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 K^+p REGGEIZED PHASE SHIFT ANALYSIS

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A B S T R A C T

We performed KN and $\bar{K}N$ absorptive Regge fits at high energies, and phase shift analysis of K^+p at 26 energies below 2 GeV/c. They used the same Veneziano formula for the high partial waves. The shortest path solution turned out to be that closest to the Regge extrapolation in low waves also. We thus have a consistent picture of K^+p from 0 to 16 GeV/c. Agreement with the KN^* channel is also better than before. Though an exotic resonance is not completely excluded, K^+p scattering can be accurately described without one. The $K\Lambda N$ coupling constant is $g_{\Lambda}^2 = 10.5 \pm 1.5$.

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1. - INTRODUCTION

The search for exotic resonances has led to great experimental interest in K^+p scattering, and to many phase shift analyses ^{1),2)}. Some have announced a Z_1^* resonance, others denied it. K^+p is also interesting because exchange degeneracy (EXD) predicts ³⁾ that the Regge pole terms should be pure real, except for the Pomeron. Unlike similar predictions in $\bar{K}N \rightarrow \pi\Lambda$ and $\bar{K}N \rightarrow \pi\Sigma$, this does not depend on exact SU(3). Therefore, if duality is going to work anywhere, it must work in K^+p . At least for small t , the Pomeron-Regge cut should also be real ⁴⁾. This means that the mysterious Pomeron itself can be easily isolated here.

There have been three previous attempts to unify K^+p phase shift analysis and Regge fits, but all are open to serious objection. Kato et al. ⁵⁾ tried phase shift analysis with high partial waves taken from the Regge fit of Blackmon and Goldstein ⁶⁾. Unfortunately they seem to have completely omitted Λ exchange, which is the longest range force. Therefore their assumed high waves bear no resemblance to ours. Carreras and Donnachie ⁷⁾ performed a Regge fit and used it as a trial solution in phase shift analysis. Unfortunately their Regge fit was an old-fashioned one. This led them to prefer an implausible ²⁾ phase shift solution with S_1 attractive at low energies. Ours gives the standard, strongly repulsive S_1 . Daum et al. ⁸⁾ compared extrapolated Regge pole fits with a previous phase shift analysis ⁹⁾ at 1.455 GeV/c. Unfortunately the latter had assumed high waves to be zero, which resulted in a pathologically inelastic D_5 . This was then reflected into the Regge fit as an anomalous helicity dependence of the Pomeron. Also the pure Regge pole model of Daum et al. is not an adequate representation of absorptive Regge cuts.

We start with a $\bar{K}N$ Veneziano formula satisfying various Y^* constraints, with three remaining adjustable parameters (Section 2). Unlike most authors, we do not suppose that this represents the whole $\bar{K}N$ scattering amplitude, but only its high partial waves. This philosophy has been explained previously ¹⁰⁾. At high energies the low waves are given by an absorptive Regge model. By using a formula suggested by dual loops ¹¹⁾, only a normalization parameter is needed for the Pomeron.

This four-parameter Regge fit explains the whole of the high energy data, except that the K^-p cross-over is not moved far enough. However, to check we were not biased, we also tried alternative formulae with more parameters for the Pomeron, and a λ factor for the cut.

Below 2 GeV/c, we then switch off Pomeron and cut, and determine the low partial waves from phase shift analysis at each of 26 energies (Section 3). The high waves are taken from the same Veneziano formula, with its three coupling constants varied in the fits. The phase shift solutions at these 26 energies are then linked up continuously and smoothly by the shortest path method (Section 4). Astonishingly, the shortest path solution is also the one closest to the extrapolated Regge fit even in the low waves.

We thus achieve a consistent picture of K^+p scattering over its entire observed range. P_3 describes a slow semi-circle unrelated to the total cross-section bump, and is therefore called non-resonant. We give a resonant alternative, but it is definitely inferior (Section 5).

2. - REGGE FITS

In a previous paper ¹⁰⁾, we proposed an eight term Veneziano formula for $\bar{K}N$

$$A^0 = -\gamma_a^0 \frac{\Gamma(\frac{3}{2}-\alpha_\lambda)\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_\lambda-\alpha_t)} + \gamma_b^0 \frac{\Gamma(\frac{1}{2}-\alpha_\lambda)\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_\lambda-\alpha_t)},$$

$$B^0 = \beta_a^0 \frac{\Gamma(\frac{1}{2}-\alpha_\lambda)\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_\lambda-\alpha_t)} - \beta_b^0 \frac{\Gamma(\frac{3}{2}-\alpha_\lambda)\Gamma(2-\alpha_t)}{\Gamma(\frac{5}{2}-\alpha_\lambda-\alpha_t)},$$

$$A^1 = -\gamma_a^1 \frac{\Gamma(\frac{3}{2}-\alpha_\Sigma)\Gamma(1-\alpha_t)}{\Gamma(\frac{3}{2}-\alpha_\Sigma-\alpha_t)} + \gamma_b^1 \frac{\Gamma(\frac{3}{2}-\alpha_\Sigma)\Gamma(1-\alpha_t)}{\Gamma(\frac{5}{2}-\alpha_\Sigma-\alpha_t)},$$

$$B^1 = \beta_a^1 \frac{\Gamma(\frac{3}{2} - \alpha_\Sigma) \Gamma(1 - \alpha_t)}{\Gamma(\frac{5}{2} - \alpha_\Sigma - \alpha_t)} - \beta_b^1 \frac{\Gamma(\frac{3}{2} - \alpha_\Sigma) \Gamma(2 - \alpha_t)}{\Gamma(\frac{5}{2} - \alpha_\Sigma - \alpha_t)} . \quad (2.1)$$

The trajectories are completely determined by well-known particle masses, plus the universal slope constraint ¹²⁾

$$\begin{aligned} \alpha_\Lambda &= -0.62 + 0.9s, & \alpha_\Sigma &= -0.22 + 0.9s, \\ \alpha_t &= 0.482 + 0.9t. \end{aligned} \quad (2.2)$$

The Σ trajectory is that of $Y_1^*(1382)$; Σ_α is weakly coupled to $\bar{K}N$ ³⁾. Since then, similar $\bar{K}N$ studies have been performed by Meyer and Salin ¹³⁾ and by Namyslowski et al. ¹⁴⁾.

