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ON THE INVESTIGATION OF NUCLEON-NUCLEON CORRELATIONS
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A B S T R A C T

The possible effects of short range dynamical nucleon-nucleon correlations on high energy hadron scattering on ^4He are examined. The Glauber theory of multiple scattering is used as the basis for the computations. The conclusion is that very small effects are to be expected for elastic and sum total inelastic scattering of commonly available projectiles.

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Some of de Shalit's last published works ¹⁾ were devoted to investigating the effects of short range nucleon-nucleon correlations in nuclei. As both, he and Brown, in accompanying papers ²⁾ stressed, the disentangling of such effects from experiment is fraught with difficulty. In the present note we should like to illustrate this point with explicit calculations of the expected influence of short range correlations on the high energy scattering of particles from nuclei. One of the problems in interpreting any such experiment is that the desired information may be masked by the Pauli and centre-of-mass correlations (which everybody believes in and hence are uninteresting) as well as nuclear deformations and the long range Coulomb correlation. These complications are considerably diminished if we look at the special case of ⁴He. Since, to a good approximation, the nucleons in the alpha particle can be taken as distinguishable ³⁾, the Pauli principle plays only a minor role, influencing the small spin-flip or charge exchange scattering terms. The consequence of Coulomb correlations is also unimportant because of the small size and charge of the alpha particle; the spin zero closed shell structure makes strong deformations unlikely. Furthermore, the small number of constituents makes a proper inclusion of the centre-of-mass constraint feasible.

It is clear that in order to investigate correlations, we must look at expectation values of two-body operators. The charge on the alpha particle being small, high energy elastic electron scattering from it may be considered as due to just one photon exchange. Thus, the information content of such cross-sections is simply a form factor for momentum transfer $\vec{q}$,

$$\rho^{(1)}(\vec{q}) = \langle 0 | e^{i\vec{q} \cdot \vec{r}} | 0 \rangle \quad (1)$$

the expectation value of a one-body operator.

It is a very old remark ⁴⁾ that inelastic scattering might contain measurable information about correlations. The cross-section for the excitation of a final nuclear state f , without pion production, depends again only on a one-body operator:

$$\left(\frac{d\sigma}{d\Omega}\right)_{0 \rightarrow f} \propto \left| \sum_i \langle f | e^{i\vec{q} \cdot \vec{r}_i} | 0 \rangle \right|^2 \quad (2)$$

However, if we sum over all the final nuclear states (including the ground state) then:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{total}} \propto \sum_f \sum_{i,j} \langle 0 | e^{-i\vec{q} \cdot \vec{r}_j} | f \rangle \langle f | e^{i\vec{q} \cdot \vec{r}_i} | 0 \rangle \quad (3)$$

For a high energy electron and not too large q^2 , it is perhaps reasonable to forget that there is insufficient energy to excite some of the final states f and make use of closure

$$\sum_f |f\rangle \langle f| = I \quad (4)$$

to eliminate the explicit dependence on the final states. The resulting expression

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{total}} \propto \sum_{i,j} \langle 0 | e^{i\vec{q} \cdot (\vec{r}_i - \vec{r}_j)} | 0 \rangle \quad (5)$$

for the total nuclear cross-section (i.e., elastic plus break-up) is proportional to the expectation value in the ground state of a two-body operator and can therefore lead to correlation phenomena.

Performing the sums over i and j ,

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{total}} \propto [Z + Z(Z-1) \langle 0 | e^{i\vec{q} \cdot (\vec{r}_1 - \vec{r}_2)} | 0 \rangle] \quad (6)$$

Clearly, in the absence of any correlations

$$\rho^{(2)}(\vec{q}, -\vec{q}) \equiv \langle 0 | e^{i\vec{q} \cdot (\vec{r}_1 - \vec{r}_2)} | 0 \rangle = (\rho^{(1)}(\vec{q}))^2 \quad (7)$$

The correlation function $C(q)$ is therefore defined by

$$C(\vec{q}) \equiv \rho^{(2)}(\vec{q}, -\vec{q}) - (\rho^{(1)}(\vec{q}))^2 \quad (8)$$

In terms of C the cross-section becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{total}} \propto Z \left[1 + (Z-1)(\rho^{(1)}(q))^2 + (Z-1)C(q) \right] \quad (9)$$

Given that $\rho^{(1)}(q)$ is known from elastic electron scattering, in principle we can deduce C from the summed cross-section. It should be noted that even in the absence of dynamical correlations, C will be non-zero because of the centre-of-mass constraint.

