESIDUE OPERATOR

a. The signature operator

Let $|\psi\rangle$ be a vector in $H_{N,\ell}^k$, then we know that there exists a vector $|\phi\rangle$ in $H_{N-k,\ell}^0$ such that

$$|\psi\rangle = (L_+)^k |\phi\rangle \quad (3.1)$$

The corresponding non-spurious state is $|\tilde{\psi}\rangle = \Omega^+ |\psi\rangle$ and we have

$$\begin{aligned} P^+ \Omega^+ |\tilde{\psi}\rangle &= P^+ |\psi\rangle \\ &= P^+ (L_+)^k |\phi\rangle \\ &= P^+ L (L_+)^{k-1} |\phi\rangle \end{aligned}$$

In the last equation we have used the relation ⁴⁾

$$P^+ L = P^+ L_+$$

Since $(L_+)^{k-1} |\phi\rangle$ is an eigenstate of R with eigenvalue $N-1$ we have

$$P^+ \Omega^+ |\tilde{\psi}\rangle = (\alpha_0 - 1) P^+ (L_+)^{k-1} |\phi\rangle$$

By repeating this procedure we find

$$P^+ \Omega^+ |\tilde{\psi}\rangle = d_k P^+ |\phi\rangle \quad (3.2)$$

where

$$d_k = (\alpha_0 - 1)(\alpha_0 - 2) \dots (\alpha_0 - k)$$

Now, we can write $|\tilde{\psi}\rangle$ in the form

$$|\tilde{\psi}\rangle = P^+ |\tilde{\psi}\rangle = (P^+ \Omega^+)^k |\tilde{\psi}\rangle$$

Therefore, from (3.2) we obtain

$$\begin{aligned} |\tilde{\psi}\rangle &= d_k P^+ \Omega^+ P^+ |\phi\rangle \\ &= d_k P^+ \Omega^+ |\phi\rangle \\ &= d_k P^+ (-1)^k e^{L_-} |\phi\rangle \end{aligned}$$

Since $L_- |\phi\rangle = 0$ we have

$$|\tilde{\psi}\rangle = (-1)^{N-k} d_k P^+ |\phi\rangle \quad (3.3)$$

It follows from (3.2) and (3.3) that

$$P^+ \Omega^+ |\tilde{\psi}\rangle = (-1)^{N-k} |\tilde{\psi}\rangle \text{ for any } |\tilde{\psi}\rangle \text{ in } \tilde{H}_{N,k}^r \quad (3.4)$$

which completes our proof.

This result shows the importance of the basis of non-spurious states and the usefulness of our decomposition of $\tilde{H}_{N,l}$ since the eigenvalues of the signature operator $P^+ \Omega^+$ are determined completely without knowing the explicit form of the projection operator P^+ .

b. The residue of the twisted propagator

Now, for the non-spurious twisted propagator we have

$$P^+ \Omega^+ D = D \Omega P = \int_0^1 dx x^{L-\alpha_0-1} (1-x)^{\alpha_0-1} \Omega (1-x)^A$$

and, therefore,

$$P^+ \Omega^+ D |\tilde{\psi}\rangle = \int_0^1 dx x^{L-\alpha_0-1} (1-x)^{\alpha_0-1} \Omega |\tilde{\psi}\rangle$$

By using the relation $\Omega \Omega^+ = e^{L_+} e^{L_-}$ we immediately obtain the residue at $R = \alpha(m_N^2)$

$$\tilde{\beta}^T |\tilde{\psi}\rangle = \left\{ \sum_{k=0}^{N-l} \binom{k-\alpha_0}{k} P_{N-k} \right\} e^{L_+} e^{L_-} |\psi\rangle \quad (3.5)$$

where P_i is the projection operator on the eigenspace of $R=i$ and we use the fact that

$$P_i e^{L_+} e^{L_-} |\psi\rangle = 0 \quad \text{if} \quad i < l$$

Equation (3.5) reads also

$$\tilde{\beta}^T |\tilde{\psi}\rangle = \sum_{k=0}^{N-l} \binom{l-l_0}{k} \sum_{j=0}^{N-l-k} \frac{1}{j!} \frac{1}{(j+k)!} (L_+)^j (L_-)^{j+k} |\psi\rangle \quad (3.6)$$

On the other hand,

$$|\tilde{\psi}\rangle = \Omega^+ |\psi\rangle = \sum_{k=0}^{N-l} (-1)^{N-k} \frac{1}{k!} (L_-)^k |\psi\rangle \quad (3.7)$$