The γ_b^1 term is non-leading in both channels, and therefore of no interest for Regge fits. The other seven coupling constants must satisfy four constraints: (a) the parity doublets of $\Lambda(1116)$, $\Lambda(1535)$ and $\Sigma(1382)$ should cancel; (b) the f must decouple from the A amplitude, as follows by factorization from the known πN Regge formula ¹⁵⁾. To satisfy these constraints, we define the remaining three parameters g_Λ^2, a_f, c_n thus

$$\begin{aligned} \gamma_a^0 &= 3a_f, & \gamma_b^0 &= -4.01075 g_\Lambda^2, \\ \beta_a^0 &= 22.61947 g_\Lambda^2, \\ \beta_b^0 &= -15.89597 g_\Lambda^2 - 5.02778 a_f, \\ \gamma_a^1 &= -a_f, \\ \gamma_b^1 &= 3.20480 a_f - 1.49966 c_n + 13.69523 g_\Lambda^2, \end{aligned} \quad (2.3)$$

$$\beta_a^1 = -6.20278 a_p + 3.37772 c_n - 30.84611 g_\Lambda^2,$$

$$\beta_b^1 = -0.43095 a_p.$$

(2.3)

g_Λ^2 is the usual ¹⁶⁾ $\Lambda\bar{K}N$ coupling constant. Only its coefficients contain the $\Lambda(1116)$ pole. Therefore g_Λ^2 dominates K^+p backward scattering (Fig. 1h) a_p is the coupling of the $\rho+A_2$ to the A amplitude. It determines K^+n and K^-p charge exchange (Fig. 1g) (except very near forward), and also elastic polarization (Fig. 1e-f) since the P, f and ω all couple to B only. Equation (2.3) was so chosen that the g_Λ^2 and a_p terms cancel in the $K^\pm p$ total cross-section difference and the K^+p forward real part. These are therefore entirely determined by c_n . So with this arrangement, all three parameters g^2, a_p, c_n can be measured experimentally in a clear and uncorrelated way. For KN we used the full Veneziano formula (2.1), but for $\bar{K}N$ its Regge limit ¹⁰⁾.

The Pomeron is also known to decouple from the A amplitude ^{15), 10)}. Almost all recent Regge fits have concluded that the Pomeron is a moving Regge singularity. In particular, the Serpukhov pp experiment ²⁴⁾ gives $\alpha'_P = 0.40 \pm 0.09 \text{ GeV}^{-2}$. This number is explained by dual loops ²⁵⁾, which predict $\alpha'_P = \frac{1}{2}\alpha' = 0.45 \text{ GeV}^{-2}$. Dual loops also suggest ¹¹⁾ a formula for the Pomeron residue ^{*)}.

$$B(s, t) = \beta_P \frac{\gamma_P(t)}{\gamma_P(0)} \left[-e^{-i\pi\alpha_P/2} \right] \left[\frac{1}{2}(s-u) \right]^{\alpha_P-1},$$

(2.4)

*) Due to the small angle approximation, this expression suffers from backward blow up at energies $>20 \text{ GeV}/c$.

$$\sigma_p(t) = (4\alpha')^{\alpha_P} \cdot 4^{-\alpha_f} \cdot \frac{\Gamma(1 - \frac{\alpha_P}{2})}{\Gamma(\alpha_P/2)} \left\{ \frac{\Gamma(\frac{\alpha_P}{2} - \frac{\alpha_f}{2})}{\Gamma(\frac{\alpha_P}{2} - \frac{\alpha_f}{2} + \frac{1}{2})} \right\}^2 \quad (2.5)$$

where

$$\begin{aligned} \alpha_P &= 1 + \frac{1}{2}\alpha' t, \\ \alpha_f &= \frac{1}{2} + \alpha' t, \\ \alpha' &= 0.9 \text{ GeV}^{-2}. \end{aligned} \quad (2.6)$$

It has the property that $\text{Im } B$ remains positive for all t , despite the moving trajectory. The single parameter b_P here is determined by K^+p total cross-sections (Fig. 1a). Equations (2.5), (2.6) fit the width, shrinkage and shape of the K^+p diffraction peak out to $t = -2 \text{ GeV}^2$, without adjustable parameters (Fig. 1c). This Pomeron was not unitarized, but a Pomeron-Regge cut was included by the usual absorption model formula ¹⁰⁾ with $\lambda = 1$

$$T_\ell = T_\ell^P + T_\ell^R + 2i\lambda T_\ell^P T_\ell^R, \quad (2.7)$$

where T_ℓ is a partial wave projection with Argand normalization,

We thus have four parameters to explain the whole of the forward and backward KN and $\bar{K}N$ data. The solid curves in Fig. 1 show the result with (GeV units)

$$\begin{aligned} b_P &= 44, & g_1^2 &= 10.5, \\ a_f &= -34, & c_n &= 27. \end{aligned} \quad (2.8)$$

The fit is very satisfactory, except for the usual ^{10),13)} trouble with the K^-p cross-over (Fig. 1d). Similar data not shown are fitted equally well. $K^+p \rightarrow K^0p$ is virtually identical to $K^-p \rightarrow \bar{K}^0n$, as is true experimentally above 5 GeV/c ²⁶⁾. The K^+p forward real/imaginary part ratio at 5 GeV/c is -0.42, compared to the experimental ²⁷⁾ -0.52 ± 0.11 .

The dashed curves in Fig. 1 are a five-parameter formula, identical to (2.8) except that Pomeron-Regge cut in (2.7) was multiplied by $\lambda = 1.5$. Backward K^+p polarization would be very sensitive to λ . In EXD cut models, K^+p and pp Regge terms remain pure real, and so do not interfere with the Pomeron for small t . These processes thus rapidly approach pure Pomeron exchange, and their differential cross-sections are insensitive to secondary terms (Fig. 1c). Experimentally pp and K^+p have the strongest shrinkage of all elastic processes. This shows immediately that a moving Pomeron is essential in any EXD theory. The Pomeron-Regge cut thus moves the $K^\pm p$ cross-over from $t = -0.55$ by changing K^-p and leaving K^+p alone (Fig. 1c,d). In contrast, breaking EXD by shifting trajectories would change both, and also produce K^+p Argand loops. The two mechanisms are decidedly not equivalent.

We also tried other Pomerons. In particular a square root Pomeron trajectory

$$\alpha_P = 1 + 2\alpha'_P \sqrt{t_0} [\sqrt{t_0} - \sqrt{t_0 - t}], \quad (2.9)$$

$$\gamma_P(t) = (2M)^{-\alpha_P(t)} \exp[-a \sqrt{t_0 - t}], \quad (2.10)$$

with the values

$$\alpha'_P = 0.5, \quad t_0 = 0.16, \quad a = 5.76, \quad b_P = 50, \quad (2.11)$$

works well after eikonal unitarization ¹⁰⁾. It also fits πN with only the normalization adjusted to $b_p = 68$, which is not the case for (2.5). Despite their slope, partial wave analysis of either Pomeron does not give Schmid loops in K^+p , but only slow curves similar to the experimental phase shifts.

