Test of Special Relativity and Equivalence principle from K Physics

T. Hambye\(^{(a)}\)\(^1\), R.B. Mann\(^{(b)}\)\(^2\) and U. Sarkar\(^{(c)}\)\(^3\)

(a) Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany
(b) Physics Department, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1
(c)Theory Group, Physical Research Laboratory, Ahmedabad, 380 009, India

Abstract

A violation of Local Lorentz Invariance (VLI) and hence the special theory of relativity or a violation of equivalence principle (VEP) in the Kaon system can, in principle, induce oscillations between \(K^0\) and \(\bar{K}^0\). We construct a general formulation in which simultaneous pairwise diagonalization of mass, momentum, weak or gravitational eigenstates is not assumed. We discuss this problem in a general way and point out that, as expected, the VEP and VLI contributions are indistinguishable. We then insist on the fact that VEP or VLI can occur even when CPT is conserved. A possible \(CP\) violation of the superweak type induced by VEP or VLI is introduced and discussed. We show that the general VEP mechanism (or the VLI mechanism, but not both simultaneously), with or without conserved CPT, could be clearly tested experimentally through the energy dependence of the \(K_L - K_S\) mass difference and of \(\eta^{++}, \eta^{00}, \delta\). Constraints imposed by present experiments are calculated.

\(^1\)email: hambye@hal1.physik.uni-dortmund.de
\(^2\)email: mann@avatar.uwaterloo.ca
\(^3\)email: utpal@prl.ernet.in
1 Introduction.

A few of the basic building blocks of particle physics are the assumptions that nature preserves local Lorentz invariance and hence the special theory of relativity, the product of the discrete symmetries CPT and the equivalence principle. It is also true that to date we have not seen any violation of any of these laws. In recent times many new attempts have been made to obtain new and quantifiable information on the degree of validity of these basic laws. It is in this connection that we plan to investigate the Kaon–system.

Many experiments have tested the special theory of relativity to a high degree of precision \([1]\). These experiments probe for any dependence of the (non-gravitational) laws of physics on a laboratory’s position, orientation or velocity relative to some preferred frame of reference, such as the frame in which the cosmic microwave background is isotropic. Failure to observe such dependence further enhances the validity of (respectively) Local Position Invariance and Local Lorentz Invariance (LLI), and hence of the Einstein Equivalence Principle (EEP) \([2]\). However, these empirical results have been obtained primarily in the baryon-photon sector of the standard model. There is no logically necessary reason to conclude from these results that the special theory of relativity must be valid in all sectors of the standard model of elementary particle physics. Its validity must be empirically checked for each sector (gauge boson, neutrino, massive lepton, etc.) separately \([3]\).

A characteristic feature of LLI violation (VLI) is that every species of matter has its own maximum attainable speed. This yields several novel effects in various sectors of the standard model \([3]\), including vacuum Cerenkov radiation \([4]\), photon decay \([5]\) and neutrino oscillations \([6, 7, 8]\). Recently we extended these arguments and pointed out that violation of special relativity will in general induce an energy dependent \(K_L - K_S\) mass difference \([9]\); an empirical search for such effects can therefore be used to obtain bounds on VLI in the kaon sector of the standard model. As we shall discuss later VLI in the kaon sector can occur in a manner that may or may not violate CPT.

The EEP implies universality of gravitational coupling for all forms of mass-energy, thereby ensuring that spacetime is described by a unique operational geometry. An extreme converse of this principle is that every form of stress-energy couples to its own metric, so that the Lagrangian for the standard model is modified to be one of the form

\[
\mathcal{L} = \mathcal{L}_G(g_I) + \sum_I \mathcal{L}_M(g_I, \Phi_I) + \mathcal{L}_C
\]  

(1)

where each matter field \(\Phi_I\) couples to its own metric \(g_{\mu\nu}^I\). The gravitational Lagrangian density \(\mathcal{L}_G\) describes the behaviour of all of these metrics in the absence of any matter fields. The Lagrangian density \(\mathcal{L}_C\) describes the interaction between the different matter fields; it will in general include at least some subset of the metric fields \(g_{\mu\nu}^I\). Although such a Lagrangian is generally covariant, spacetime no longer has a unique operational geometry, since clocks and measuring rods constructed out of different types of matter fields will in general yield different results for a given set of experiments that depend on the choice of coordinate frame. Furthermore, while it is possible for any given metric \(g_{\mu\nu}^I\) to interpret a diffeomorphism of the manifold as a gauge transformation of the linearized tensor \(h_{\mu\nu}^I = g_{\mu\nu}^I - \bar{g}_{\mu\nu}\) where \(\bar{g}_{\mu\nu}\) is some reference metric (typically chosen to be a flat metric), this cannot be done simultaneously for all the metrics (unless they are all the same). This means
that the spin modes of all the other metrics will in general be excited. It is then a theoretical
challenge to ensure that the excitations of the additional degrees of freedom of the other
metric do not yield unacceptable pathologies such as runaway negative energy solutions,
tachyons, etc. One might imagine doing this by giving, say, the gravitons associated with
the metrics a tiny mass, save for the metric associated with ordinary stable matter. More
general theoretical mechanisms than that given in (1) can also be considered: for example
some of the metrics may not be describable by second rank tensor fields, or some sectors of
the theory may not even be Lagrangian-based. For an overview and further discussion of
the different possibilities, see ref. [2].

