CERN LIBRARIES, GENEVA

CM-P00058696

Ref.TH.1225-CERN

REGGE CUTS AND ASYMPTOTIC SUM RULES FOR π N FORWARD SCATTERING

A.D. Martin ^{*)} and R. Wit ^{**)}

CERN - Geneva

A B S T R A C T

A dispersion representation for the inverse of the forward scattering is derived, which leads to a useful sum rule constraint on the high energy data. The sum rule is applied to an analysis of pion-nucleon forward scattering data.

^{*)} Present address: University of Durham, England.

^{**)} On leave of absence from the Jagellonian University, Cracow, Poland.

1. INTRODUCTION

The recent measurements of high energy total cross-sections at Serpukhov ¹⁾ have stimulated interest in the possible asymptotic behaviour of scattering amplitudes. The validity of the Pomanchuk theorem has been widely discussed ²⁻⁸⁾ and the necessity for Regge-cut-like terms has got further support ⁹⁾. Regge cuts, which can be regarded as arising from multiple scattering or absorptive effects, have provided a plausible explanation of various deficiencies in the pure Regge pole model ¹⁰⁾. However, the theoretical basis of the prescriptions for calculating the effect of cut contributions is at a rudimentary stage. Their inclusion therefore introduces further ambiguities in a phenomenological analysis of high energy data and so constraints are highly desired. Finite energy sum rules, continuous moment sum rules, finite contour dispersion relations - all these techniques reflect the need of (more or less) rigorously obtained equations which may constrain the high energy parameters in terms of the known low energy behaviour of the amplitudes. A deficiency of these analyticity constraints is that they are often to a large extent automatically satisfied. In an attempt to overcome this we propose the use of an asymptotic sum rule for the inverse of the forward scattering amplitude. We derive the relation in Section 2 and apply it to the analysis of the high energy forward πN scattering data in Section 3.

2. THE ASYMPTOTIC SUM RULE

We consider the use of analyticity constraints for πN scattering because the high energy data (total cross-sections and the real parts of the forward scattering amplitudes) are much better for this reaction than for other reactions. Moreover, the analytic structure of the πN amplitudes is relatively simple.

We denote the $\pi^{\pm}p$ forward scattering amplitudes in the laboratory system by

$$T_{\pm}(\omega) = D_{\pm}(\omega) + i H_{\pm}(\omega)$$

and choose the normalization, so that the optical theorem is

$$H_{\pm}(\omega) = \frac{\sqrt{\omega^2 - \mu^2}}{4\pi} \sigma_{\pm}(\omega) \quad (1)$$

where ω is the lab. energy and μ is the mass of the incoming pion. Because of the $s \leftrightarrow u$ crossing symmetry it is convenient to work with the following linear combinations

$$\begin{aligned} T_1 &= \frac{1}{2} (T_- + T_+), & \sigma_1 &= \frac{1}{2} (\sigma_- + \sigma_+), \\ T_2 &= \frac{1}{2} (T_- - T_+), & \sigma_2 &= \frac{1}{2} (\sigma_- - \sigma_+). \end{aligned}$$

In deriving a constraint we wish to make use of the property that the imaginary parts of $T_{\pm}$, and therefore of T_1 , are positive for physical values of ω . The Froissart bound, $|T(\omega)| \leq C\omega(\ln \omega)^2$, allows to write the dispersion formula for $T_1(\omega)$ with only one subtraction

$$T_1(z) = T_1(\mu^2) + \frac{f^2}{M} \frac{z - \mu^2}{(z - \omega_0^2) (1 - (\frac{\mu}{2M})^2)} + \frac{(z - \mu^2)}{\pi} \int_{\mu^2}^{\infty} \frac{dx H_2(x)}{(x - \mu^2)(x - z)} \quad (2)$$

where $z = \omega^2$ and $\omega_0 = \mu^2/2M$. Now provided that $T_1(\mu^2) < 0$ it follows using the positivity in Eq. (2) that

the function

$$F(z) = T_1(z) - \frac{f^2}{M} \frac{z - \mu^2}{(z - \omega_0^2) \left(1 - \left(\frac{\mu}{2M}\right)^2\right)}$$

does not have any zero ¹¹⁾. On the other hand, if the total cross-section $\sigma_1(\omega)$ tends to a non-zero limit as $\omega \rightarrow \infty$ we have ¹²⁾

$$\lim_{\omega \rightarrow \infty} i \frac{\omega}{F(\omega)} = \frac{4\pi}{\sigma_1(\infty)}.$$

Combining these two results one can write the following Cauchy formula

$$\frac{i\sqrt{z-a^2}}{F(z)} = \frac{4\pi}{\sigma_1(\infty)} + \frac{1}{\pi} \int_{\mu^2}^{\infty} \frac{dx \operatorname{Im} [F^*(x) \sqrt{x-a^2}]}{|F(x)|^2 (x-z)} \quad (3)$$

where a is an arbitrary real constant satisfying $\mu \leq a < \infty$ and the function $\varphi(z) = i\sqrt{z-a^2}$ is chosen in such a way that

- i) $\varphi(z) = \varphi^*(z^*)$
- ii) $\varphi(x) < 0$ for $x < a^2$.