The Veneziano formula (2.1) with (2.8) gives reasonable $\bar{K}N$ partial widths for the leading Λ 's : $D_{03} = 5.7$ MeV (experiment 7.4), $F_{05} = 27.1$ MeV (experiment 52 ± 12), $G_{07} = 28.8$ MeV (experiment 25), D_{05} parity doublet = -7 MeV. The Σ 's are too narrow, e.g., $D_{25} = 14.1$ MeV (experiment 50). However, it is well known ²⁸⁾ that πp backward Regge fits extrapolate to Δ widths that are too small, so this disagreement does not mean we have underestimated Y_1^* exchange in the $\bar{K}N$ channel.

3. - PHASE SHIFT ANALYSIS

From the high energy fits we concluded that pessimistic errors on the Veneziano parameters would be

$$g_\Lambda^2 = 9 \pm 2, \quad a_g = -28 \pm 6, \quad c_n = 27 \pm 3. \quad (3.1)$$

The Pomeron formula has an incorrect centrifugal barrier in the imaginary part. At low energies, we therefore switched the Pomeron off completely, on the grounds that being mainly imaginary it cannot contribute to high waves. The high partial waves in the phase shift analyses were determined from the Veneziano formulae (2.1)-(2.3). Its three parameters g_Λ^2 , a_g , c_n were varied separately at each energy but using (3.1) as data. Table I shows the Veneziano partial waves for $g_\Lambda^2 = 10.5$, $a_g = -34$, $c_n = 27$. It gives real parts in Argand units.

The large low waves in Table I were not used. Instead these were subjected to normal phase shift analysis. The Veneziano formula makes all $I = 1$ K^+p partial waves repulsive, especially S_{21} in contrast to a previous Regge fit ⁷⁾. However, P_{01} in K^+n is strongly attractive (last column of Table I). KN dynamics is therefore much more favourable to a Z_0 resonance than to a Z_1 , which would have to depend entirely on inelastic channels. Table I would make very suitable left-hand cut contributions for partial wave dispersion relation fits. For KN the right-hand terms should be dual to the Pomeron and Regge cuts.

We then performed single-energy phase shift analyses of K^+p at 26 energies from 140 to 1900 MeV/c. This was done in February and March 1970. The angular data included are shown in Table II ²⁹⁾⁻⁴⁴⁾ (columns 3, 4). The shortest path linkage took place in August 1970. Some additional data ⁴⁵⁾⁻⁴⁸⁾ which had arrived in the meantime, were therefore used to discriminate among the solutions found (see below).

The total cross-sections were taken from Cool et al. ⁴⁹⁾ and Bugg et al. ⁵⁰⁾. The forward real parts were Martin and Perrin ⁵¹⁾ with 10% errors.

Each angular experiment had a normalization variable assigned. These were varied in the fits, subject to the quoted or estimated normalization errors. For bubble chamber experiments we used the tau decay normalization error, but also fitted their total inelastic as well as elastic differential cross-section (DCS), so that renormalization to the counter total cross-sections took place in the fit.

Much of the DCS data was obtained in LMN or butanol polarized targets. In general these check with each other and with hydrogen measurements. An exception is Rebka et al. ⁴³⁾, whose DCS at 1540 MeV/c disagrees badly with two other experiments (see Fig. 6a below), and was therefore omitted. Their DCS at 1710 MeV/c was included, but with a normalization error of 40% instead of the quoted 5%. The polarization of Rebka et al. ⁴³⁾ was used and gave no trouble.

The backward DCS of Carroll et al.⁴⁰⁾ were all multiplied by 1.6, and their normalization error increased to 115%, instead of the quoted 15%. This renormalization is clearly needed at the lower energies from comparison with bubble chamber experiments. We assumed the 1.6 factor was energy-independent, but since it was varied in the fits, this is not significant except for graph plotting.

The DCS of Barrelet et al.³¹⁾ at 735 MeV/c seems to be permanently preliminary, and gave no fit with $P(\chi^2) > 0.0036$. However, this low energy is not of much importance, so we simply lowered the $P(\chi^2)$ acceptance level here.

At each energy, the partial waves were divided into "low" and "high". The low waves were varied freely, subject to unitarity. The high waves were taken from the Veneziano formula, but with the three parameters varied. The upper limit of the high waves was when they became smaller than 10^{-6} . The dividing line between low waves and high waves changed with energy, and above 1 GeV/c we tried two alternatives. The last low waves are shown in column 5 of Table II. For each alternative and each energy a large number of trial solutions were used (column 6). These consisted of all single-energy solutions (not just shortest paths) found in the previous CERN phase shift analyses^{9), 52)}, together with many new random starting points. The method has been described in detail elsewhere⁵³⁾. The minimum distance d_0 between distinct solutions (column 7 of Table II) was defined so as to give 200 solutions at each energy. A small d_0 means that the solution is relatively unique at that energy. At the higher energies in KN, $d_0 \sim 5$ corresponds to marked grouping on the Argand diagrams [as illustrated in Refs. 9), 52)].

The best energy is 865 MeV/c, where all 200 solutions lie in a single small patch on the Argand diagram. This has a large repulsive S_1 . Previous analyses^{6), 9)} found alternative solutions with positive S_1 or with dominant P waves. They are eliminated by the Veneziano D_5 and F waves.

4. - SHORTEST PATHS

We have described our version of the shortest path technique in detail elsewhere ⁵³⁾. Here we just define some technical terms. Each solution at each energy has a shortest path up to it. This passes through one solution at each lower energy, and minimizes a certain length ⁵³⁾ chosen to impose smoothness as well as continuity in the Argand diagram. Since there are 200 solutions at each energy, there are 200 shortest paths up to that energy, but most only differ near the end. A branch is a type of shortest path distinct from all others for at least five energies. A leaf is the single-energy solution at the top of a shortest path. The different branches thus compete at each energy for the 200 available "leaves". A live branch is one which continues to the highest energy (2193 MeV mass = 1900 MeV/c). A dead branch is one which stopped before the last energy, because it lost all its leaves to other branches. Planting a dead branch means continuing it artificially by removing all the competing branches, thus forcing all paths to the last energy to pass through it. This is a trick we frequently tried to produce more alternatives. Usually the end of a dead branch is obviously unreasonable, so we lopped it before planting.

