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ON THE HELICITY FLIP PROPERTY OF A_2 NN COUPLING

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A B S T R A C T

Using backward and forward dispersion relations for pion photoproduction amplitudes we are able to relate the ratio of s channel helicity non-flip to helicity flip coupling constants of A_2 to nucleons to the electromagnetic vertex $\Delta(1236)N\gamma$ according to

$$\frac{G_{A_2 NN}^{(1)} + G_{A_2 NN}^{(2)}}{G_{A_2 NN}^{(2)}} = \frac{1}{4} \frac{m_{A_2}^2}{m_N^2} \frac{x}{1+x}$$

where x is the ratio of the Gourdin-Salin coupling constants :

$$x \equiv \frac{(C_4 + C_5)m_N}{C_3}$$

Experimentally, x is small (≈ 0.16) such that A_2 predominantly flips the nucleon spin on its mass shell, exhibiting a surprising smoothness of the ratio of flip to non-flip A_2 coupling to nucleons, when continued from the forward direction.

Evidence is piling up in favour of the hypothesis that the more important meson Regge trajectories P, f, ω and ρ, A_2 are coupling to nucleons predominantly to s channel helicity non-flip ¹⁾⁻³⁾ and helicity flip amplitudes ²⁾, respectively. The interesting point about this hypothesis is that the on-mass shell coupling constants of the particles lying on these trajectories appear to possess the same properties, indicating a surprisingly smooth behaviour of the flip to non-flip ratio when continuing t from the forward direction to the meson mass.

Consider πN scattering. There the assumption of absence of P and f trajectory in the helicity flip amplitude has the consequence that both invariant amplitudes $A^{(+)}$ and $B^{(+)}$ obey unsubtracted dispersion relations in the forward direction. Then using the additional information on the values of $A^{(+)}$ and $B^{(+)}$ at threshold supplied by an unsubtracted backward dispersion relation, Engels and Höhler ⁴⁾ have derived the estimates for the coupling of f to nucleons ^{*)}:

$$\frac{G_{fNN}^{(1)2}}{4\pi} = 53 \pm 10 \quad ; \quad \frac{G_{fNN}^{(2)2}}{4\pi} = 3 \pm 7 \quad (1)$$

compatible with pure non-flip amplitude ^{**) .} It is interesting to note that this property allows the second of the gravitational form factors of the nucleons to be dominated by an f meson via an unsubtracted dispersion relation ^{***)}.

*) Note also H. Schaile (Karlsruhe Thesis, 1970) using fixed u dispersion relations, and R. Strauss (Karlsruhe Thesis, 1970) using fixed angle dispersion relations have obtained similar values for these coupling constants, however, they quote larger errors.

**) For f as well as A_2 couplings to protons we use the Lagrangian

$$\mathcal{L} = \left[\frac{G_{fNN}^{(1)}}{m} \frac{i}{4} \bar{\psi} (\vec{\partial}_\mu \gamma_\nu + \vec{\partial}_\nu \gamma_\mu) \psi + \frac{G_{fNN}^{(2)}}{m^2} \partial_\mu \bar{\psi} \partial_\mu \psi \right] f^{\mu\nu}$$

(where $m = m_N$ is used here and in the rest of the paper). Then the helicity flip to non-flip ratio in πN scattering is given for $\nu \rightarrow \infty$ close to the f pole by

$$f_{+-}/f_{++} \approx \frac{\sqrt{-t}}{2m} G_{fNN}^{(2)} / G_{fNN}^{(1+2)}$$

***) If

$$\langle p' | \Theta_{\mu\nu} | p \rangle = \bar{u}(p') \left[(\gamma_\mu p_\nu + \gamma_\nu p_\mu) F_1(t)/4 + p_\mu p_\nu F_2(t)/4m + (q_\mu q_\nu - t g_{\mu\nu}) F_3(t) \right] u(p)$$

mass and spin normalization determine $F_1(0) = 1$ and $F_2(0) = 0$, respectively. f dominance of F_2 gives $F_2(t) \propto G_{fNN}^{(2)}(m_f^2 - t)^{-1}$, hence $G_{fNN}^{(2)} = 0$. Note, however that a similar f dominance assumption for F_1 gives $G_{fNN}^{(1)}$ about 1/3 in magnitude of that given in (1), see Ref. 5).

