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SUM RULES FOR MASSIVE VECTOR GAUGE FIELDS

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A B S T R A C T

Assuming the canonical commutation relations between bare and dressed massive vector gauge fields, sum rules are derived and their relevance to elementary particle symmetry schemes is discussed.

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constant gauge transformations and consequently the current $\vec{J}_\mu$ remains conserved. The field equation for $\vec{\varphi}_\nu$ is

$$: \partial_\mu \vec{G}_{\mu\nu} - m_0^2 \vec{\varphi}_\nu - \vec{J}_\nu : = 0 \quad (3)$$

from which we obtain the field $\vec{\varphi}_0$, which is an explicit function of the independent fields

$$\vec{\varphi}_0 = : \frac{\partial_\mu \vec{G}_{\mu 0} - \vec{J}_0}{m_0^2} :$$

and the current $\vec{J}_\nu$ is found to be :

$$\begin{aligned} \vec{J}_\nu = & -i g_0 \bar{N} \gamma_\nu \vec{\tau} N + 2g_0 \vec{\phi} \times (\partial_\mu \vec{\phi} + 2g_0 \vec{\varphi}_\mu \times \vec{\phi}) \\ & + 2g_0 \vec{\varphi}_\mu \times \vec{G}_{\mu\nu} \end{aligned} \quad (4)$$

The momenta conjugate to φ_μ^i , ϕ^i , where superscripts denote isotopic spin indices, are

$$\begin{aligned} \pi_\mu^i &= G_{\mu 0}^i \quad \pi_0^i = 0 \quad \rho_i = \partial_0 \phi_i + 2g_0 (\vec{\varphi}_0 \times \vec{\phi})^i \\ [\varphi_k^i(x), \pi_\ell^j(y)]_{x_0=y_0} &= i \delta_{ij} \delta_{kl} \delta(\vec{x} - \vec{y}) \end{aligned} \quad (5)$$

and consequently

$$\begin{aligned} \vec{J}_0 = & -i g_0 \bar{N} \gamma_0 \vec{\tau} N + 2g_0 \vec{\phi} \times \vec{\rho} \\ & + 2g_0 \vec{\varphi}_\mu \times \vec{\pi}_\mu \end{aligned} \quad (6)$$

which, as stated before, is the density of the generator of a rotation in isospin space. One can then construct the Hamiltonian density $:H(x):$ and obtain

$$\begin{aligned} \dot{\vec{\varphi}}_k &= [\vec{\varphi}_k, :H:] = : [\vec{\varphi}_k, H] : \\ &= : \vec{G}_{0k} + \partial_k \vec{\varphi}_0 - 2g_0 \vec{\varphi}_0 \times \vec{\varphi}_k : \end{aligned}$$

We wish to consider a spinor field N of isotopic spin $\frac{1}{2}$ coupled to a pseudoscalar field $\vec{\phi}$ of isotopic spin 1. A theory, invariant under gauge transformations which are functions of both space and time can be constructed by introducing a massless vector field $\vec{\varphi}_\mu$ with isotopic spin one ¹⁾. Under gauge transformations, the field $\vec{\varphi}_\mu$ undergoes a rotation as well as a translation.

$$\begin{aligned}\vec{\varphi}_\mu(x) &\rightarrow \vec{\varphi}_\mu(x) - \partial_\mu \vec{\Lambda}(x) + 2g_0 \vec{\Lambda}(x) \times \vec{\varphi}_\mu(x) \\ N(x) &\rightarrow N(x) - i g_0 \vec{\tau} \cdot \vec{\Lambda}(x) N(x) \\ \vec{\phi}(x) &\rightarrow \vec{\phi}(x) + 2g_0 \vec{\Lambda}(x) \times \vec{\phi}(x)\end{aligned}\tag{1}$$

Introducing then

$$\vec{G}_{\mu\nu} \equiv \partial_\mu \vec{\varphi}_\nu - \partial_\nu \vec{\varphi}_\mu + 2g_0 \vec{\varphi}_\mu \times \vec{\varphi}_\nu$$

we can construct a fully invariant Lagrangian [see Appendix]

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4} G_{\mu\nu} \cdot G_{\mu\nu} - \bar{N} (\gamma_\mu \partial_\mu + M_0 - i g_0 \gamma_\mu \vec{\tau} \cdot \vec{\varphi}_\mu) N \\ & - \frac{1}{2} (\partial_\mu \vec{\phi} + 2g_0 \vec{\varphi}_\mu \times \vec{\phi}) \cdot (\partial_\mu \vec{\phi} + 2g_0 \vec{\varphi}_\mu \times \vec{\phi}) \\ & - \frac{M_0^2}{2} \vec{\phi} \cdot \vec{\phi}\end{aligned}\tag{2}$$

From this Lagrangian one obtains a current to which $\vec{\varphi}_\mu$ is coupled, $\vec{j}_\mu$, whose fourth component, integrated over volume, is proportional to the total isotopic spin of the system. If we now introduce a common mass term for the vector mesons

$$-\frac{M_0^2}{2} \vec{\varphi}_\mu \cdot \vec{\varphi}_\mu$$

the theory becomes unrenormalizable ²⁾ and is no longer invariant under general space-time dependent gauge transformations. It is still invariant, however, under

The sum rules for the pseudoscalar mesons

$$\delta_{ij} = \int \omega_{ij}(m^2) dm^2 \quad \delta_{ij} \mu_0^2 = \int \omega_{ij}(m^2) m^2 dm^2 \quad (11)$$

can also be obtained easily, ω_{ij} being defined by

$$\langle 0 | \phi_i(x) \phi_j(y) | 0 \rangle = \int \omega_{ij}(m^2) \Delta^{(+)}(x-y, m^2) dm^2 \quad (12)$$

We have derived these sum rules for the Yang-Mills field, but they are also valid for suitable generalizations⁴⁾ of this field, such as the case of an octet of vector mesons coupled to an octet of pseudoscalar mesons and an octet of baryons, where F spin now plays the role of I spin⁵⁾.