Suppose now that $|\tilde{\psi}\rangle$ is an eigenstate of $\tilde{\beta}^T$ then

$$\tilde{\beta}^T |\tilde{\psi}\rangle = \tilde{\lambda}^T |\tilde{\psi}\rangle \quad (3.8)$$

In order to satisfy (3.8) we see, by comparing the terms with the smallest value of R in (3.6) and (3.7) multiplied by $\tilde{\lambda}^T$, that if $(L_-)^k |\psi\rangle \neq 0$ but $(L_-)^{k+1} |\psi\rangle = 0$, or equivalently if $|\psi\rangle$ belongs to $\sum_{j=0}^k H_{N,l}^j$ then, we must have $\tilde{\lambda}^T = \tilde{\lambda}_k^T$ where

$$\tilde{\lambda}_k^T = (-1)^{N-k} \binom{l-l_0}{k} \quad 0 \leq k \leq N-l \quad (3.9)$$

It will not be difficult to see that the number of eigenvectors with the eigenvalue $\tilde{\lambda}_k^T$ is equal to the dimension of $H_{N,l}^k$ and, hence, to that of $\tilde{H}_{N,l}^k$. Clearly, any eigenvector with the eigenvalue $\tilde{\lambda}_0^T$ belongs to $\tilde{H}_{N,l}^0$. We assumed in the beginning that $\alpha_0 < 1$ and $\alpha_0 \neq \frac{1}{2}$. Then, all the eigenvalues $\tilde{\lambda}_k^T$ are different from each other except when $\alpha_0 = 0$. Since two eigenvectors of the Hermitian operator $\tilde{\beta}^T$ corresponding to different eigenvalues must be orthogonal to each other, it follows that eigenvectors

with eigenvalue $\tilde{\lambda}_1^T$ belong to $\tilde{H}_{N,l}^1$. By repeating this argument, we see for $\alpha_0 \neq 0$ that eigenvectors with the eigenvalue $\tilde{\lambda}_k^T$ belong to $\tilde{H}_{N,l}^k$. For $\alpha_0 = 0$ we have only two eigenvalues $+1$ and -1 . But we can choose a set of eigenvectors so that the above statement remains true for $\alpha_0 = 0$ too.

To summarize, we have

$$\tilde{\beta}^T |\tilde{\psi}\rangle = \tilde{\lambda}_k^T |\tilde{\psi}\rangle \text{ for any } |\tilde{\psi}\rangle \in \tilde{H}_{N,l}^k \quad (3.10)$$

where $0 \leq k \leq N-l$ and $\tilde{\lambda}_k^T$ is given by (3.9).

In terms of the result (3.4) we find that for any $|\tilde{\psi}\rangle$ in $\tilde{H}_{N,l}^k$

$$\tilde{\beta} |\tilde{\psi}\rangle = \tilde{\lambda}_k |\tilde{\psi}\rangle \text{ with } \tilde{\lambda}_k = (-1)^{N-k} \tilde{\lambda}_k^T = \begin{pmatrix} k-2\alpha_0 \\ k \end{pmatrix} \quad (3.11)$$

For $k=0$ we have $\tilde{\lambda}_0 = 1$ regardless the value of α_0 . For $1 \leq k \leq N-l$ we have

$$\begin{aligned} \tilde{\lambda}_k &> 0 \text{ for } \alpha_0 < \frac{1}{2} \\ \tilde{\lambda}_k &< 0 \text{ for } \frac{1}{2} < \alpha_0 < 1 \end{aligned}$$

To sum up we have found that the linear subspaces $\tilde{H}_{N,l}^k$ of the non-spurious space $\tilde{H}_{N,l}$, provide us with simultaneous eigenspaces of the signature operator $\rho^+ \Omega^+$ and the residue operator $\tilde{\beta}$.

4. DISCUSSION

We have seen in Section 3 that the signature operator has the eigenvalues $(-1)^{N-k}$ on the subspace $\tilde{H}_{N,\ell}^k$; so, for $k=0$, namely for the generating spaces, the signaturized residue $\tilde{\beta}^s$ vanishes when N is odd. There exists one state with abnormal parity on the second daughter trajectory except for $\ell=0$. This state belongs to $\tilde{H}_{N,\ell=N-2}^0$ because there is no state with abnormal parity on the first daughter ^{2),8)}. Therefore, no $J^P = (\text{odd})^+$ state on the second daughter trajectory appears in the signaturized propagator. We also note that the signature of the generated spaces $\tilde{H}_{N,\ell}^k$ ($k>0$) is equal to that of the generating space $\tilde{H}_{N-k,\ell}^0(m_N)$ which is the same as that of $\tilde{H}_{N-k,\ell}^0(m_{N-k})$.