From an empirical perspective, the validity of the EEP must therefore be checked sector-
by-sector in the standard model, since it cannot be imposed on grounds of logic. Although
the EEP has been tested to impressive levels of precision, virtually all such tests have been
carried out with matter fields. The possibility that matter and antimatter may have different
gravitational couplings remains a fascinating open question. The strongest bound on matter-
antimatter gravitational universality comes from the $K^0 - \bar{K}^0$ system. Recent studies of
this system have considered a straightforward violation of the weak equivalence principle (VEP)
in which it is assumed that $K^0 - \bar{K}^0$ mass and gravitational eigenstates can be simultaneously
diagonalised but with differing eigenvalues (i.e. differing $K^0$ and $\bar{K}^0$ masses) [10, 11, 12], in
which case violation of gravitational universality also means violation of CPT.

However, more generally, a violation of the EEP (VEP) in the Kaon system will not
assume simultaneous pairwise diagonalization of mass, gravitational or weak eigenstates. We
shall consider in this article the consequences of such a general VEP mechanism, showing that
it can provide a source of $CP$ violation whilst conserving CPT. In this context, previously
investigated mechanisms of EEP violation in the Kaon system may be considered either
as special cases of maximal $CPT$ violation in the gravitational sector [10, 11, 12] or else
$CPT$ conserved VEP, which is the other extreme case [13]. Our analysis is more general,
including all the earlier analyses as special cases and in addition allows us to compare with
the VLI bounds. We consider constraints imposed on this general VEP mechanism by present
experiments.

In section 2 and 3 we derive the general mass matrix including VLI and VEP effects
respectively. In both sections we point out that VLI (VEP) allows for a phase $\alpha_v$ ($\alpha_G$)
responsible for $CP$ violation whilst conserving $CPT$. At the end of section 3 we compare
both general mass matrices noticing that VEP and VLI effects are indistinguishable as
expected [2] (for the neutrino sector this similarity was pointed out in ref.[6]). In section 4 we
discuss the general case where these phases are taken to be 0. We consider first the
$CPT$-conserving case and examine the energy dependence VLI and VEP induce in the mass
difference $m_L - m_S$. Constraints on VEP parameters (and hence on VLI parameters) from
experiments on $m_L - m_S$ are discussed. We also give constraints on the interesting maximally
$CPT$—violating case where matter or antimatter states are the velocity or gravitational
eigenstates with differing eigenvalues. Then in section 5 we discuss the effect of the phases
$\alpha_v$ and $\alpha_G$ and constraints on VEP and VLI parameters from $CP$ violation experiments. We
point out that the $CP$ violation induced by VEP or VLI is of the superweak type and has an
inherent energy dependence. Consequently, although this mechanism cannot fully account
for observed $CP$ violation in the Kaon system, it yields a definite testable prediction for the

3
energy dependence of $CP$ violation parameters. This can then be used to put a qualitatively new bound on VEP or VLI. We summarise our results in section 6.

2 Violation of LLI.

The maximum attainable velocities of particles and antiparticles can differ if there is violation of LLI [5]. Here we take a phenomenological approach to this problem and assume that neither the mass nor the weak eigenstates are a priori simultaneously diagonalisable with the momentum eigenstates.

Then the general form of the effective Hamiltonian associated with the Lagrangian in the $(K^0, \bar{K}^0)$ basis will be

$$H = U_W H_{SEW} U_W^{-1} + U_v H_v U_v^{-1}$$

(2)

with,

$$H_{SEW} = \frac{(M_{SEW})^2}{2p} = \frac{1}{2p} \begin{pmatrix} m_1 - i \frac{\Gamma_1}{2} & 0 \\ 0 & m_2 - i \frac{\Gamma_2}{2} \end{pmatrix}^2$$

(3)

and

$$H_v = \begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix} p$$

(4)

to leading order in $\bar{m}^2/p^2$ with $p$ the momentum and $\bar{m} = (m_1 + m_2)/2$ the average mass. From now on we define $\delta X \equiv (X_1 - X_2)$, $\bar{X} \equiv (X_1 + X_2)/2$ for any quantity $X$. $H_{SEW}$ refers to the strong and electroweak part of the Hamiltonian. The constants $v_1$ and $v_2$ correspond to the maximum attainable speeds of each eigenstate. If special relativity is valid within the Kaon sector these are both equal to their average $\bar{v} = (v_1 + v_2)/2$, which we normalize to unity. Hence $v_1 - v_2 = \delta v$ is a measure of VLI in the Kaon sector. If $\bar{v}$ corresponds to the speed of electromagnetic radiation then special relativity is valid within the Kaon–photon sector of the standard model. In the limit $v_1 = v_2$, $m_{1,2}$ and $\Gamma_{1,2}$ are interpreted as the masses and the decay widths of the physical states $\tilde{K}_{1,2}$. These states are usually denoted as $K_{L,S}$, but since we shall be representing the physical states including VLI effects with the same notation we shall refer to them as $\tilde{K}_{1,2}$. The transformation matrix $U_W$ which relates the states $\tilde{K}_{1,2}$ to the states $K^0, \bar{K}^0$ can be written as

$$U_W = \frac{1}{\sqrt{2(1 + |\tilde{\varepsilon}|^2)}} e^{i\chi_W} \begin{pmatrix} 1 + \tilde{\varepsilon} & 1 + \tilde{\varepsilon} \\ -1 - \tilde{\varepsilon} & 1 - \tilde{\varepsilon} \end{pmatrix} \begin{pmatrix} e^{-i\beta_W} & 0 \\ 0 & e^{i\beta_W} \end{pmatrix}.$$  

