ABSTRACT

Assuming the canonical commutation relations between bare and dressed massive vector gauge fields, sum rules are derived and their relevance to elementary particle symmetry schemes is discussed.

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constant gauge transformations and consequently the current \( \mathcal{J}_\mu \) remains conserved. The field equation for \( \mathcal{G}_\nu \) is
\[
\mathcal{C}_\mu \mathcal{G}_{\mu \nu} - m_0^2 \mathcal{G}_\nu - \mathcal{J}_\nu = 0
\]
(3)
from which we obtain the field \( \mathcal{G}_\nu \), which is an explicit function of the independent fields
\[
\mathcal{G}_\nu = \mathcal{C}_\mu \mathcal{G}_{\mu \nu} - \mathcal{J}_\nu \frac{m_0^2}{m_0^2}
\]
and the current \( \mathcal{J}_\nu \) is found to be:
\[
\mathcal{J}_\nu = -i q_0 \chi_\nu \xi \mathcal{G}_\nu + 2 q_0 \mathcal{G}_\nu \left( \mathcal{C}_\mu \mathcal{G}_\mu + 2 q_0 \mathcal{G}_\nu \times \mathcal{G}_\nu \right)
\]
+ 2 q_0 \mathcal{G}_\nu \times \mathcal{G}_\mu \nu
(4)
The momenta conjugate to \( \mathcal{G}_\nu \), \( \mathcal{F}_i \), where superscripts denote isotopic spin indices, are
\[
\Pi_{\mu i} = \mathcal{G}_{\mu i} \quad \Pi_{\sigma i} = 0 \quad \rho_i = \mathcal{C}_\sigma \mathcal{F}_i + 2 q_0 \left( \mathcal{G}_{\sigma i} \times \mathcal{G}_i \right)
\]
(5)
and consequently
\[
\mathcal{J}_\nu = -i q_0 \chi_\nu \xi \mathcal{G}_\nu + 2 q_0 \mathcal{G}_\nu \times \mathcal{F}_\nu
\]
+ 2 q_0 \mathcal{G}_\nu \times \Pi_{\mu i}
(6)
which, as stated before, is the density of the generator of a rotation in isospin space. One can then construct the Hamiltonian density \( :H(x) : \) and obtain
\[
\mathcal{G}_\nu = \left[ \begin{array}{c} \mathcal{G}_\nu \\ :H: \end{array} \right] = \left[ \begin{array}{c} \mathcal{G}_\nu \\ :H: \end{array} \right]
\]
\[
\mathcal{G}_\nu = \mathcal{G}_\sigma \mathcal{G}_\nu - 2 q_0 \mathcal{G}_\sigma \times \mathcal{G}_\nu \]
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We wish to consider a spinor field $\Psi$ of isotopic spin $\frac{1}{2}$ coupled to a pseudoscalar field $\phi_0$ of isotopic spin 1. A theory, invariant under gauge transformations which are functions of both space and time can be constructed by introducing a massless vector field $\vec{\Phi}_\mu$ with isotopic spin one $^1$. Under gauge transformations, the field $\vec{\Phi}_\mu$ undergoes a rotation as well as a translation.

\[
\begin{align*}
\vec{\Phi}_\mu (x) & \rightarrow \vec{\Phi}_\mu (x) - \partial_\mu \vec{\Lambda} (x) + 2 q_0 \vec{\Lambda} (x) \times \vec{\Phi}_\mu (x) \\
N (x) & \rightarrow N (x) - i q_0 \vec{\tau} \cdot \vec{\Lambda} (x) N (x) \\
\vec{\phi} (x) & \rightarrow \vec{\phi} (x) + 2 q_0 \vec{\Lambda} (x) \times \vec{\phi} (x)
\end{align*}
\]

Introducing then

\[
\vec{G}_{\mu \nu} \equiv \partial_\mu \vec{\Phi}_\nu - \partial_\nu \vec{\Phi}_\mu + 2 q_0 \vec{\Phi}_\mu \times \vec{\Phi}_\nu
\]

we can construct a fully invariant Lagrangian $\mathcal{L}$ see Appendix $^7$

\[
\mathcal{L} = - \frac{i}{4} G_{\mu \nu} \cdot G_{\mu \nu} - \bar{N} \left( \gamma^\mu \partial_\mu + m_0 - i q_0 \gamma_5 \vec{\tau} \cdot \vec{\Phi}_\mu \right) N
- \frac{i}{2} \left( \partial_\mu \vec{\phi} + 2 q_0 \vec{\Phi}_\mu \times \vec{\phi} \right) \cdot \left( \partial_\mu \vec{\phi} + 2 q_0 \vec{\Phi}_\mu \times \vec{\phi} \right)
- m_0^2 \phi \cdot \vec{\phi}
\]

From this Lagrangian one obtains a current to which $\vec{\Phi}_\mu$ is coupled, $\vec{J}_\mu$, whose fourth component, integrated over volume, is proportional to the total isotopic spin of the system. If we now introduce a common mass term for the vector mesons

\[
- \frac{m_0^2}{2} \vec{\Phi}_\mu \cdot \vec{\Phi}_\mu
\]

the theory becomes unrenormalizable $^2$ and is no longer invariant under general space-time dependent gauge transformations. It is still invariant, however, under
The sum rules for the pseudoscalar mesons

\[ \delta_{i,j} = \int \omega_{i,j}(m^2) \, d m^2 \quad \delta_{i,j} \mu_{c}^{2} = \int \omega_{i,j}(m^2) \, m^2 \, d m^2 \]  

(11)

can also be obtained easily, \( \omega_{ij} \) being defined by

\[ \langle 0 | \phi_{i}(x) \phi_{j}(y) | 0 \rangle = \int \omega_{i,j}(m^2) \, \Delta_{i,j}(x-y, m^2) \, d m^2 \]  

(12)

We have derived these sum rules for the Yang-Mills field, but they are also valid for suitable generalizations \(^4\) of this field, such as the case of an octet of vector mesons coupled to an octet of pseudoscalar mesons and an octet of baryons, where \( F \) spin now plays the role of \( I \) spin \(^5\).