In our approach the residue of the scattering amplitude at $p^2 = m_N^2$ where p_μ is the total four-momentum of the intermediate state is written as

$$\sum_n \frac{\langle f | \tilde{\varphi}_n \rangle \langle \tilde{\varphi}_n | \tilde{\beta} \tilde{\varphi}_n \rangle \langle \tilde{\varphi}_n | i \rangle}{\{ \langle \tilde{\varphi}_n | \tilde{\varphi}_n \rangle \}^2}$$

Then, we notice that in the case $|i\rangle \equiv |f\rangle$, the sign of the contribution from $|\tilde{\varphi}_n\rangle$ to the residue is determined by that of the product $\tilde{\lambda}_n \langle \tilde{\varphi}_n | \tilde{\varphi}_n \rangle$. But we know, from our results of Section 3, that $\tilde{\lambda}_n$ is always positive if $\alpha_0 < \frac{1}{2}$. Then a ghost ⁹⁾ will be, in this case, a negative norm state. When $\frac{1}{2} < \alpha_0 < 1$ we still have the same situation if $|\tilde{\varphi}_n\rangle$ belongs to $\tilde{H}_{N,\ell}^0$, but if $|\tilde{\varphi}_n\rangle$ belongs to $\tilde{H}_{N,\ell}^k$ ($k>0$) a ghost corresponds to a positive norm state because $\tilde{\lambda}_n$ is now negative. The situation might look rather complicated; this is not the case however. We denote by $|\tilde{\varphi}\rangle$ one of the $|\tilde{\varphi}_n\rangle$ in $\tilde{H}_{N,\ell}^k$ and we have

$$\langle \tilde{\varphi} | \tilde{\varphi} \rangle = (-1)^k \tilde{\lambda}_k \langle 4 | 4 \rangle$$

where we have used the fact that

$$\tilde{P} \Omega^+ = P^+ \Omega^+ \quad \text{on} \quad \tilde{H}_{N,l}$$

We know that $|\psi\rangle$ can be written as

$$|\psi\rangle = (L_+)^k |\phi\rangle \quad \text{where} \quad |\phi\rangle \in H_{N-k,l}^0(m_N)$$

By simple calculations we obtain

$$\langle \psi | \psi \rangle = (-1)^k g_k \langle \phi | \phi \rangle$$

where

$$g_k = 2^k k! (1 - \alpha_0) \left(\frac{3}{2} - \alpha_0\right) \dots \left(\frac{k+1}{2} - \alpha_0\right)$$

which is always positive for $\alpha_0 < 1$. Therefore,

$$\tilde{g}_k \langle \tilde{\psi} | \tilde{\psi} \rangle = g_k (\tilde{g}_k)^2 \langle \phi | \phi \rangle$$

We thus see that the sign of the contribution from $|\tilde{\psi}\rangle$ to the residue is determined by that of the norm of $|\phi\rangle$. The problem of the sign of the norm in the generating spaces $H_{N-k,l}^0(m_N)$ is an important one which deserves more investigation. At the present, we can say that $\tilde{H}_{N,l}^{N-l}$ and $\tilde{H}_{N,l}^{N-l-1}$ give positive contributions, due to the well-known fact that there is no ghost on the mother and the first daughter trajectories. As a final remark we would like to emphasize that in this work, we have classified non-spurious resonance states into groups of, so to speak, Regge families. The generating states correspond to mothers of these families and the generated ones to their daughters. The dimension of $\tilde{H}_{N,l}^k$ is equal to the numbers of linearly independent Regge families contained in it.

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together with their main properties.
- 4) M. Ida, H. Matsumoto and S. Yazaki, Tokyo preprint (1970).
- 5) The operators $\underline{Q}^+$, $L_{\pm}$, L , as well as the spaces we shall
introduce, are depending upon a four-vector which is taken to be
 $p = (m_N, 0, 0, 0)$; we shall omit this argument unless confusion may
arise.
- 6) The scalar product $\langle | \rangle$ is identical to the one $\langle | \rangle'$ defined
in Ref. 1).
- 7) By using the explicit form of L_+ and going to the limiting case
 $\alpha_0 \rightarrow -\infty$ i.e., $m_N \rightarrow +\infty$ we see that we have a creation operator
 $a_0^{+(1)}$ having a vanishing eigenvalue: this is not possible.
- 8) J.L. Hopkinson and Chan, Hong-Mo, Nuclear Phys. 14B, 28 (1969).
- 9) We only discuss the usual type of ghost with imaginary coupling
constant, but we are perfectly aware that, in addition to it,
another type of ghost, the so-called tachyon, arises when $\alpha_0 > 0$.