(5)

We have assumed that there is no $CPT$ violation in the non-VLI part of the Hamiltonian, but only that $CP$ is violated, parametrized by $\tilde{\varepsilon}$. The phases $\chi_W$ and $\beta_W$ can be eliminated by a redefinition of the $\tilde{K}_{1,2}$ states in such a way that we have the usual formula:

$$\tilde{K}_2 = \frac{1}{\sqrt{2(1 + |\tilde{\varepsilon}|^2)}} [(1 + \tilde{\varepsilon}) K^0 \mp (1 - \tilde{\varepsilon}) \bar{K}^0]$$

(6)

For the VLI part if we assume that the velocity eigenstates are orthogonal they are related to the $K^0, \bar{K}^0$ by a unitary matrix $U_v$ which can be written in the general form

$$U_v = e^{i\chi_v} \begin{pmatrix} e^{-i\alpha_v} & 0 \\ 0 & e^{i\alpha_v} \end{pmatrix} \begin{pmatrix} \cos \theta_v & \sin \theta_v \\ -\sin \theta_v & \cos \theta_v \end{pmatrix} \begin{pmatrix} e^{-i\beta_v} & 0 \\ 0 & e^{i\beta_v} \end{pmatrix}.$$  

(7)
The phases $\chi_v$ and $\beta_v$ can be absorbed in a redefinition of the velocity eigenstates. The phase $\alpha_v$ (which is similar to $\text{Im} \tilde{\varepsilon}$ in Eq. (5) which to this order in $\tilde{\varepsilon}$ can be written in the form of such a phase) cannot be absorbed because $K^0 - \bar{K}^0$ are by definition charge conjugate states. The phase $\alpha_v$ is a new source of $CP$ violation which can be present even though the velocity states are still orthogonal.

From the form of the transformation matrix $U_W$ and $U_v$, the total Hamiltonian in the $K^0 - \bar{K}^0$ basis is

$$H = pI + \frac{1}{2p} \begin{pmatrix} M_+ & M_{12} \\ M_{21} & M_- \end{pmatrix}^2$$

with

$$M_{\pm} = \bar{m} - i \frac{\bar{\Gamma}}{2} \pm \frac{p^2 \cos 2\theta_v}{\bar{m}} \delta v$$

$$M_{12} = -\frac{1}{2} \frac{1 + \varepsilon}{1 - \varepsilon} (\delta m - i \frac{\delta \Gamma}{2}) - e^{-2i\alpha_v} \frac{p^2 \sin 2\theta_v}{\bar{m}} \delta v$$

$$M_{21} = -\frac{1}{2} \frac{1 - \varepsilon}{1 + \varepsilon} (\delta m - i \frac{\delta \Gamma}{2}) - e^{2i\alpha_v} \frac{p^2 \sin 2\theta_v}{\bar{m}} \delta v$$

(8)

The mass matrix above is the general formula from which we will discuss different special cases.

3 General mass matrix for VEP.

To formulate the VEP mechanism in the Kaon system, we first study the energy of the particles under consideration, taking the kaons to be relativistic. The gravitational part of the Lagrangian to first order (linearized theory) in a weak gravitational field $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ (where $h_{\mu\nu} = 2\phi \text{diag}(1,1,1,1)$) can be written as $L = -\frac{1}{2} (1 + g_{\mu\nu}) T_{\mu\nu}$ where $T_{\mu\nu}$ is the stress-energy in the gravitational eigenbasis. The principle of equivalence says that the gravitational couplings $g_{\mu\nu}$ are equal.

We can now write down the effective Hamiltonian including the strong, electromagnetic, weak, and gravitational interactions in the $(K^0, \bar{K}^0)$ basis:

$$H = pI + U_W H_{\text{SEW}} U_W^{-1} + U_G H_G U_G^{-1}$$

(9)

with $I$ the identity matrix,

$$H_{\text{SEW}} = \frac{(M_{\text{SEW}})^2}{2p} = \frac{1}{2p} \begin{pmatrix} m_1 - i \frac{\Gamma_1}{2} & 0 \\ 0 & m_2 - i \frac{\Gamma_2}{2} \end{pmatrix}^2$$

(10)

and

$$H_G = \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} = \begin{pmatrix} -2(1 + g_1) \phi (p + \bar{m}^2/2p) & 0 \\ 0 & -2(1 + g_2) \phi (p + \bar{m}^2/2p) \end{pmatrix}$$

(11)

in physical time and length units [12] to first order in $\bar{m}^2/p^2$ with $p$ the momentum. In formalisms where the weak equivalence principle is assumed [11, 12], one starts with $U_G$ proportional to $U_W$ (in the case considered where $CP$-violating effects in $U_W$ are taken to
be 0), which leads to a violation of CPT if VEP is operative, that is to say if \( g_1 \neq g_2 \). More generally when VEP is operative, \( U_G \) is not necessarily proportional to \( U_W \). Note that in the gravitational Hamiltonian \( H_G \) we have neglected terms proportional to \( \delta m \), and \( \phi \) is the gravitational potential on the surface of earth, which is constant over the range of terrestrial experiments.