Previous analyses ^{9),52)} found a large inelastic D_5 , which was even proportional to the total inelastic cross-section at low energies (~ 1 GeV/c). In contrast P_1 remained elastic up to 1600 MeV/c. Both conclusions disagreed with the KN^* channel ³⁴⁾, and seem unphysical, since D_5 would be a D wave for KN^* and should be suppressed by the centrifugal barrier. We think that this large imaginary D_5 was a spurious effect, representing the neglected high partial waves and especially the Λ exchange pole. Table I shows that the Veneziano high waves, though individually small, are quite important cumulatively. Table II proves that, with them included, D_5 might remain elastic even up to 1500 MeV/c. However, if D_5 is held to the Veneziano value too long, the solutions get more unique with increasing energy, showing they have been over-constrained. Also D_5 then jumps in the shortest path as soon as it is let go. After trials, we found that the best place to release D_5 was 1350 MeV/c. This pushes its inelasticity into P_1 , without producing these other symptoms of bias.

Otherwise we left as many waves loose as possible. The asterisks in column 5 of Table II show the low/high wave boundaries which were finally chosen. Other choices were tried, but they made all the shortest paths much more jumpy and wobbly showing that the fits were either under- or over-constrained.

In the shortest path analysis, smoothness was imposed on groups of K consecutive energies, with K variable. As explained elsewhere ⁵³⁾, this was achieved by calculating the distances from solutions at one energy to a linear extrapolation of K preceding energies. $K \geq 2$ imposes smoothness as well as continuity. We found that $K = 3$ or 4 was needed for really neat paths.

The $K = 4$ analysis gave 11 live branches (A to K), plus five more (L to P) which died between 1940 and 2145 MeV mass. The $K = 3$ analysis gave eight live and five dead branches. Each $K = 3$ branch was visually the same as some $K = 4$ branch, though typically half the single-energy solutions on them were different ones. This shows encouraging stability of the physical results. Three branches (A, B, C) were very popular. Of the 200 "leaves" in the $K = 4$ analysis, 72, 24 and 48 were on branches A, B and C respectively. The $K = 3$ analysis gave 25, 99 and 18 leaves to A, B and C. The shortest path of all belonged to branch A for $K = 4$, and to B for $K = 3$. Branches I, K, L, O were found only for $K = 4$, but none of them is at all plausible.

The branches A - M are arranged on four "trunks" : A, C, G, L on trunk A ; B, D, E, F on trunk B ; H, I, J, K on trunk J ; M on trunk M. These trunks already diverge near 1700 MeV mass, but the branches on each trunk only separate between 1900 and 2000 MeV.

Just by inspecting the Argand diagrams, it was easy to decide between the different branches on each trunk. A, B, J and M are certainly to be preferred.

Branches A and B look very satisfactory, and are variants of the same non-resonant solution. They are shown as solutions IA and IB in Fig. 2 and Tables III, IV. Visually, A and B seem so close that it is surprising to find the shortest path separation so deep and stable. Branches D, F, I were poor relations of this solution I. D had P_1 elastic in disagreement with the KN^* data ³⁴⁾, and a clockwise semi-circle in D_5 . F differed mainly in being more wobbly. I had P_1 elastic and a hairpin in D_3 .

Most of the other branches could be excluded for obvious physical reasons. The commonest pathology was a sharp hairpin bend in P_3 , which started resonating, suddenly changed its mind, and retraced its steps. This was especially marked in branch C, but G, L, O, P also had P_3 hairpins. In branches E, H, K, D_3 was wobbly and P_1 had unreasonable looking structure. N died at only 1963 MeV and already had jumps and sharp angles in P_3 .

Thus only two branches gave natural alternatives to solution 1. J differed in that P_1 stayed elastic to 1940 MeV, and then moved rapidly to the centre of the Argand diagram. Its P_3 described nearly a semi-circle and then stopped still after 1900 MeV.

M was a dead branch ending at 2002 MeV in the $K = 4$ analysis, and at 2039 MeV for $K = 3$. It had a nice little resonance in P_3 agreeing with the Cool bump ^{49),50)}, but its D_5 showed sudden breaks.

Figure 2 also shows the partial wave projection of the Regge solution at 2278 - 2687 MeV, calculated with the Pomeron (2.9) and eikonal unitarization. It is obviously very close to solution A. In fact the shortest path up to the Regge solution (the version shown) gave branch A, with only the last four (of 26) energies slightly changed from the shortest path with Regge omitted. It even chose an experimentally better solution at 1780 MeV/c. The Pomeron (2.5) with Regge-Pomeron cut but no unitarization again gave a shortest path on branch A. However, changing to $\lambda = 1.5$ in (2.7), moved the Regge fit to branch B. Note that this convergence of the shortest path to the Regge solution at

2 GeV/c is very non-trivial, since only waves $\geq G9$ were held to the Regge values in the phase shift analysis here.

In calculating these shortest paths, we excluded only solutions with $P(\chi^2) < (1/1000) P_{\max}(E)$ where $P_{\max}(E)$ is the best $P(\chi^2)$ at the same energy. However we also tried omitting everything with $P(\chi^2) < (1/10) P_{\max}(E)$. This had the following effects :

- (1) - the branches F, H, I, J, K, N, P disappeared completely ; thus the entire J trunk depends on poor solutions ;
- (2) - M acquired a continuation to the top energy, though not a good one since P_1 and D_3 were both very wobbly ; its P_3 looked rather like solution 2 below ;
- (3) - D split ;
- (4) - the top end of B changed appreciably, so that even the $\lambda = 1.5$ Regge fit was now closer to A.

In all our solutions, and in almost all other recent analyses ^{1),2)}, P_3 is attractive at low energies and describes at least a semi-circle anticlockwise. However, we would still distinguish two cases. In resonant solutions, P_3 has a small fast circle which agrees with the Cool bump in the total cross-section ^{49),50)}. In non-resonant solutions, P_3 moves towards the centre of the Argand diagram. In doing so, it describes a very slow circular arc, but this bears no relation to the total cross-section bump. Paths A, B and J are all non-resonant by this definition.

We made a determined effort to continue and improve M by "planting", since it was the only branch with a clear exotic resonance. This produced six variants with fairly reasonable Argand plots. However, only the two in Fig. 3 survived comparison with new data (see below). Their speed plots (Fig. 4) do not favour a resonance interpretation.

We also tried lopping and planting branches J and F, but this gave nothing new.

