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RELATION BETWEEN REGGE CUT THEORIES

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A B S T R A C T

Gribov and Migdal argued that elastic Regge cuts must always be stronger than the absorption model. The ordinary Veneziano formula is a counterexample to a crucial step in their derivation. We show that, at least for exotic cuts, the naive absorption model corresponds to non-dual double counting in the Schwarz sum rule. Correct formulae are given.

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By the reggeon calculus ¹⁾, a Regge-Regge cut contributes at large s

$$(i/16\pi^2 |s|) \int d^2 k_{\underline{w}} [N(\underline{k}_{\underline{w}}, \underline{\Delta} - \underline{k}_{\underline{w}})]^2 R(-k_{\underline{w}}^2, s) R(-[\underline{\Delta} - \underline{k}_{\underline{w}}]^2, s) \quad (1)$$

where $\underline{\Delta}$, $\underline{k}$ are two-dimensional transverse momenta ($\underline{\Delta}^2 = -t$), $R(t, s)$ are reggeon propagators, and $N(\underline{k}_1, \underline{k}_2)$ is the Gribov vertex (or impact factor). Equation (1) has recently been derived both from dual loops ²⁾, and from Feynman graphs ³⁾. N is the residue at a fixed pole of the reggeon-particle scattering amplitude, evaluated on the "angular momentum shell"

$$N(\underline{k}_1, \underline{k}_2) = N_{l_1, l_2}(t),$$

$$\text{at } l_1 = \alpha_1(-k_1^2), \quad l_2 = \alpha_2(-k_2^2). \quad (2)$$

In perturbation theory, this fixed pole residue is projected by the infinite-momentum frame limit ³⁾. The Schwarz sum rule ⁴⁾ relates N to the absorptive part of the reggeon-particle scattering amplitude

$$N_{l_1, l_2}(t) = \frac{2}{\pi} \int_{s_0}^{\infty} dx \cdot \text{Im} A_{l_1, l_2}^{(s)}(x, t), \quad (3)$$

if this superconverges, i.e.,

$$\alpha_t < l_1 + l_2 - 1. \quad (4)$$

Equation (1) is represented by Fig. 1a, where the hatched lines are fixed poles. The absorption model corresponds instead to Fig. 1b - the Regge cut contribution is the absorptive part of a box diagram

$$(i/16\pi^2 s) \cdot g^4 \int d^2 k_{\underline{w}} R(-k_{\underline{w}}^2, s) R(-[\underline{\Delta} - \underline{k}_{\underline{w}}]^2, s). \quad (5)$$

Gribov and Migdal ⁵⁾ claimed that (3) could be analytically continued to $t = 0$. It would then be a reggeon unitarity equation, with all terms positive, and the one-particle intermediate states contributing the lower bound

$$N(\underline{0}, \underline{0}) > \sum_n g_n^2. \quad (6)$$

It is easily seen that $N(\underline{0}, \underline{0})$ determines Eq. (1) at $t = 0$, $s \rightarrow \infty$. Inserting the right-hand side of (6) into (1), would then give (5). This argument would imply :

(a) that all possible intermediate states must be included in the absorption model

(i.e., strong absorption) ; (b) that, for elastic scattering, the one-particle intermediate states give a lower bound on the strength of the Regge cut. This has been applied to estimate cuts phenomenologically ⁶⁾, to study renormalization effects ^{5),7)}, to reassure believers in the strong absorption model ⁸⁾, and to inspire calculators of exotic cuts ⁹⁾.

We will prove : (i) that the lower bound (6) is systematically violated by the RR cuts extracted from dual loops ²⁾ ; (ii) that the ordinary Veneziano formula is a counterexample to a crucial step in Gribov and Migdal's derivation ^{*}). The validity of the strong absorption model for the cut, will be seen to depend on a non-dual interference model for the reggeon scattering amplitude. Our counterexample is especially relevant to exotic cuts, since it is the actual Gribov vertex for one of them.

The non-planar single loop duality diagram contains an RR cut ¹¹⁾. To $O(s^{-1})$ it is given ²⁾ by (1) with propagator

$$R(t, s) = [\alpha' s e^{-i\pi}]^{\alpha(t)} \Gamma(-\alpha(t)), \quad (7)$$

and vertex

$$N(\underline{k}_1, \underline{k}_2) = g^2 \cdot 2^{2(\underline{k}_1, \underline{k}_2)}. \quad (8)$$

Several people ¹²⁾ showed that the Veneziano (s,u) term

$$g^2 \Gamma(M - \alpha_s) \Gamma(P - \alpha_u) / \Gamma(Q - \alpha_s - \alpha_u) \quad (9)$$

has a fixed pole in the t channel at $\ell = -1$, with residue

$$F(\underline{k}_1, \underline{k}_2) = (\mu^2 - \frac{1}{4}t)^{-1} \cdot g^2 \cdot 2^{-(M+P+x)} \cdot x \times \Gamma(M+P+x) / \Gamma(Q+x), \quad (10)$$

where, for two particles off-shell,

$$x = -\alpha_s - \alpha_u = -\alpha_s(\mu^2) - \alpha_u(\mu^2) - 2\alpha'(\underline{k}_1, \underline{k}_2). \quad (11)$$

^{*}) Landshoff ¹⁰⁾ has also given reasons why Gribov and Migdal's result "must be treated with some suspicion".