In the absence of gravity, \( m_1, m_2 \) and \( \Gamma_1, \Gamma_2 \) are interpreted as the masses and the decay widths of the physical states \( \tilde{K}_1, \tilde{K}_2 \) defined by Eq.(6) as in section 2. For the gravitational part if we assume that the gravitational states are orthogonal they are related to the \( K_0, \bar{K}_0 \) by a unitary matrix \( U_G \) which can be written in the general form

\[
U_G = e^{i\chi_G} \begin{pmatrix} e^{-i\alpha_G} & 0 \\ 0 & e^{i\alpha_G} \end{pmatrix} \begin{pmatrix} \cos \theta_G & \sin \theta_G \\ -\sin \theta_G & \cos \theta_G \end{pmatrix} \begin{pmatrix} e^{-i\beta_G} & 0 \\ 0 & e^{i\beta_G} \end{pmatrix}.
\] (12)

The phases \( \chi_G \) and \( \beta_G \) can be absorbed in a redefinition of the gravitational states but the phase \( \alpha_G \) cannot like in the VLI case\(^4\) and is a new source of CP violation like \( \alpha_v \).

From \( U_W \) and \( U_G \) we then get

\[
M_{\pm} = \tilde{m} - i\frac{\bar{\Gamma}}{2} + \frac{p}{\tilde{m}} \bar{G}_G \pm \frac{p}{\tilde{m}} \cos 2\theta_G \delta G
\]

\[
M_{12} = -\frac{1}{2} + \frac{\varepsilon}{2} (\delta m - i\frac{\bar{\Gamma}}{2}) - e^{-2i\alpha_G} \frac{p}{\tilde{m}} \sin 2\theta_G \delta G
\]

\[
M_{21} = -\frac{1}{2} + \frac{\varepsilon}{2} (\delta m - i\frac{\bar{\Gamma}}{2}) - e^{2i\alpha_G} \frac{p}{\tilde{m}} \sin 2\theta_G \delta G
\] (13)

The mass matrix above is the general formula from which we will discuss different cases like Eq.(8) in the VLI case.

Comparing Eq.(8) and Eq.(13) we see that both the VLI and VEP mass matrices are similar. Putting \( \theta_v = \theta_G \) and \( \alpha_v = \alpha_G \), VLI and VEP effects are indistinguishable to lowest order in \( \tilde{m}^2/p^2 \) providing one identifies the VLI parameter \( \delta v \) with the VEP parameter \( -2\phi \delta g \). From now on we will discuss the VEP case knowing that any corresponding VLI formula can be obtained straightforwardly from this identification.

### 4 Testing the equivalence principle in the case \( \alpha = 0 \).

In this section we restrict ourselves to tests of the equivalence principle from \( m_L - m_S \) data in the case where gravitational states are related to the states \( K_0 - \bar{K}_0 \) by a simple orthogonal matrix (i.e. \( \alpha_G = 0 \)). In a first step we neglect the decay widths in Eq.(10)-(13). In the basis of the physical states \( K_L \) and \( K_S \), the hamiltonian of Eq.(13) becomes

\[
H = \begin{pmatrix} \frac{p + \frac{m_L^2}{2p}}{2p} & 0 \\ 0 & \frac{p + \frac{m_S^2}{2p}}{2p} \end{pmatrix}
\] (14)

\[^4\text{This is in contrast to the VEP mechanism for neutrinos [7, 15, 16] where charge conjugation plays no role. Relative phases } \alpha \text{ between neutrino flavour eigenstates can be absorbed in at least one sector, e.g. the weak sector [16].}\]
with \((m_L + m_S)/2 = \bar{m} + \frac{\bar{p}}{m}G\) and

\[
m_L - m_S = \left[ (\delta m)^2 + \left( 2\phi \delta g \frac{p}{m} (p + \bar{m}^2) \right)^2 - 4\delta m \phi \delta g \frac{p}{m} (p + \bar{m}^2) \cos(\frac{\pi}{2} - 2\theta_G) \right]^{1/2}
\] (15)

where \(m_L\) and \(m_S\) are the experimentally measured masses of \(K_L\) and \(K_S\) respectively. From this expression it is clear that the mass difference \(m_L - m_S\) is energy dependent. (The possibility of energy dependence of the various parameters in the Kaon system has been previously considered in different contexts [3,9-13]).

From Eq.(13) we can define the amount of CPT violation induced by VEP as follows

\[
\Delta_{CPT} = M_+ - M_- = -\cos(2\theta_G)2\phi \delta g \frac{p}{m} (p + \bar{m}^2)
\] (16)

Recent studies of VEP in the Kaon system [11]-[12] assumed CPT violation in the gravitational sector, from which it was argued that empirical bounds can be placed on the difference between the gravitational couplings to \(|K^0\rangle\) and \(|\bar{K}^0\rangle\). The difference in gravitational eigenvalues then corresponds to a difference \((\Delta M_g)\) in the masses of \(|K^0\rangle\) and \(|\bar{K}^0\rangle\):

\[
|M_+ - M_-| = \phi \Delta M_g = 2\phi |\delta g| \frac{p}{m} (p + \bar{m}^2)
\] (17)

and is entirely attributable to the amount of CPT violation. The first equality in Eq.(17) was given by Kenyon [11] and the second by Hughes [12], who specified the energy dependence of \(\Delta M_g\). From the experimental upper bound on \(M_+ - M_-\) [17] the bound \(|\delta g| < 2.5 \times 10^{-18}\) may be obtained, where the potential \(\phi\) is taken to be that due to the local supercluster \((\phi \simeq 3 \times 10^{-5})\) and \(p \simeq 100\) GeV [17]. In this approach CPT conservation implies no gravitational mass difference and hence no VEP. However it is clear from the expression (16) for \(\Delta_{CPT}\) that the bound obtained on \(\Delta M_g\) is actually on some combination of VEP parameters and not on \(\delta g\) and \(\cos(2\theta_G)\) separately. When \(\theta_G = 0\), Eq.(17) agrees with Eq.(16). More recent experiments [18] find \(|M_+ - M_-|/m_K < 9 \times 10^{-19}\), yielding the bound \(|\delta g| < 3.8 \times 10^{-19}\) for the same values of \(\phi\) and \(p\).