5. - DATA AND AMPLITUDE COMPARISONS

Graphical comparisons with the whole of the K^+p data ²⁹⁾⁻⁵¹⁾ (including experiments ⁴⁵⁾⁻⁴⁸⁾ which arrived too late to be included in the fits) were performed for the branches A, B, J and all six variants of M. There are no good χ^2 whatsoever at 735 MeV/c, where the data ³¹⁾ seem to be permanently preliminary. At 1087 MeV/c, A and B fit the polarization rather better than M or J. These differences become very marked at 1124 MeV/c, where unfortunately there is no polarization data. At 1256 MeV/c the new polarization measurements of Hughes et al. ⁴⁶⁾, which were not included in the fit, completely eliminated three of the six M variants. Also J was rather too large in the backward direction, and B in the middle here (Fig. 5). At 1317 MeV/c, J and B were similarly poor fits to the polarization used ³⁵⁾. At 1367 MeV/c, another M variant was eliminated by Hughes et al. ⁴⁶⁾ new backward polarization. Above 1500 MeV/c, the big normalization errors on the DCS experiments become an important cause of uncertainty (Figs. 6a, 7a). At 1786 MeV/c, two of the already eliminated M variants gave a large backward polarization, in disagreement with Hill et al. ⁴⁸⁾. Also B, J and some of the M variants went through solutions which disagreed with the polarization near $\cos\theta = -0.3$ (Fig. 7b).

Thus both B and J contain some single energy solutions in poor agreement with polarization data. As mentioned in Section 4, omitting these bad χ^2 solutions destroys branch J completely. Branch B is not destroyed but merely has its top end shifted, though in a direction away from the Regge fit.

A further argument against J and two of the eliminated M variants (as well as D, G, I and L), is that they made P_1 almost elastic up to 1350 MeV/c, in disagreement with KN^* experiments ³⁴⁾.

Next we calculated the A and B amplitudes for comparison with Regge predictions. Duality requires that K^+p be pure real in the backward direction ³⁾. The backward imaginary parts were small for branches A and B (Fig. 9), but large for E, J and M. Now, if an exotic

exists as in branch M, then exchange degeneracy may be seriously broken. However, it seems very reasonable to require non-resonant K^+p solutions to be real backwards. This gives further evidence against J and E.

In the forward direction, s channel helicity conservation for the Pomeron would require the forward peak to occur in Im B but not in Im A ^{15),10)}. Branch A comes nearest to satisfying this (Fig. 8). J in particular would decouple the Pomeron from Im B instead of Im A.

As we noted in Section 4, only A, B, J and M give physically reasonable Argand diagrams. New data clearly exclude four of the six M variants. We thought J now had enough black marks to be dropped also.

The four remaining solutions are IA and IB, corresponding to the shortest paths A and B, and II, III, which are the survivors from the "planting" of branch M. They are shown in Figs. 2, 3 and Tables III-VI. III has a clear P_3 resonance in the Argand diagram (Fig. 3), which agrees with the Cool bump in the total cross-section ^{49),50)}. II is "semi-resonant", breaking off half way round the circle. The last four energies of II are identical to IB. Figure 4 shows speed plots of P_3 for the four solutions. They do not favour a resonance interpretation. In fact the only solution with a regular peak in the speed is IB at 1850 MeV. This looks the least resonant of all on the Argand diagram !

The following experiments are needed (all equally urgent) :

- (1) - polarized target measurements at 1124 and 1170 MeV/c, especially the former where there are 0.4 differences between solutions. Only solution I interpolates smoothly between the existing measurements at 1087 and 1215 MeV/c.
- (2) - better backward polarization measurements ($\cos\vartheta < -0.75$) at 1455, 1540, 1636, 1710, 1900 MeV/c. The most important angle is $\cos\vartheta = -0.9$. At 1455 MeV/c, 0.1 accuracy would be needed to distinguish solutions, but even cruder measurements would help at the higher energies.

- (3) - bubble chamber experiments at 1540, 1636, 1710, 1786 MeV/c, mainly to determine the relative normalization of total and elastic cross-sections ;
- (4) - backward polarizations ($u > -1$) in the 2 - 6 GeV/c region, and also better backward differential cross-sections (and at higher energies if possible). As pointed out by Berger and Fox²⁸⁾, these are crucial for Regge cut theories ;
- (5) - more polarized target experiments above 1900 MeV/c, at least at 2.11 and 2.31 GeV/c, are essential if phase shift analysis is to be extended upwards.

Apart from (1), further interpolation of the data below 1900 MeV/c does not seem very urgent. The accuracy of the experiments, rather than their spacing, is at present the limiting factor in detecting structure. However, we issue the usual warning to owners of bubble chambers and of polarized targets to correlate their plans, so that they measure at the same energies.

5. - CONCLUSIONS

We emphasize that the non-resonant solutions IA and IB (Fig. 2) emerged naturally from the shortest path method, while II and III (Fig. 3) are artificial constructions. Comparison with Regge fits at fixed ℓ or at fixed u also independently favours solution I. Furthermore the polarization predictions of II and III do not interpolate smoothly at 1124 MeV/c. The existing K^+p experiments therefore do not require an exotic resonance, and even disfavour a clear circle. Polarization fits and Regge comparison taken together, slightly prefer IA to IB.

The three Veneziano coupling constants g_Λ^2 , a_g and c_n of Eqs. (2.1)-(2.3) were varied in the single energy fits, subject to the high energy constraints (3.1). We found that the solutions on the shortest paths all gave very consistent results. a_g and c_n stayed at the

values (3.1), but g_{Λ}^2 almost invariably increased from the input 9 ± 2 , indicating that it is being independently determined. Since a Λ exchange peak is clearly visible in the data from 700 MeV/c to 7 GeV/c, it is hardly surprising that the Λ coupling constant should be measurable in K^+p phase shift analysis. The smallest errors are obtained near 1 GeV/c, giving $g_{\Lambda}^2 = 10.5 \pm 1.5$. In fact the neglect of Λ exchange in previous phase shift analyses ^{1),2),5),9),52)} was directly responsible for their pathologically inelastic D_5 in this region. Since altogether we have 26 single energy determination, plus one from high energy backward K^+p , $g_{\Lambda}^2 = 10.5 \pm 1.5$ would be a realistic rather than statistical error. The extrapolation can be checked from the Y_0^* widths, which would increase g_{Λ}^2 slightly (Section 2). Values from 3.9 ± 0.8 to 13.5 ± 2.1 have been claimed from Kp forward dispersion relations ¹⁶⁾. A recent analysis ⁵⁴⁾ of backward $K^-n \rightarrow \pi^- \Lambda$ gave $g_{\Lambda}^2 = 14.5 \pm 5.0$.