Up till now we have been discussing bare fields; for renormalized fields the mass sum rules are

$$\underline{Z}^{-1} = \int \underline{\rho}_R(m^2) dm^2 \quad \frac{\underline{Z}^{-1}}{m_0^2} = \int \frac{\underline{\rho}_R(m^2)}{m^2} dm^2 \quad (13)$$

where $\underline{\rho}$ is a diagonal matrix with positive definite elements, $\rho_R = \rho Z^{-1}$, and $\underline{Z}^{-1} = Z^{-1} 1$, Z being the renormalization constant. Because of (13), having the bare mass $m_0^2 \rightarrow \infty$ is only consistent with $Z \rightarrow 0$, as was shown by Gell-Mann and Zachariasen⁶⁾, for the ρ and the ω mesons by examining form factors to lowest order in e .

If an additional interaction which preserves (7) and (8), and consequently also the sum rules, is introduced, a new spectral function $\underline{\rho}_R(m^2)$ is obtained, related to $\underline{\rho}_R(m^2)$ by

We will now derive our sum rules for the vector mesons, following closely K. Johnson's ³⁾ derivation of the analogous rules for a neutral vector meson field coupled to a current with which it commutes. Using the commutation relations we find

$$\begin{aligned} \langle 0 | [\varphi_0^i(x), \varphi_K^j(y)] | 0 \rangle_{x_0=y_0} &= \frac{-i \delta_{ij} \partial_K \delta(\vec{x}-\vec{y}) - \langle 0 | [: \vec{J}_0^i : , \varphi_K^j] | 0 \rangle}{M_0^2} \\ &= -\frac{i}{M_0^2} \delta_{ij} \partial_K \delta(\vec{x}-\vec{y}) \end{aligned} \quad (7)$$

$$\begin{aligned} \langle 0 | [\varphi_K^i(x), \dot{\varphi}_\pi^j(y)] | 0 \rangle_{x_0=y_0} &= i \delta_{ij} \left(\delta_{K\pi} - \frac{\partial_K \partial_\pi}{M_0^2} \right) \delta(\vec{x}-\vec{y}) \\ &\quad + \langle 0 | [\varphi_K^i(x), -\partial_\pi : \vec{J}_0^j : - 2g_0 : (\vec{\varphi}_0 \times \vec{\varphi}_\pi)^j] | 0 \rangle_{(8)} \\ &= i \delta_{ij} \left(\delta_{K\pi} - \frac{\partial_K \partial_\pi}{M_0^2} \right) \delta(\vec{x}-\vec{y}) \end{aligned}$$

as

$$\begin{aligned} \langle 0 | [\varphi_K^i, : (\vec{\varphi}_\pi \times \vec{\pi}_\pi \times \vec{\varphi}_\pi)^j] | 0 \rangle &= (\dots) \delta(\vec{x}-\vec{y}) \langle 0 | : \varphi_K \varphi_\pi : | 0 \rangle \\ &= 0 \end{aligned}$$

On invariance grounds, taking into account the transversality of the vector meson field, we may write

$$i \langle 0 | [\varphi_\mu^i(x), \varphi_\nu^j(y)] | 0 \rangle = (\Box^2 \delta_{\mu\nu} - \partial_\mu \partial_\nu) \int \frac{\rho_{ij}(m^2) \Delta(x-y; m^2)}{m^2} dm^2 \quad (9)$$

As $\partial_0 \Delta(x) = -\delta(\vec{x})$ for $x_0 = 0$, by using Eqs. (7), (8) and (9), we immediately obtain the sum rules

$$\delta_{ij} = \int \rho_{ij}(m^2) dm^2 \quad \frac{\delta_{ij}}{M_0^2} = \int \frac{\rho_{ij}(m^2)}{m^2} dm^2 \quad (10)$$

$$\int \rho_R(m^2) = \frac{Z'^{-1}}{Z^{-1}} \int \rho'_R(m^2) \quad \int \frac{\rho_R(m^2)}{m^4} = \frac{Z'^{-1}}{Z^{-1}} \int \frac{\rho'_R(m^2)}{m^4} \quad (14)$$

to lowest order in the additional interaction, $Z/Z' = 1$, and then (14) becomes the relations which are at the basis of the so-called "vector mixing approximation" ⁷⁾.

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A P P E N D I X

The purpose of the two points in the Lagrangian is to give a well-defined meaning to the vacuum expectation values of the field operators and the field equations. For, e.g., a scalar field :

$$:\phi: = \phi - \langle 0|\phi|0\rangle$$

$$:\phi^2: = \phi^2 - \langle 0|\phi^2|0\rangle - 2:\phi:\langle 0|\phi|0\rangle$$

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the vacuum expectation value of $:\phi^n:$ equals zero as does the equal time commutator of π , the momentum conjugate to ϕ , with $:\phi^n:$ for $n > 1$. Operations, such as variation with respect to a field, are understood to be performed inside the two points

$$\frac{\delta}{\delta\phi} : \phi^n : = : \frac{\delta}{\delta\phi} \phi^n :$$

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