In the case of VLI with \(\theta_v = 0\), the amount of CPT violation associated with VLI is given by,

\[
|M_+ - M_-| = |\delta v| \frac{p^2}{m}
\] (18)

The same experimental results can be used to constrain the VLI parameter: \(|\delta v| < 2.3 \times 10^{-23}\). To leading order this has exactly the same energy dependence as the VEP mechanism.

Next we shall consider a scenario in which CPT is conserved, so that \(\Delta_{CPT} = 0\). From the above it is clear that, even if CPT is conserved, there is still a VEP-induced difference between the masses of the physical states. As a result bounds can be placed on the VEP parameter \(\phi \delta g\) without the assumption that locality in quantum field theory is violated.

From the expression of \(\Delta_{CPT}\) it is clear that it is possible to conserve CPT for all momentum taking \(\theta_G = \frac{\pi}{4}\) (modulo \(\frac{\pi}{2}\)). In this case the mass difference is

\[
m_L - m_S = \delta m - 2\phi \delta g \frac{p}{m} (p + \bar{m}^2)
\] (19)
which as noted above is energy dependent and to the leading order similar to the VLI expression in the case \( \theta_e = \pi/4 \): \( m_L - m_S = \delta m + \delta v p^2/m \). It is possible to put a bound on the VEP parameter \( \delta g \) if we know the value of \( \phi \) and the mass difference at various given energies. Alternatively, if mass measurements at two different energies were different, the differing values for \( m_L - m_S \) could be used to extract a value for the VEP parameter \( \delta g \).

We now proceed to find out constraints on the parameters \( \delta m \) and \( \delta g \) (or \( \delta v \)). In the review of particle properties \([18]\) six experiments were taken into account. Two of them \([19, 20]\) are with the kaon momentum \( p_K \) between 20 GeV and 160 GeV. The weighted average of these two experiments is \([20]\): \( \Delta m_{LS} = m_L - m_S = (0.5282 \pm 0.0030) \times 10^{10}\text{hs}^{-1} \).

The four other experiments \([21, 22, 23, 24]\) are at lower energy, with \( p_K \approx 5 \) GeV, or less with a weighted average \( \Delta m_{LS} = (0.5322 \pm 0.0018) \times 10^{10}\text{hs}^{-1} \). A fit of equation (19) with the high and low energy value of \( \Delta m_{LS} \) gives: \( \delta m = (3.503 \pm 0.012) \times 10^{-12}\text{MeV} \) and \( \phi \delta g = (8.0 \pm 7.0) \times 10^{-22} \times (\frac{90}{E_{av}})^2 \), (where \( E_{av} \) is the average energy for the high energy experiment). All these bounds on the VEP parameter \( \phi \delta g \) are also bounds of the VLI parameter \(-\delta v/2\) with the same energy dependence. We shall not explicitly present the VLI bounds.

Taking \( \phi \) to be the earth’s potential (\( \phi \approx 0.69 \times 10^{-9} \)), we find \( \delta g = (1.2 \pm 1.0) \times 10^{-12} \) whereas if \( \phi \) is due to the local supercluster then \( \delta g = (2.7 \pm 2.3) \times 10^{-17} \). These values differ from zero by 1.15 standard deviations. A precise fit of mass difference per energy bin in present and future high energy experiments would be extremely useful in constraining the energy dependent VEP or VLI parameters. Improvement on the low energy experiments can also change the bounds. One of the low energy experiments published last year found \( \Delta m_{LS} = (0.5274 \pm 0.0029 \pm 0.0005) \times 10^{10}\text{hs}^{-1} \) \([24]\); when fitted with the high energy experiments, a value of \( \delta g \) consistent with 0 at less than 1 standard deviation is obtained. On the other hand, without this new experiment, a similar fit of the other five experiments yields \( \phi \delta g = (1.38 \pm 0.77) \times 10^{-21}(90/E_{av})^2 \). In this case \( \delta g \) is different from 0 by 1.8 standard deviations.