To obtain an order of magnitude estimate for the effect, we have taken two specific configuration space models of ${}^4\text{He}$. Nuclear density A is constructed to have short range nucleon-nucleon repulsion as well as the centre-of-mass correlation. The free parameters in it are chosen so as to reproduce reasonably well the measured elastic form factor ⁵⁾. A second density B , with only the centre-of-mass correlation, is adjusted so as to reproduce as accurately as possible the form factor predicted by the first density. For simplicity of calculation, we took for the first density a superposition of Gaussians

$$\rho_A = N_A \prod_{i=1}^4 [e^{-a_A \vec{r}_i^2} \prod_{j>i} (1 - b_A e^{-d_A \vec{r}_{ij}^2})] \delta(\frac{1}{4} \sum_{i=1}^4 \vec{r}_i) \quad (10)$$

The appropriate Fourier transforms can then be done analytically, the computer being required merely for bookkeeping. Similarly, for the second density

$$\rho_B = N_B \prod_{i=1}^4 [e^{-a_B \vec{r}_i^2} (1 - b_B e^{-d_B \vec{r}_i^2})] \delta(\frac{1}{4} \sum_{i=1}^4 \vec{r}_i) \quad (11)$$

In units ^{*)} of $a_A = 1.0$, the best agreement with the measured form factor ⁵⁾ is obtained with $b_A = 1.0$, $d_A = 0.6$. The B parameters were then adjusted to give a form factor which is essentially identical up to quite large momentum transfers (Fig. 1). This gave $a_A = 1.086$, $b_B = 0.949$ and $d_B = 0.158$.

The correlation functions associated with these densities can also be calculated analytically (Fig. 2). Although the $C(q)$ thus derived have a maximum value of about 13%, a large part of this is due to the centre-of-mass effect and when the form factors $g^{(1)}(q)$ are so accurately matched the difference between C_A and C_B never exceeds about 3%. To distinguish between the two cases, the total cross-section [Eq. (9)] must be measured to much better than this 3%. There seems little prospect of measuring the break-up cross-section to anything approaching this accuracy; the bremsstrahlung corrections alone give an uncertainty of about 5%.

Because of their larger mass, bremsstrahlung effects are much less for pions. Furthermore, if the pion-proton and pion-neutron amplitudes were equal and relatively weak, then the total nuclear cross-section would be given by Eq. (9), but with Z replaced by A . Thus, for the alpha particle the 3% is enhanced to $\sim 10\%$. The difficulty is that in real life the pion-nucleon interaction is sufficiently strong for absorption effects to be important. To get an estimate of the importance of these effects, we shall calculate the high energy scattering of a hadron from the alpha particle by means of the Glauber multiple scattering model ⁶⁾. If we neglect spin then this predicts the elastic hadron-alpha cross-section

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \left| \frac{ik}{2\pi} \int e^{i\vec{q} \cdot \vec{b}} \langle 0 | 1 - \prod_{j=1}^4 \{1 - f_j(\vec{b} - \vec{s}_j)\} | 0 \rangle d^2b \right|^2 \quad (12)$$

and, using closure, the elastic plus break-up

^{*)} Throughout this exercise we shall use these units. Experimentally $a_A \approx 0.96 \text{ f}^{-2}$.

$$\left(\frac{d\sigma}{d\Omega}\right)_{total} = \frac{k^2}{4\pi^2} \int e^{i\vec{q} \cdot (\vec{b} - \vec{b}')} \langle 0 | \left[\prod_{j=1}^4 (1 - \Gamma_j(\vec{b} - \vec{z}_j)) \right] \left[\prod_{i=1}^4 (1 - \Gamma_i^*(\vec{b}' - \vec{z}_i)) \right] | 0 \rangle d^2b d^2b' \quad (13)$$

where the integrations over the impact parameters b are perpendicular to the beam momentum k and the $\vec{s}_i$ are the components of the nucleon co-ordinates $\vec{r}_i$ in this plane. The profile functions Γ are Fourier transforms of the hadron-nucleon amplitudes

$$\Gamma(b) \equiv \frac{1}{2\pi i k} \int e^{-i\vec{q} \cdot \vec{b}} f(q) d^2q \quad (14)$$