Similarly, ω does not flip the nucleon spin on its mass shell since $\alpha^s = -0.06$ and flip to non-flip ratio is, for large ν and close to the ω pole,

$$f_{+-}/f_{++} \approx \frac{\sqrt{-t}}{2m} \left(\frac{A+mB}{A+\nu B} \right) \approx -\frac{\sqrt{-t}}{2m} 2\alpha^s$$

For ρ the same argument shows that on-shell ρ mostly flips the nucleon spin (since $2\alpha^v = 3.7$).

No such direct on-shell argument has, until now, been presented for the coupling of A_2 to nucleons. We shall show in this note that, indeed, the A_2 meson couples on-shell predominantly to the helicity-flip amplitude.

Consider the standard CGLN⁶⁾ basis of photoproduction. The amplitudes $(1/s-u)A^{(-)}$ and $(1/s-u)D^{(-)}$ are even functions in $s-u$. They behave for large energy in forward direction according to $s^{\alpha_{A_2(0)}-2}$ and therefore certainly obey unsubtracted dispersion relations (since $\alpha_{A_2(0)} \approx 0.5$). At large energy in backward direction they are dominated by

$$s^{\alpha_{\Delta(0)}-3/2} \approx (-t)^{\alpha_{\Delta(0)}-3/2}$$

Since $\alpha_{\Delta(0)} \approx 0.2$ we can write also here an unsubtracted dispersion relation. Equating both relations at threshold we obtain two sum rules. The nucleon Born term does not contribute to either one of them^{*)}. Due to the strong fall off for large energies these sum rules should saturate extremely quickly and we can be content with inserting only the lowest resonances which can contribute^{**) : Δ in the s channel and A_2 in the t channel. In this way we find from $(1/s-u)A^{(-)}$ and $(1/s-u)D^{(-)}$, respectively^{***),****)} :}

*) Nor does the Roper (or other $J^P = \frac{1}{2}^+$ baryons) contribute.

**) The error arising by neglecting higher meson resonances is hard to estimate ; the error introduced by leaving out higher baryon resonances can be shown to be small. A more complete treatment will be given in a future publication.

***) Here g^* defined by

$$\mathcal{L}_{\Delta N \pi} = \frac{g^*}{m_\pi} \bar{\Delta}_{\mu a} N \partial^\mu \pi_a$$

such that from

$$\Gamma_{\Delta N \pi} = \frac{1}{3} \frac{g^{*2}}{4\pi} p^3 \frac{E^* + m}{M^* m_\pi} : g^{*2}/4\pi \approx 0.37$$

Taking finite width into account, one estimates $g^{*2}/4\pi \approx 0.26$ ⁷⁾. The Gourdin-Salin^{8),9)} coupling constants C_3, C_4, C_5 are defined at $t=0$ by

$$\langle \Delta(p') | j^\mu | N(p) \rangle = \bar{u}_\nu(p') \gamma_5 [C_3 (K g^{\mu\nu} - K^\nu \gamma^\mu) + C_4 (K p^\mu g^{\mu\nu} - K^\nu p^\mu) + C_5 (K p^\mu g^{\mu\nu} - K^\nu p^\mu)] u(p)$$

The $A_2 \pi \sigma$ coupling we use :

$$\langle \pi(q) | j_{\mu\tau}^{A_2} | \sigma(k) \rangle = i e \frac{g_{A_2 \pi \tau}}{2m_{A_2}} [\epsilon_{\mu\nu\lambda\tau} q_\nu p_\tau (E^\sigma K^\lambda - E^\lambda K^\sigma) + (\mu \leftrightarrow \tau)]$$

****) We neglect corrections to the equations of order m_π/m .

$$g^* \frac{m}{m_\pi} [m^2(C_4+C_5)+mC_3] = \frac{3}{2} G_{A_2NN}^{(2)} g_{A_2\pi\pi} \frac{m_\Delta^2}{m_{A_2}^2} \quad (2)$$

$$g^* \frac{m}{m_\pi} m^2(C_4+C_5) = 6 G_{A_2NN}^{(1+2)} g_{A_2\pi\pi} \frac{m_\Delta^2 m^2}{m_{A_2}^4} \quad (3)$$

Therefore, we find for the ratio of flip to non-flip couplings

$$G_{A_2NN}^{(1+2)} / G_{A_2NN}^{(2)} = \frac{1}{4} \frac{m_{A_2}^2}{m^2} \frac{x}{1+x} \approx .44 \frac{x}{1+x} \quad (4)$$

with

$$x \equiv m(C_4+C_5)/C_3 \quad (5)$$

The ratio x can be taken from experiment by relating it to the ratio of electric quadrupole and magnetic dipole amplitudes E_{1+}/M_{1+} of CGLN at the Δ resonance. One finds

$$-E_{1+}/M_{1+} = (1 - \frac{m_\Delta}{m} x) / (\frac{3m_\Delta+m}{m_\Delta-m} - \frac{m_\Delta}{m} x) \approx 6.5(1-1.32x)\% \quad (6)$$