In the above analysis we have not included the effect of the absorptive part of the Hamiltonian, i.e of the decay widths in Eqs.(10-13). Including them we now obtain

\[
\begin{align*}
m_L - m_S &= \frac{1}{\sqrt{2}} \left[ \sqrt{F^2 + G^2} + F \right]^{1/2} \quad (20) \\
\Gamma_L - \Gamma_S &= \sqrt{2} \left[ \sqrt{F^2 + G^2} - F \right]^{1/2} \quad (21)
\end{align*}
\]

\[
F = (\delta m)^2 + (2\phi \delta g \frac{p}{m} (p + \frac{\bar{m}^2}{2p})^2 - 4\delta m \phi \delta g \frac{p}{m} (p + \frac{\bar{m}^2}{2p}) \cos(\frac{\pi}{2} - 2\theta_G) - (\frac{\delta \Gamma}{2})^2
\]

\[
G = -(\delta m \delta \Gamma) + 2 \cos(\frac{\pi}{2} - 2\theta_G) [\delta \Gamma \phi \delta g \frac{p}{m} (p + \frac{\bar{m}^2}{2p})]
\]

In deriving these equations we neglected terms in \( \delta m \Gamma, \delta m \delta \Gamma \) and \( \Gamma^2 \) with respect to the terms in \( m \delta m \) or \( \delta m \delta \Gamma \). It can be shown that in the CPT-conserving case the mass difference given in Eq. (20) reduces to Eq. (19). So in the CPT—conserving case the results above are not affected by inclusion of the widths. In this case the difference \( \Gamma_S - \Gamma_L = \delta \Gamma \) is independent of energy. This is consistent with experiment, which indicates that the low
and high energy measurements of $\Gamma_S - \Gamma_L$ are fully compatible \[18\]. For $\theta_G \neq \pi/4$, an examination of (21) indicates that $\Gamma_S - \Gamma_L$ is energy dependent; however this is small and measurements of $\Gamma_S - \Gamma_L$ do not constrain $\delta g$ more than measurements of $\Delta m_{LS}$ even though they are relatively more precise. We note that measurements of $\Gamma_S - \Gamma_L$ would more strongly constrain a possible absorptive part coming from the gravitational sector which presumably would induce a larger energy dependence. We shall not consider this possibility here. For $\theta_G \neq \pi/4$, width effects in Eq.(20) are small and (15) remains valid to within a few percent.

In Fig.1, for completeness, we plot as a function of $\cos(2\theta_G)$ the upper bounds we get on $|\phi \delta g|$ by fixing $\delta m$ to the central value of the world average \[18\], $\Delta m_{LS} = (0.5310 \pm 0.0019) \times 10^{10} \text{fs}^{-1}$ and requiring that $m_L - m_S$ in (15) does not differ from $\delta m$ by more that $\pm 2$ standard deviations. Note that in the case of maximal CPT violation ($\theta_G = 0$), $m_L - m_S$ can only increase with energy, as is clear from Eq.(15) or Eq. (20). The actual difference between low and high energy experiments, if valid, could not be explained in this case except for complex values of $\delta g$ (and similarly for values of $\theta_G$ very close to 0).

In Fig.1 we also show the bound coming from Eq.(16) requiring that at high experimental energy ($p \approx 100 \text{GeV}$) the experimental upper bound $|M_+ - M_-|/m_K < 9 \times 10^{-19}$ \[18\] on CPT violation is satisfied.

5 Testing the equivalence principle from CP violation experiments.

Now we consider the effect of the CP-violating phase $\alpha_G$. Let us note first that in the maximally CPT-violating case, $\theta_G = 0$, there is no effect of the phase $\alpha_G$ in the mass matrix Eq.(13) and consequently no CP-violating effect coming from this phase (similarly in the VLI case when matter and antimatter states are the velocity eigenstates). In the following we will restrict ourself to the most interesting CPT-conserving case with $\theta_G = \pi/4$. The total Hamiltonian can then be diagonalised

$$H = \begin{pmatrix} p + \frac{1}{2p}(m_L - i\Gamma_L)\xi^2 & 0 \\ 0 & p + \frac{1}{2p}(m_S - i\Gamma_S)\xi^2 \end{pmatrix}$$

with the physical eigenstates $K_L$ and $K_S$ being given by

$$K_L = \frac{1}{\sqrt{2(1 + |\epsilon|^2)}} \left[ (1 + \epsilon)K^0 \mp (1 - \epsilon)\tilde{K}^0 \right]$$

Defining

$$G_c = \frac{p}{m}\delta G \cos 2\alpha_G = -2g_c\left(\frac{p^2}{m} + \frac{\bar{m}}{2}\right); \quad g_c = \phi \delta g \cos 2\alpha_G$$

$$G_s = \frac{p}{m}\delta G \sin 2\alpha_G = -2g_s\left(\frac{p^2}{m} + \frac{\bar{m}}{2}\right); \quad g_s = \phi \delta g \sin 2\alpha_G$$

the new CP-violating parameter $\epsilon$ is now defined in terms of the CP-violating parameters $\bar{\epsilon}$ and $G_s$ via

$$\epsilon = \frac{\bar{\epsilon}(\delta m - i\frac{\Delta}{2}) - \frac{i}{2}G_s}{(\delta m + G_c) - i\frac{\Delta}{2}}$$
to first order in $\tilde{\varepsilon}$ and $G_s$. Similarly we have to first order in $\tilde{\varepsilon}$ and $G_s$

$$\Delta m \equiv m_L - m_S = (\delta m + G_s) \quad (25)$$

$$\Delta \Gamma \equiv \Gamma_L - \Gamma_S = \delta \Gamma. \quad (26)$$

Since in the mechanism considered here there is no $\varepsilon'$ type CP violation coming from VEP we will neglect other possible $\varepsilon'$ effects and the relevant CP-violating quantities are

$$\phi_{+-} = \phi_{00} = -2\Delta m/\Delta \Gamma \simeq 43.5^\circ$$

and also consequently $|\eta_{+-}| = |\eta_{00}| \simeq \frac{G_s}{\sqrt{2}}$. The fact that the superweak phase is obtained is due to the fact that the VEP mechanism considered here respects the hermiticity of the interaction between the CP-eigenstates $K_1$ and $K_2$ (i.e. the numerator of Eq.(24) is purely imaginary).