Having neglected spin and isospin, we now parametrize the high energy amplitude f by

$$f(q) = \frac{ik\sigma}{4\pi} (1 - i\alpha) e^{-q^2/4c} \quad (15)$$

for which σ is the hadron-nucleon total cross-section, α is the relative real part (assumed to be angle independent) and $1/c$ is a measure of the "size" of the nucleon. It follows that

$$\Gamma(b) = \frac{c\sigma}{2\pi} (1 - i\alpha) e^{-cb^2} \quad (16)$$

Keeping the lowest order terms in Γ in Eqs. (12) and (13) leads to results equivalent to the Born approximation that we discussed earlier. The expansion parameter is of the order of $\sigma ac/(a+c)$.

With the explicit alpha-particle densities (10) and (11) the elastic and total nuclear cross-sections were computed for a range of parameters c , σ and α . From this we extracted five quantities which typify these two cross-sections; the initial logarithmic slope of the elastic cross-section with respect to q^2 , the

position of the first minimum of the elastic cross-section, the value of the elastic cross-section at the first diffraction maximum, the value of the inelastic cross-section at its maximum and the value of the inelastic cross-section at the position of the first diffraction maximum in the elastic cross-section.

Nature does not allow us to choose c and σ at will; in Fig. 3 are plotted the ranges of these parameters for possible high-energy projectiles. Unitarity limits one to $\sigma c < 16\pi$, but the expansion in powers of Γ_j i.e., the Glauber approximation we would expect not to be valid unless $\sigma c < 8(a+c)/(3a)$ indicated in Fig. 3. Now, for small σ , we can expect no differences for the elastic scattering because single scattering dominates and the form factors are constructed to be similar. The inelastic scattering maximum will differ by $\sim 10\%$. Furthermore, towards the right in Fig. 3 σ becomes so large that each nucleon becomes black and the probing particle rarely penetrates to the nuclear centre where short-range correlations are important. So, in this limit, we should also expect very small effects. Finally, in the limit of small c or large interaction ranges (as predicted at high energy by Regge models) one measures only average nuclear properties and so again one loses sensitivity. The width of a tram rail is measured much more accurately with a bicycle rather than a car tyre.

The detailed numerical analysis in the interesting region of the σ, c plane shows that nothing important is to be expected from the differences of the initial slopes and the positions of the first minimum in elastic scattering. Since the differences in the break-up cross-section do not change very fast as a function of angle, we present results only on the differences in height of the second diffraction maximum (Fig. 4) and the height of the break-up maximum (Fig. 5). In all cases the predicted effects are rather small - the inelastic differences being always smaller than or of the same order as those predicted by the Born approximation. Pushing the nucleons apart reduces the interference terms from the scattering off different nucleons but also reduces the absorptive effects and the two contributions tend to cancel. The biggest effect for the elastic maximum would

seem to be for relatively low energy K^+ scattering, but even then the difference is only of the order of 5%.

These calculations were carried out with zero real part in the hadron-nucleon amplitude. The introduction of $\alpha \neq 0$ tends to reduce any differences between the predictions of densities A and B.

The questions one must ask now are can one do such experiments to this level of accuracy (say 2%) and if one did, would the theory be sufficiently reliable. Our answers are probably not and certainly not. Any differential cross-section is hard to measure to an absolute accuracy of 2%, although some of the uncertainties can be removed by making measurements relative to the cross-sections on hydrogen. Even so it seems unlikely that the height of the second diffraction maximum in the elastic scattering ($d\sigma/d\Omega \sim 1$ mb/sr) could be obtained to better than 5-10%. To obtain the break-up cross-section, one must sum over all the energy loss spectrum of the fast hadron but exclude any of the pions produced in the hadron-nucleon collisions. This is much more troublesome than in the case of electron scattering because pions can be produced with much lower energy losses via a double scattering of the hadron on two nucleons. The resultant error in the sum rule depends upon q^2 but is probably larger than any differences predicted from densities A and B. It is interesting to note that even if one could introduce extra counters to veto pion production, one would get the wrong answer. Pions can be produced in final state interactions among the target nucleons and this part of the cross-section must be included in the sum rule. Nucleons by themselves do not form a complete set of states for the nucleus, we must include nucleons plus pions if the energy is sufficient to excite them.