The present work of course assumes Argonne Regge phenomenology ^{6) 10),12)}. The pole residues are exchange degenerate. In K^+p they are pure real ³⁾, but in other processes they will generate dips by nonsense zeros. The contrary Michigan view that dips are due to pole-cut cancellation would predict one in K^+p also at $u = -0.15$. The Argonne and Michigan mechanisms lead to completely different amplitudes ^{28),55)}, and can therefore be distinguished by comparison with phase shift analysis, if one believes duality. Despite some qualitative speculation about making it dual, we note that in practice Michigan authors do not seem to trust their model below 8 GeV/c. In contrast, we have explained K^+p scattering from threshold to the horizon, and much of $\bar{K}N$ as well.

We are grateful to Drs. C. Schmid and C. Michael for discussions, and to the CERN - Holland group for data communication.

Mass	1505.1	1607.0	1689.2	1730.1	1779.5	1834.3
S_1	-0.82183	-1.06021	-1.11091	-1.11195	-1.09895	-1.07135
P_1	-0.07045	-0.19917	-0.28897	-0.32637	-0.36478	-0.39892
P_3	-0.00381	-0.00931	-0.01384	-0.01638	-0.01980	-0.02405
D_3	-0.00417	-0.02592	-0.05256	-0.06739	-0.08590	-0.10651
D_5	-0.00191	-0.01211	-0.02505	-0.03245	-0.04187	-0.05261
F_5	-0.00027	-0.00366	-0.01015	-0.01462	-0.02096	-0.02901
F_7	-0.00005	-0.00054	-0.00138	-0.00193	-0.00271	-0.00369
G_7	-0.00002	-0.00051	-0.00192	-0.00310	-0.00499	-0.00771
G_9	-0.00001	-0.00017	-0.00065	-0.00106	-0.00173	-0.00270
H_9		-0.00007	-0.00036	-0.00066	-0.00118	-0.00203
H_{11}		-0.00002	-0.00007	-0.00012	-0.00022	-0.00036
I_{11}		-0.00001	-0.00007	-0.00014	-0.00028	-0.00053
I_{13}			-0.00002	-0.00004	-0.00008	-0.00015
J_{13}			-0.00001	-0.00003	-0.00007	-0.00014
J_{15}				-0.00001	-0.00001	-0.00003
K_{15}				-0.00001	-0.00002	-0.00004
K_{17}						-0.00001
L_{17}						-0.00001

- TABLE I -

K^+_p partial waves from Veneziano fit

- TABLE I -

Continued

Mass	1893.5	1962.6	2039.4	2112.9	2192.7	1834.3 I = 0
S ₁	-1.03073	-0.97432	-0.90546	-0.83741	-0.76425	-0.07405
P ₁	-0.42654	-0.44779	-0.45953	-0.46114	-0.45452	0.41882
P ₃	-0.02919	-0.03576	-0.04360	-0.05138	-0.05984	-0.26400
D ₃	-0.12818	-0.15188	-0.17549	-0.19490	-0.21224	0.06121
D ₅	-0.06422	-0.07733	-0.09090	-0.10257	-0.11357	-0.04230
F ₅	-0.03863	-0.05062	-0.06437	-0.07742	-0.09096	0.02275
F ₇	-0.00490	-0.00648	-0.00845	-0.01053	-0.01300	-0.02146
G ₇	-0.01136	-0.01650	-0.02316	-0.03027	-0.03856	0.00505
G ₉	-0.00405	-0.00599	-0.00859	-0.01146	-0.01492	-0.00422
H ₉	-0.00328	-0.00523	-0.00803	-0.01129	-0.01543	-0.00143
H ₁₁	-0.00057	-0.00090	-0.00136	-0.00191	-0.00261	-0.00145
I ₁₁	-0.00094	-0.00165	-0.00276	-0.00418	-0.00614	0.00035
I ₁₃	-0.00028	-0.00049	-0.00083	-0.00128	-0.00192	-0.00033
J ₁₃	-0.00027	-0.00052	-0.00094	-0.00153	-0.00240	0.00009
J ₁₅	-0.00006	-0.00011	-0.00019	-0.00030	-0.00047	-0.00010
K ₁₅	-0.00008	-0.00016	-0.00032	-0.00056	-0.00093	0.00002
K ₁₇	-0.00002	-0.00004	-0.00009	-0.00015	-0.00026	-0.00002
L ₁₇	-0.00002	-0.00005	-0.00011	-0.00020	-0.00036	0.00001
L ₁₉	-0.00001	-0.00001	-0.00002	-0.00004	-0.00008	-0.00001
M ₁₉	-0.00001	-0.00002	-0.00004	-0.00007	-0.00014	
M ₂₁		-0.00000	-0.00001	-0.00002	-0.00004	
N ₂₁		-0.00001	-0.00001	-0.00003	-0.00006	
N ₂₃				-0.00001	-0.00001	
O ₂₃				-0.00001	-0.00002	
O ₂₅					-0.00001	
B ₂₅					-0.00001	

Mass MeV	P _{lab} MeV/c	Data references		Last low wave	Trials	d ₀
		DCS	POL			
1444.8	140	29	--	S ₁	60	0.1
1451.7	175	29	--	S ₁	176	0.1
1458.6	205	29	--	S ₁	187	0.1
1466.4	235	29	--	S ₁	175	0.1
1475.1	265	29	--	S ₁	199	0.1
1505.1	355	29	--	P ₁	415	0.1
1571.6	520	29,30	--	P ₁	1085	0.1
1607.0	600	29	--	P ₁	485	0.1
1669.1	735	31,32	--	D ₃	916	0.5
1689.2	778	33,32	(33)	D ₃	379	2
1730.1	865	34,35	35,(36)	D ₃	648	1.5
1746.6	900	37,38	(38)	D ₃	544	5
1779.5	970	34,35,37,39,40	35	D ₃	344	4
1834.3	1087	35,37,40	35	D ₃ *	529	2
				D ₅	481	5
1851.5	1124	37	--	D ₃ *	424	20
				D ₅	408	25
1872.8	1170	39,40	--	D ₃ *	421	20
				D ₅	349	20
1893.5	1215	34,35,37,40	35	D ₃ *	534	5
				D ₅	504	12
1910.9	1253	37	--	D ₃ *	494	15
				D ₅	390	15
1940.0	1317	35,37,40	35	D ₃ *	652	3
				F ₅	463	10
1962.6	1367	34,35,37,40	35,41	D ₃	542	3
				F ₅ *	531	15
2001.9	1455	35,37,40,42	35,41	D ₃	600	5
				F ₅ *	360	15
2039.4	1540	35,40	35	F ₅	602	15
				F ₇ *	415	15
2081.1	1636	35,40	35	F ₅	718	12
				F ₇ *	525	20
2112.9	1710	35,40,43	35,41,43	F ₅	598	12
				F ₇ *	559	20
2145.1	1786	35,40	35	F ₅	714	12
				F ₇ *	558	20
2192.7	1900	39,40,44	41	F ₅	506	15
				G ₇ *	344	20