Consider first the case $\tilde{\varepsilon} = 0$, i.e. there is no CP violation induced from the weak interaction. Can we interpret the observed CP violation parameters above as originating purely due to the relative phase $\alpha_G$? In other words does the superweak mechanism have a gravitational origin? Eq. (24), with $\tilde{\varepsilon} = 0$, can be written as

$$\varepsilon = -\frac{i}{2} \frac{G_s}{(\Delta m - i \frac{\Delta \Gamma}{2})}. \quad (27)$$

Equating the real and the imaginary parts we get

$$\text{Re} \varepsilon = \frac{G_s \Delta \Gamma}{2} \left[ (\Delta m)^2 + \left( \frac{\Delta \Gamma}{2} \right)^2 \right]^{-1}; \quad \text{Im} \varepsilon = -\frac{G_s}{2} \Delta m \left[ (\Delta m)^2 + \left( \frac{\Delta \Gamma}{2} \right)^2 \right]^{-1}.$$

The above equations reproduce the results of the superweak theory: $\phi_{+-} = \phi_{00} = -2\Delta m/\Delta \Gamma \simeq 43.5^\circ$ and also consequently $|\eta_{+-}| = |\eta_{00}| \simeq \frac{G_s}{\sqrt{2}}$. The fact that the superweak phase is obtained is due to the fact that the VEP mechanism considered here respects the hermiticity of the interaction between the CP-eigenstates $K_1$ and $K_2$ (i.e. the numerator of Eq.(24) is purely imaginary). Interestingly, we see that by assuming the gravitational, weak and mass eigenstates are all related by unitary transformations, the physical states are still of the superweak type; in particular they are no longer related to the other states by a unitary transformation and are no longer orthogonal.

Taking the experimental value $\delta = (0.327 \pm 0.012)^\circ$ [18] as input we obtain $G_s \simeq -2 \times 10^{-14}$. This value of $G_s$ yields a consistent fit to all the CP violation parameters above, as with any superweak mechanism. However this does not provide us with positive evidence

\[5\]Note that insofar as there is no gravitational coupling to the imaginary part of the energy (as we have assumed here), a CP violation of the Re$\epsilon$ type coming from gravitation (which would render $U_G$ nonunitary) does not respect the hermiticity of the $K_1$-$K_2$ interaction and is consequently experimentally suppressed. We do not consider the possibility of such an effect here.
for VEP-induced $CP$ violation because $|\eta_{+-}|$ and $|\eta_{00}|$ have been observed experimentally with good accuracy to be constant over a large energy range [18]. Hence is not possible to reproduce the data for all energies with $\tilde{\epsilon} = 0$ since $G_s$ is proportional to $p^2$. An observed energy dependence in these parameters that is consistent with (24) would be a definitive signature of a VEP-mechanism operative in this sector.

We now demonstrate how a bound on the VEP parameter $\phi \delta g$ can be obtained independently of the phase $\alpha_G$. We can extract a bound on $g_c$ from the experimental constraint on the energy dependence of $\Delta m$, Eq. (25) in the same way that for $\phi \delta g$ above in Eq.(19) substituting $\delta g$ by $\delta g \cos 2\alpha_G$. The bound obtained on $\phi \delta g$ in Eq.1 is now a bound on $g_c$:

$$|g_c| < 9 \times 10^{-22}.$$  \hspace{1cm} (28)

From $CP$ violation parameter data we can also obtain a bound on $g_s$. Defining

$$A = \frac{G_c}{\delta m}, \quad B = \frac{\Re \varepsilon}{\Im \varepsilon} - 1, \quad C = -\frac{\delta \Gamma}{2\delta m} - 1$$

where the magnitudes of $A$, $B$ and $C$ are all much smaller than unity, implies from (24)

\[
\begin{align*}
\Re \varepsilon &= \Re \tilde{\varepsilon}(1 - A) + \frac{1}{2} G_s \left( \frac{\delta \Gamma}{\delta m} \frac{\delta \varepsilon}{\sqrt{2}} + \left( \frac{\delta \varepsilon}{\sqrt{2}} \right)^2 \right) \\
\Im \varepsilon &= \Im \tilde{\varepsilon} - \frac{1}{2} G_s \left( \frac{\delta \varepsilon}{\sqrt{2}} \right)^2 + \left( \frac{\delta \varepsilon}{\sqrt{2}} \right)^2
\end{align*}
\]  \hspace{1cm} (29) \hspace{1cm} (30)

where only terms linear in $A$, $B$ and $C$ have been retained. We observe that $g_s$ changes the value of $|\varepsilon|$ but not the phase of $\varepsilon$. In addition, $\Im \varepsilon$ depends on $g_s$ but not on $g_c$. From the experimental values of $\phi_{+-} = (43.7 \pm 0.6)^\circ$ and $|\eta_{+-}| = (2.284 \pm 0.018)10^{-3}$ [18] we obtain $\Im \varepsilon = (1.58 \pm 0.02).10^{-3}$. Fixing $\Im \tilde{\varepsilon}$ to the central value of $\Im \varepsilon$ and requiring that $\Im \varepsilon$ in Eq.(30) differs from the central value by no more than 2 standard deviations at high experimental energies ($p \simeq 70$ GeV[18]) yields

$$|g_s| < 3 \times 10^{-23}$$

This value hardly varies when we calculate $\Im \varepsilon$ from the experimental value of $|\eta_{00}|$ and $\phi_{00}$ instead of $|\eta_{+-}|$ and $\phi_{+-}$. From the bounds on $g_c$ and $g_s$ we then get

$$|\phi \delta g| < 9 \times 10^{-22}.$$  \hspace{1cm} (29)