Up till now we have been discussing bare fields; for renormalized fields the mass sum rules are

\[ \mathcal{Z}^{-1} = \int \rho_{R} \left( m^2 \right) \, d m^2 \quad \mathcal{Z}_{\mu}^{-1} = \int \frac{\rho_{R} \left( m^2 \right)}{M_{0}^{2}} \, d m^2 \]  

(13)

where \( \rho \) is a diagonal matrix with positive definite elements, \( \rho_{R} = \rho \mathcal{Z}^{-1} \), and \( \mathcal{Z}^{-1} = Z^{-1} \mathcal{Z}^{-1}_{\mu} \), \( Z \) being the renormalization constant. Because of (13), having the bare mass \( m_{0}^{2} \to \infty \) is only consistent with \( Z \to 0 \), as was shown by Gell-Mann and Zachariasen \(^6\), for the \( \rho \) and the \( \omega \) mesons by examining form factors to lowest order in \( e \).

If an additional interaction which preserves (7) and (8), and consequently also the sum rules, is introduced, a new spectral function \( \rho_{R}(m^2) \) is obtained, related to \( \rho_{R}(m^2) \) by
We will now derive our sum rules for the vector mesons, following closely K. Johnson's derivation of the analogous rules for a neutral vector meson field coupled to a current with which it commutes. Using the commutation relations we find

$$\langle 0 | \left[ \Phi_\mu^i(x), \Phi_\nu^j(y) \right] | 0 \rangle = \frac{i \delta_{ij}}{\mathcal{M}_0^2} \delta(x^2 - y^2)$$

$$\langle 0 | \left[ \Phi_\mu^i(x), \Phi_\nu^j(y) \right] | 0 \rangle = \frac{i \delta_{ij}}{\mathcal{M}_0^2} \left( \delta(x^2 - y^2) - \frac{\partial \delta(x^2 - y^2)}{\partial x^2} \right)$$

$$\langle 0 | \left[ \Phi_\mu^i(x), \Phi_\nu^j(y) \right] | 0 \rangle = \frac{i \delta_{ij}}{\mathcal{M}_0^2} \left( \delta(x^2 - y^2) - \frac{\partial \delta(x^2 - y^2)}{\partial x^2} \right)$$

Equation (7)

$$\langle 0 | \left[ \Phi_\mu^i(x), \Phi_\nu^j(y) \right] | 0 \rangle = \frac{i \delta_{ij}}{\mathcal{M}_0^2} \left( \delta(x^2 - y^2) - \frac{\partial \delta(x^2 - y^2)}{\partial x^2} \right)$$

Equation (8)

As

$$\langle 0 | \left[ \Phi_\mu^i, \Phi_\nu^j \right] | 0 \rangle = \delta(x^2 - y^2)$$

$$\langle 0 | : \Phi_\mu^i \times \Phi_\nu^j : | 0 \rangle = 0$$

On invariance grounds, taking into account the transversality of the vector meson field, we may write

$$i \langle 0 | \left[ \Phi_\mu^i(x), \Phi_\nu^j(y) \right] | 0 \rangle = \left( \partial^2 \delta_{\mu \nu} - \partial_{\mu} \partial_{\nu} \right) \int \frac{\rho_{ij}(m^2) \Delta(x^2 - y^2) \mathcal{M}^2}{m^2} \, dm^2$$

Equation (9)

As $\mathcal{O}_0 A(x) = -\delta(x^2)$ for $x = 0$, by using Eqs. (7), (8) and (9), we immediately obtain the sum rules

$$\delta_{ij} = \int \rho_{ij}(m^2) \, dm^2$$

$$\frac{\delta_{ij}}{\mathcal{M}_0^2} = \int \frac{\rho_{ij}(m^2) \, dm^2}{\mathcal{M}_0^2}$$

Equation (10)
\[
\int \mathcal{D}_R(\Lambda \xi^t) = \frac{Z'}{Z} \int \mathcal{D}_R(\Lambda \xi^t) \\
\int \mu_R(\Lambda \xi^t) = \frac{Z'}{Z} \int \mu_R(\Lambda \xi^t)
\]

To lowest order in the additional interaction, \(Z/Z' = 1\), and then (14) becomes
the relations which are at the basis of the so-called "vector mixing approximation". 7)

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APPENDIX

The purpose of the two points in the Lagrangian is to give a well-defined meaning to the vacuum expectation values of the field operators and the field equations. For, e.g., a scalar field:

\[
: \phi : = \phi - \langle \phi | \phi \rangle \\
: \phi^2 : = \phi^2 - \langle \phi^2 | \rangle - 2 : \phi : \langle \phi | \phi \rangle
\]

the vacuum expectation value of \( : \phi^n : \) equals zero as does the equal time commutator of \( \Pi \), the momentum conjugate to \( \phi \), with \( : \phi^n : \) for \( n > 1 \). Operations, such as variation with respect to a field, are understood to be performed inside the two points

\[
\frac{\delta}{\delta \phi} : \phi^n : = : \frac{\delta}{\delta \phi} \phi^n :
\]
REFERENCES


7) S. Coleman and H. Schnitzer (to be published).