- TABLE II -

Single energy searches

Mass	$10+\log P(\chi^2)$	S_1	P_1	P_3	D_3	D_5	F_5	F_7	G_7	G_9	H_9	H_{11}
1432	8.960	0	0	0	0	0	0	0	0	0	0	0
1445	8.335	-11	0	0	0	0	0	0	0	0	0	0
1452	8.971	-17	0	0	0	0	0	0	0	0	0	0
1459	8.261	-20	-1	0	0	0	0	0	0	0	0	0
1466	9.430	-20	-2	0	0	0	0	0	0	0	0	0
1475	9.301	-25	-2	0	0	0	0	0	0	0	0	0
1505	8.394	-31	-4	0	0	0	0	0	0	0	0	0
1570	8.754	-43	-9	0	0	0	0	0	0	0	0	0
1607	8.574	-46	-10	0	-1	0	0	0	0	0	0	0
1660	6.200	-47	-14	0	-2	0	0	0	0	0	0	0
1689	6.164	-49	-15	0	-2	0	0	0	0	0	0	0
1730	8.370	-49	-18	1	-6	0	-1	0	0	0	0	0
1747	8.524	-48	-21	1	-2	0	-1	0	0	0	0	0
1780	8.365	-49	-24	3	-4	0	-1	0	0	0	0	0
1824	8.295	-43	-31	4	-7	0	-2	0	0	0	0	0
1851	9.012	-45	-34	9	-9	0	-2	0	0	0	0	0
1873	6.101	-31	-35	9	-13	0	-2	0	0	0	0	0
1893	8.709	-30	-42	11	-12	0	-2	0	0	0	0	0
1911	8.403	-23	-42	14	-13	0	-3	0	0	0	0	0
1940	6.103	-22	-45	14	-11	0	-3	0	-1	0	0	0
1963	6.626	-26	-45	14	-14	0	-3	0	-1	0	0	0
2002	6.521	-34	-33	14	-14	0	-2	0	-2	0	0	0
2039	6.463	-43	-42	15	-17	3	-6	0	-2	0	0	0
2081	6.297	-48	-30	13	-18	5	-8	6	-2	-1	0	0
2113	6.557	-46	-27	14	-19	6	-6	6	-2	0	0	0
2145	7.941	-49	-25	15	-16	9	-8	7	-2	-1	-1	0
2193	8.422	-45	-28	13	-19	10	-9	6	-2	-1	-1	0

- TABLE III -

SOLUTION IA

100 x real and imaginary parts of partial waves

Mass	$10+\log P(\chi^2)$	S_1	P_1	P_3	D_3	D_5	P_5	P_7	G_7	G_9	H_9	H_{11}
1432	0.910	0	0	0	0	0	0	0	0	0	0	0
1445	3.319	-11, 2	0	0	0	0	0	0	0	0	0	0
1452	3.574	-17, 3	0	0	0	0	0	0	0	0	0	0
1459	3.206	-20, 4	0	0	0	0	0	0	0	0	0	0
1456	5.405	-20, 5	0	0	0	0	0	0	0	0	0	0
1475	9.103	-25, 6	0	0	0	0	0	0	0	0	0	0
1505	5.484	-31, 11	0	0	0	0	0	0	0	0	0	0
1572	8.354	-43, 25	0	0	0	0	0	0	0	0	0	0
1607	8.576	-46, 39	0	0	0	0	0	0	0	0	0	0
1669	6.368	-47, 35	0	0	0	0	0	0	0	0	0	0
1639	9.164	-49, 40	0	0	0	0	0	0	0	0	0	0
1750	6.370	-49, 42	0	0	0	0	0	0	0	0	0	0
1747	6.479	-49, 45	0	0	0	0	0	0	0	0	0	0
1787	9.264	-48, 46	0	0	0	0	0	0	0	0	0	0
1834	6.414	-46, 48	0	0	0	0	0	0	0	0	0	0
1851	9.608	-49, 47	0	0	0	0	0	0	0	0	0	0
1873	6.434	-46, 43	0	0	0	0	0	0	0	0	0	0
1893	6.554	-41, 46	0	0	0	0	0	0	0	0	0	0
1911	6.574	-25, 49	0	0	0	0	0	0	0	0	0	0
1940	7.372	-15, 53	0	0	0	0	0	0	0	0	0	0
1963	6.555	-2, 55	0	0	0	0	0	0	0	0	0	0
2002	6.106	4, 57	0	0	0	0	0	0	0	0	0	0
2039	6.394	3, 61	0	0	0	0	0	0	0	0	0	0
2081	6.421	-2, 70	0	0	0	0	0	0	0	0	0	0
2113	6.451	0, 66	0	0	0	0	0	0	0	0	0	0
2145	7.175	-2, 65	0	0	0	0	0	0	0	0	0	0
2193	6.645	-2, 60	0	0	0	0	0	0	0	0	0	0