$g_c$ can also be bounded from $CP$ violation. Indeed the $CP$-conserving parameter $g_c$ is present in $\Re \varepsilon$ and a bound on it can be obtained by considering the phase of $\varepsilon$

$$\tan \phi_{+-} = \frac{\Im \varepsilon}{\Re \varepsilon} = \frac{\Im \tilde{\varepsilon}}{\Re \tilde{\varepsilon}}(1 + A)$$  \hspace{1cm} (31)

which doesn’t depend on $g_s$. Taking the low energy value of $\phi_{+-}$ to equal its central value above we similarly obtain (requiring that $\phi_{+-}$ not differ at high energy by more than two standard deviations from its low energy value):

$$|g_c| < 7 \times 10^{-21}.$$  \hspace{1cm} (32)
This value is of the same order of magnitude as the upper bound obtained above by looking for energy dependence in $\Delta m$. CP violation measurements consequently are a useful means for searching both for $CP$–conserving VEP effects (through the parameter $g_c$) and $CP$–violating VEP effects (through $g_s$).

We shall not consider the case where $CPT$ is violated ($\theta_G \neq \pi/4$) with $\alpha_G \neq 0$. Equations similar to Eqs.(24)-(26) can be straightforwardly obtained from Eq.(13) but they are lengthy and do not provide any new interesting physical results which have not already been discussed above.

In ref.[13], the effect of a tensorial field $f^{\mu\nu}_{ij}$ whose $CPT$–violating interactions with the kaons is given by the lagrangian $L=f^{\mu\nu}_{ij}d^\mu\phi_i d^\nu\phi_j$ has been considered with $\phi_{1(2)}$ the $CPT$–eigenstates and $f^{\mu\nu}_{11}, f^{\mu\nu}_{22}, f^{\mu\nu}_{12} = f^{\mu\nu}_{21}$ real parameters. Writing $f^{\mu\nu}_{ii}$ as $\frac{f_T}{f} \phi \eta^{\mu\nu}$ and $f^{\mu\nu}_{12}$ as $\frac{f_T}{f} \eta^{\mu\nu}$, constraints on $(f_1 - f_2)/f$ and $f_T/f$ have been obtained from experimental energy constraints on the energy dependence of $\Delta m_{LS}$ and $\eta_{+-}$ respectively. We observe that, except for a different experimental situation, the constraints obtained on $|(f_1 - f_2)/f|$ and $|f_T/f|$ are similar to ours on the corresponding quantities, $|4\delta g \cos 2\alpha_G|$ and $|\delta g \sin 2\alpha_G|$ respectively with $\theta_G = \pi/4$. From our treatment, we see consequently that provided the tensorial interaction is of gravitational origin the bounds on the parameters $|(f_1 - f_2)/f|$ and $|f_T/f|$ are in fact bounds on a combination of the VEP parameters, the difference $\delta g$ in the gravitational couplings and the phase $\alpha_G$.

We close this section by noting that the bounds on $|g_c|$, $|g_s|$ and $|\phi \delta g|$ above are also bounds on the corresponding VLI parameters $|\delta v_2 \cos 2\alpha_v|$, $|\delta v_2 \sin 2\alpha_v|$ and $|\delta v_5/2|$ respectively.

6 Summary and Conclusion.

The Kaon system provides us with an interesting physical situation in which we can empirically check the validity of special relativity and/or the equivalence principle in a matter/antimatter sector of the standard model that includes 2nd generation matter. A variety of interesting combinations of VLI/VEP effects exist which can be associated with $CP$ violation and/or $CPT$ violation. Violations of the equivalence principle in the Kaon system need not violate $CPT$ (which in turn implies a loss of locality in quantum field theory) as considered in recent studies.

A general feature of the VLI/VEP mechanisms is that they predict an energy dependence in $m_L - m_S$ and in the $CP$ violation parameters which can be empirically tested to obtain bounds on the relevant parameters (such as $\phi \delta g$ for VEP or $\delta v$ for VLI). Since both VEP and VLI have the same energy dependence, although we can obtain bounds for both, it will not be possible to experimentally distinguish between the two mechanisms. Under the assumption that all such parameters are within two standard deviations over the energy scales at which they have been measured, present experiments provide rather stringent bounds on the VEP (or VLI) mechanism. A more systematic search for energy dependence in $m_L - m_S$ and in the $CP$-violating parameters (such as $\varepsilon$) will provide us with more definitive information about the VEP (or VLI) mechanism in this sector. In addition in our formalism, we observe that the $CPT$-conserving case [13] and the $CPT$-violating case of recent VEP studies [11, 12] are special cases of the same general mechanism.
Acknowledgements

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

References


Fig.1: Upper bounds on $|\phi\delta g|$ (for $p \approx 90$ GeV) obtained from Eq.(15) (solid line), Eq.(16) (dashed line) as explained in the text.