Putting $M = P = Q = 0$, $\alpha(\mu^2) = 0$, we see that

$$N(\underline{k}_1, \underline{k}_2) = (\mu^2 - \frac{1}{4}t) \cdot F(\underline{k}_1, \underline{k}_2), \quad (12)$$

so the Gribov vertex is essentially the fixed pole residue, as anticipated. For the RR cut, the Veneziano $N_{l_1 l_2}(\underline{k}_1, \underline{k}_2)$ is independent of l_1, l_2 , as can be checked directly using excited particle formulae. [This is not true for vertices with pomerons, which have poles in the external masses ²⁾, and therefore depend non-trivially on l_1, l_2 before (2).]

The $\pi\pi$ Veneziano formula ¹³⁾ is (9) with $M = P = Q = 1$, $\alpha(\mu^2) = \frac{1}{2}$. Thus (10) and (12) give the Gribov vertex (impact factor) of the $\pi\pi$ cut as

$$N_{\pi\pi}(\underline{k}_1, \underline{k}_2) = f^2 \cdot \alpha'(\underline{k}_1, \underline{k}_2) \cdot 2^{2\alpha'(\underline{k}_1, \underline{k}_2)} \quad (13)$$

We see that it has the Adler zero ^{*)}. Now for $\mu \rightarrow 0$, the $\pi\pi$ Veneziano model is free of ghosts ¹⁴⁾. (The ε' minighost decouples in this limit.) Therefore the right-hand side of (6) is certainly positive and non-vanishing. However, by (13) the left-hand side is zero. Obviously something terrible has happened to the Gribov-Migdal bound.

To see more clearly what goes wrong, we take the original Veneziano formula

$$B_4(-\alpha_s, -\alpha_t) = \sum_{J=0}^{\infty} (J - \alpha_s)^{-1} \cdot (-1)^J \binom{-1 - \alpha_t}{J}, \quad (14)$$

with $\alpha_s + \alpha_t + \alpha_u = -1$ so that $t \rightleftharpoons u$ just removes the $(-1)^J$. The fixed pole sum rule is, by (10), (3), (14),

$$2^{-1 - \alpha_t} = \sum_{J=0}^{\infty} \binom{-1 - \alpha_t}{J}, \quad (15)$$

and the superconvergence relation

$$0 = \sum_{J=0}^{\infty} (-1)^J \binom{-1 - \alpha_t}{J}. \quad (16)$$

*) We speculate that another soft pion theorem might relate the pomeron-pion impact factor at $\underline{k}_1 = \underline{k}_2 = 0$ to a Born term (like the low energy theorem in photoproduction). If true it would be valuable, since this cut is anomalously strong experimentally.

[They can be derived heuristically by binomial expansion of $(1 \pm 1)^{-\alpha_t-1}$.] The Schwarz sum rules ⁴⁾ are the sum and difference (16) $\pm$ (15). Numerical evaluation shows that all are correct for $\alpha_t < -1$, in accord with (4), with (15) converging much faster than (16). However, the Gribov-Migdal argument requires that we continue them to $t = 0$, where the Schwarz sum rules will have all terms on the right positive. At $\alpha_t = 0$, (15) and (16) become

$$\begin{aligned} \frac{1}{2} &= 1 - 1 + 1 - 1 + \dots, \\ 0 &= 1 + 1 + 1 + 1 + \dots, \end{aligned} \quad (17)$$

and at $\alpha_t = 1$ they give

$$\begin{aligned} \frac{1}{4} &= 1 - 2 + 3 - 4 + \dots, \\ 0 &= 1 + 2 + 3 + 4 + \dots. \end{aligned} \quad (18)$$

We see that :

- (I) The first resonance contribution is not a lower bound on the fixed pole residue.
- (II) The sum of any finite number of resonances in (15) gives a wrong answer.
- (III) Not even Padé approximants could validate the superconvergence relation (16), or the Schwarz sum rules (16) $\pm$ (15), for $\alpha_t \geq -1$.

Gribov and Migdal ⁵⁾ of course realized that analytic continuation of the superconvergence relation would be needed, and proposed doing it by separating the leading t channel Regge exchange. In a non-dual theory free of ghosts (assuming such existed), their argument would be correct : the resonance and Regge contributions could be continued separately to $t = 0$, and would each be positive there. However, because of duality, when we subtract leading Regge, we also subtract part of the resonances, and there is no guarantee that what we put back after continuation will be equally large.

The strong absorption model is thus seen to reflect a classic non-dual double counting error in the fixed pole sum rule. Where superconvergence relations are valid, duality will normally require resonances to contribute with alternating signs, so that no lower bound can be inferred. Furthermore (10) shows that Veneziano fixed pole residues always decrease exponentially at positive t . (Remember $\tilde{k}$ is spacelike.) This is closely related to the decrease of the fixed angle scattering amplitude, which is obviously required physically. The Gribov-Migdal bound will therefore tend to be violated systematically for RR cuts.

For non-pomeron cuts it has already been pointed out that the absorption model gives wrong t dependence ²⁾ and wrong selection rules ¹⁵⁾. Nevertheless it is continually used to calculate ^{16),9)} or talk about ¹⁷⁾ exotic cuts. The present work gives yet one more demonstration (though probably still not enough for some people) of the wickedness of looking for RR cuts in box graphs. It is particularly senseless now that (1), (7), (10)-(12) provide complete and correct formulae for non-pomeron cuts, whenever Veneziano models exist for the subgraphs.

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(a)

(b)

RR cut according to (a) the reggeon calculus where it couples through fixed poles, (b) the absorption model.