Assuming that the theory is completely accurate, one needs to feed in hadron-nucleon amplitudes with sufficient precision. For pions below about 2 GeV the phase shift analyses give both the spin-flip amplitudes for scattering from protons and neutrons, but even then the phase shifts do not always reproduce the cross-sections to 2%.

Above this energy one has to resort to models, such as the Regge model with the addition of finite energy sum rules, to parametrise the amplitudes with unknown uncertainties. Particularly troublesome is the fact that at high energies, where the unitarity relation becomes less of a constraint, the phase variation with angle of the amplitude becomes less certain. For K mesons the situation is much worse, the information on scattering by neutrons being very sparse. The uncertainties are compounded for protons or antiprotons where one needs, in principle, to know ten complex amplitudes.

The basic scattering theory used here is derived in the context of scattering from fixed potentials, real life being incorporated by folding in initial and final wave functions, which is a further assumption ⁷⁾. There are slight inconsistencies in the theory away from the forward direction and these tend to increase with scattering angle. Even within the context of the model there are corrections to be applied for inelastic scattering. An incident π^- may charge exchange on one target nucleon and then undergo the reverse process on the next. This may easily be taken account of, but a similar process where a ρ meson is in the intermediate state is much harder. Fortunately, such excitations only produce a few percent effect below about 3 GeV. Although relativistic kinematics is used, many relativistic effects are neglected. The results in the presence of strong spin dependent terms seem to depend somewhat upon the reference frame. This is not to belittle the great success of the theory ⁸⁾, but in our case it is by no means demonstrated that it is accurate enough to differentiate between the densities A and B. Of course, it is possible that there are short range correlations which are much stronger than those that are required to adjust model A to the elastic ^4He form factor but a more positive indication of these would be required from other evidence.

In conclusion we believe that in the present state of theory and experiment elastic and break-up reactions of hadrons with the alpha particle will not provide information on short range correlations unless they are very strong. Such a negative result will probably not be affected by choices of more realistic densities than A and B.

R E F E R E N C E S

- 1) A. de Shalit, Comments on Nuclear and Particle Physics 3, 42 and 88 (1969).
- 2) G.E. Brown, Comments on Nuclear and Particle Physics 3, 48 and 78 (1969).
- 3) J.M. Blatt and V.F. Weisskopf "Theoretical Nuclear Physics", p.202, Wiley, New York (1952).
- 4) For a review see W. Czyz, M.I.T. Summer study on electron linear accelerators (1967).
- 5) R.F. Frosch et al., Phys.Rev. 160, 874 (1967).
- 6) R.J. Glauber, in Lectures in Theoretical Physics, ed., W.E. Brittin Interscience, N.Y. (1959), Vol. 1.
- 7) L. Foldy and J.D. Walecka, Ann.Phys. 54, 447 (1969).
- 8) For a review of the many successes see R.J. Glauber, Proc. of Columbia Conference (1969) (to be published).

FIGURE CAPTIONS

- Figure 1 : Form factors $(\rho^{(1)}(q))^2$ as computed from the densities of Eqs. (10) and (11) with the parameters mentioned in the text.
- Figure 2 : Correlation functions $|C(q)|$ as computed from the densities of Eqs. (10) and (11) with the parameters mentioned in the text. C_B is positive where indicated by (+). The curve A-B shows $|C_A - C_B|$.
- Figure 3 : The σ, c plane, hatched area is the region covered by computations. Rough indications of actual σ, c possibilities for mesons and nucleons as projectiles are shown.
- Figure 4 : Percentage difference in height of elastic scattering maximum versus σ, c .
- Figure 5 : Percentage difference in height of inelastic scattering maximum versus σ, c .

FIG.1

FIG. 2

FIG 3

FIG.4

56F