- TABLE IV -

SOLUTION 1B

100 x real and imaginary parts of partial waves

Mass	10+logP(χ^2)	S ₁	P ₁	P ₃	D ₃	D ₅	F ₅	F ₇	G ₇	G ₉	H ₉	H ₁₁
1432	3.500	0	0	0	0	0	0	0	0	0	0	0
1445	3.309	-11	0	0	0	0	0	0	0	0	0	0
1452	3.974	-17	0	0	0	0	0	0	0	0	0	0
1459	7.753	-17	0	0	0	0	0	0	0	0	0	0
1466	9.352	-22	-1	0	0	0	0	0	0	0	0	0
1475	8.641	-22	-2	0	0	0	0	0	0	0	0	0
1505	8.973	-30	-5	0	0	0	0	0	0	0	0	0
1572	8.354	-43	1	0	0	0	0	0	0	0	0	0
1607	7.693	-45	12	0	-1	0	0	0	0	0	0	0
1669	6.611	-47	-15	0	-2	0	0	0	0	0	0	0
1689	9.209	-48	-14	0	-2	-1	0	0	0	0	0	0
1730	8.306	-49	-13	0	-2	-2	0	0	0	0	0	0
1747	8.481	-49	-12	0	-2	-2	0	0	0	0	0	0
1780	8.364	-42	-15	0	-7	-2	0	0	0	0	0	0
1834	7.408	-24	-17	2	-13	-2	-1	0	0	0	0	0
1851	8.443	-23	-16	5	-10	-6	-2	0	-1	0	0	0
1873	8.715	-15	-21	12	-14	-2	-2	0	0	0	0	0
1893	8.679	-13	-20	11	-16	-5	-2	0	0	0	0	0
1911	8.341	-18	-26	14	-14	1	-3	0	0	0	0	0
1940	8.470	-10	-30	18	-14	-1	-2	0	-1	0	0	0
1963	8.364	-17	-26	20	-14	-2	-2	0	-1	0	0	0
2032	9.277	-8	-29	21	-18	-6	-3	-1	-2	0	0	0
2039	8.530	-9	-22	33	-16	-4	-5	0	-2	0	0	0
2081	8.420	-2	-33	36	-16	-10	-7	0	-2	-1	-1	0
2113	8.451	0	-29	34	-14	-7	-6	6	-2	-1	-1	0
2145	7.175	-2	-28	35	-17	-9	-8	5	-2	-1	-1	0
2193	8.645	-2	-23	34	-13	-6	-8	5	-2	-2	-1	0

- TABLE V -

SOLUTION II

100 x real and imaginary parts of partial waves

Mass	$10 \log P(\chi^2)$	S_1	P_1	P_3	D_3	D_5	F_5	F_7	G_7	G_9	H_9	H_{11}
1432	0.000	0	0	0	0	0	0	0	0	0	0	0
1445	5.309	-11	0	0	0	0	0	0	0	0	0	0
1452	8.974	-17	0	0	0	0	0	0	0	0	0	0
1459	7.753	-17	0	0	0	0	0	0	0	0	0	0
1466	9.332	-22	0	0	0	0	0	0	0	0	0	0
1475	8.641	-22	0	0	0	0	0	0	0	0	0	0
1505	8.978	-30	0	0	0	0	0	0	0	0	0	0
1572	8.354	-43	1	0	0	0	0	0	0	0	0	0
1607	7.698	-45	1	0	0	0	0	0	0	0	0	0
1669	6.311	-47	2	1	0	0	0	0	0	0	0	0
1689	9.209	-48	3	5	0	0	0	0	0	0	0	0
1730	8.306	-49	3	6	0	0	0	0	0	0	0	0
1747	6.461	-49	3	14	0	0	0	0	0	0	0	0
1760	8.364	-42	3	13	0	0	-1	0	0	0	0	0
1834	7.498	-24	3	10	0	0	-1	0	0	0	0	0
1851	8.443	-23	4	3	0	0	-1	0	0	0	0	0
1873	8.716	-15	10	12	0	0	-2	0	-1	0	0	0
1893	8.679	-13	16	5	0	0	-2	0	0	0	0	0
1911	8.102	-24	18	6	1	0	-2	0	0	0	0	0
1940	8.632	-19	32	13	7	0	-2	0	-1	0	0	0
1963	8.399	-19	32	8	8	0	-2	0	-1	0	0	0
2002	8.873	-15	45	6	10	0	-2	0	-2	0	0	0
2039	8.320	-8	50	6	10	0	-4	2	-2	0	0	0
2081	8.260	-4	62	3	11	0	-9	2	-2	-1	0	0
2113	8.819	0	60	1	9	0	-12	4	-2	0	-1	0
2145	7.735	-4	62	-1	8	0	-15	7	-2	-1	-1	0
2193	8.555	7	46	6	9	0	-10	5	-2	-1	-1	0

- TABLE VI -

SOLUTION III

100 x real and imaginary parts of partial waves

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FIGURE CAPTIONS

Figure_1 Regge fit. Solid line $\lambda = 1$, dotted line $\lambda = 1.5$.

There are only four other adjustable parameters.

- (a) K^+p total cross-sections ¹⁷⁾ ;
- (b) K^-p total cross-sections ¹⁸⁾ ;
- (c) K^+p elastic DCS at 2.3, 6.9, 14.8 GeV/c ¹⁹⁾ ;
- (d) K^-p elastic DCS at 5.4, 9.5, 15.91 GeV/c ²⁰⁾ ;
- (e) K^+p polarization at 6 GeV/c ²¹⁾ ;
- (f) K^-p polarization at 6 GeV/c ²¹⁾ ;
- (g) $K^-p \rightarrow \bar{K}^0 n$ at 4.2, 12.3 GeV/c ²²⁾ ;
- (h) K^+p backward DCS at 3.55, 5.2, 6.9 GeV/c ²³⁾.

Figure_2 Shortest path phase shift solutions IA (solid) and IB (dashed).
The highest mass is 2193 MeV. The dot-dashed line is the
partial wave projection of the Regge fit at 2278-2687 MeV.

Figure_3 Phase shift solutions II (solid) and III (dashed).

Figure_4 Velocity of P_3 in the four phase shift solutions.

Figure_5 Predicted polarization at 1912 MeV (1256 MeV/c).
The data ⁴⁶⁾ were not included in the fit.

Figure_6 Fit at 2039 MeV (1540 MeV/c) against used data ^{35),40),43)}
and subsequent data ^{45),46)}.
(a) DCS. All experiments have large normalization errors.
(b) Polarization.

Figure_7 Fit at 2145 MeV (1786 MeV/c) against used data ^{35),40)} and
subsequent data ^{45),46),48)}.
(a) DCS. All experiments have large normalization errors.
(b) Polarization.

Figure_8 Forward amplitudes at 2093 MeV from the four solutions :

(a) Im A,

(b) Im B.

Pomeron helicity conservation requires that only Im B have a forward peak.

Figure_9 Backward amplitudes at 2093 MeV from the four solutions :

(a) Im A,

(b) Im B.

Exchange degeneracy requires that the backward amplitudes be pure real.

FIG. 2 $\bullet\text{---}\bullet$ Sol.IA $\circ\text{---}\circ$ Sol.IB --- Regge

FIG.3 —●— Sol.II ○---○ Sol.III

FIG. 4

POLARIZATION AT 2039 MeV [1540 MeV/c]

FIG.6b

FIG. 7a