Experimentally, one has

$$-E_{1+}/M_{1+} \approx \begin{cases} 4.6 \% & 10) \\ 3.1 \% & 11) \\ 5.9 \% & 12) \end{cases}$$

and we take $C_3 m \approx 2$ ¹³⁾, giving $x \approx 0.2$ close to the original value of Gourdin and Salin ⁸⁾ of $x \approx 0.16$. This corresponds to ^{*})

^{*}) Note that this corresponds to a ratio $(A'/\nu B)_{t=m_{A_2}^2} \approx -0.1$ for the t channel $I = 1$ on amplitudes of kaon nucleon scattering at the A_2 pole at high energies. This ratio is of the same magnitude but differing in sign to the ratio of the corresponding A_2 meson Regge residue functions at $t = 0$ obtained in Regge fits ¹⁴⁾. We also remark that the same ratio at the $\mathcal{G}$ pole is estimated by

$$(A'/\nu B)_{t=m_\rho^2} \approx \frac{1}{1+2x} \approx 0.2.$$

$$G_{A_2 NN}^{(1+2)} / G_{A_2 NN}^{(2)} \approx 0.07 \quad (7)$$

such that A_2 indeed couples stronger to nucleon s channel helicity flip than to non-flip amplitudes. We would here also like to remark that by applying similar techniques to the amplitudes of Compton scattering on nucleons, one arrives at the same conclusion ¹⁵⁾. For completeness we use Eq. (2) to estimate

$$G_{A_2 NN}^{(2)} g_{A_2 \pi \pi} \approx 20 \quad (8)$$

If we take an estimate on the $A_2 \pi \pi$ coupling coming from vector meson dominance and $A_2 \rightarrow \pi \rho$ decay ^{*)}

$$g_{A_2 \pi \pi}^2 \approx 10.6 \quad (9)$$

we find

$$\frac{G_{A_2 NN}^{(1)2}}{4\pi} \approx \frac{G_{A_2 NN}^{(2)2}}{4\pi} \approx 3 \quad (10)$$

*) We use

$$g_\rho^2/4\pi = 2.5 \quad \text{and} \quad \Gamma_{A_2 \pi \rho} = \frac{4}{5} \frac{p_{cm}^5}{M_{A_2}^4} \frac{g_{A_2 \pi \rho}^2}{4\pi} = 85 \text{ MeV}$$

where we have used the total A_2 width ≈ 100 MeV and branching ratio to $\rho\pi \approx 85\%$ ¹⁶⁾ [note a slightly larger total A_2 width ≈ 125 MeV is obtained from Ref. 17].

$g_{A_2 \pi \rho}$ is defined analogously to $g_{A_2 \pi \pi}$ in footnote **) on p. 2. Equation (9) corresponds to

$$\Gamma_{A_2 \pi \pi} = \frac{2}{5} \alpha \frac{p_{cm}^5}{M_{A_2}^4} g_{A_2 \pi \pi}^2 \approx 0.113 g_{A_2 \pi \pi}^2 \text{ MeV} = 1.2 \text{ MeV}$$

agreeing with "old" estimates using vector meson dominance ¹⁸⁾. This value of the $A_2 \pi \pi$ width is much larger than estimates obtained from pion Compton scattering sum rules ¹⁸⁾, combined with the Cabibbo-Radicati ¹⁹⁾ sum rule; in this way, Harari ¹⁸⁾ estimated $\Gamma_{A_2 \rightarrow \pi \pi} = 0.3 \pm 0.3$ MeV; Singh ¹⁹⁾ estimated $\Gamma_{A_2 \rightarrow \pi \pi} = 0.4$ MeV and Sarker ²⁰⁾ estimated $\Gamma_{A_2 \rightarrow \pi \pi} = 0.3 \pm 0.1$ MeV. Such values of the widths would of course increase the estimates of the coupling constants $(G_{A_2 NN}^{(i)2})/(4\pi)$ in Eq. (10) by a factor ≈ 3 .

In conclusion we see that the simple technique of subtracting forward and backward dispersion relations from each other at threshold provides a powerful tool ^{8),15),21)} for the determination of on-shell coupling constants of particles exchanged in the t channel. We hope that a combination of this method with Regge fits of high energy scattering will supply us with direct information on the structure of Regge residues when continued from the forward direction to the particle poles.

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