We may rewrite Eq. (3) in the form

$$\begin{aligned} \frac{i\sqrt{z-a^2}}{F(z)} + \frac{1}{\pi} \int_{\mu^2}^{a^2} \frac{dx \sqrt{a^2-x} H_1(x)}{|F(x)|^2 (x-z)} = \\ = \frac{4\pi}{\sigma_1(\infty)} + \frac{1}{\pi} \int_{a^2}^{\infty} \frac{dx \sqrt{x-a^2} \operatorname{Re} F(x)}{|F(x)|^2 (x-z)} . \quad (4) \end{aligned}$$

This dispersion formula, valid for all z , yields an extremely simple sum rule for $z = a^2$:

$$\int_{\mu^2}^{a^2} \frac{dx H_1(x)}{\sqrt{a^2-x} |F(x)|^2} = \frac{4\pi^2}{\sigma_1(\infty)} + \int_{a^2}^{\infty} \frac{dx \operatorname{Re} F(x)}{\sqrt{x-a^2} |F(x)|^2} . \quad (5)$$

What are the advantages of our dispersion formula (4) as compared to relation (2)? First of all, we would like to emphasize the positivity aspect. For real $z = y > \mu^2$

$$\operatorname{Re} T_1(y) = T_1(\mu^2) + \text{pole term} + \frac{y-\mu^2}{4\pi^2} \text{P.V.} \int_{\mu^2}^{\infty} \frac{dx \sigma_2(x)}{\sqrt{x-\mu^2} (x-y)} .$$

At the same time, however,

$$\text{P.V.} \int_{\mu^2}^{\infty} \frac{dx}{\sqrt{x-\mu^2}} \frac{1}{x-y} = 0 , \quad y > \mu^2 ,$$

which means that subtracting any constant from the averaged total cross-section we do not influence at all the values of $\text{Re}T_1(y)$. Such a situation may be very convenient for the numerical determination of the real part [see, for example, Ref. 13)] but is certainly not an advantage when seeking high energy constraints. To some extent this deficiency is not present in the formula (3): we can prove that $1/F(z)$ does not have any pole under the assumption that $\sigma_1(\omega) > 0$. The additional condition $F(\mu^2) < 0$ is in fact not very restrictive since for a sufficiently large positive constant c the function $F_1(z) = F(z) - c$ will certainly fulfil the negative value at threshold requirement preserving at the same time all other important properties of $F(z)$.

Another remark is connected with the range of applications of the formulae (2) and (4). Certainly the conventional dispersion relations are well suited for any low energy analysis. Moreover, sometimes in order to emphasize the low energy self-consistency of the data one even introduces additional subtractions in order to suppress the high energy contribution. On the other hand, the conventional relations prove to be rather weak constraints as far as fitting the high energy data is concerned^{5),14),15)}: reasonable agreement with the low energy data can be achieved for quite different models. The situation is not much improved by considering moment relations. In Section 3 we shall discuss the constraints which follow from Eq. (3). Here we note that in comparison with Eq. (2) the situation is somewhat reversed: instead of a threshold subtraction the RHS of (3) contains explicitly a term corresponding to a subtraction at infinity. Moreover, Eq. (5) which follows from the dispersion relation (3) for the inverse amplitude would remain valid even if $\sigma_1(\infty) = \infty$ ^{*)}, whereas the corresponding sum rule for the direct amplitude [originally derived by Lehmann¹⁶⁾]

$$8 \int_0^{\infty} \frac{dk}{k^2} \left(\mathcal{D}_1(k) - \mathcal{D}_1(0) \right) = \sigma_1(\infty) - \sigma_1(0) + \text{pole term}$$

*) That is, it also applies in the case of "unconventional asymptotics"^{5),8)}.

(with $k^2 = \omega^2 - \mu^2$) is meaningless in this case ^{*}). Also the positivity property of $A_1(x)$, essential to the derivation of Eq. (5), is irrelevant to the derivation of Lehmann's sum rule. Therefore, our sum rule (5) would appear to be the more restrictive one. In Section 3 we illustrate the use of the sum rule in a numerical analysis of πN scattering.

Now, consider for a moment the antisymmetric combination of amplitudes. The positivity condition is obviously not ensured for $\sigma_2(\omega)$ for physical values of ω . However, from the experimental data we see that $\sigma_2(\omega) > 0$ for $1.7 \text{ GeV} \leq \omega \leq 65 \text{ GeV}$. Assuming the positivity of A_2 for all higher energies it is straightforward to construct a similar sum rule for the antisymmetric amplitude $T_2(\omega)$. This sum rule could, for example, be helpful in further detailed investigations of the validity of Pomeranchuk's theorem. Note, however, that for Serpukhov energies we are at present forced to obtain $\sigma(\pi^+p) = \sigma(\pi^-n)$ from deuterium measurements. Possible deuterium effects ¹⁷⁾ are thus crucial in an analysis of T_2 , but are of no special importance in a study of T_1 .

3. ANALYSIS OF πN SCATTERING DATA AND THE SUM-RULE CONSTRAINTS

Using a parameter form for the symmetric combination of πN amplitudes, $T_1(\omega)$, we perform a χ^2 fit to the values of σ_1 and $\alpha_1 (\equiv D_1/A_1)$ which are obtainable from the experiments above 8 GeV/c. In the second part of this section we show how the asymptotic sum rule can be used to distinguish between the solutions that emerge from such a fit to the data.

^{*}) On the other hand, Lehmann's sum rule is valid if $\sigma_1(\omega) = 0$. In this case, provided that $\sigma_1(\omega)$ does not go asymptotically to zero too fast we can obtain a sum rule for $1/F(z)$ instead of $\sqrt{z^2 - a^2}/F(z)$.

a) Parametrization of the Data

Like Barger and Phillips ⁹⁾ we use the following fairly general Regge parametrization of the symmetric combination of forward amplitudes

$$T_1(\omega) = i\gamma_p \omega - \gamma_{p'} \omega^{\alpha_{p'}} \exp(-i\pi\alpha_{p'}/2) + i\gamma_c \omega (\ln \omega - \beta - i\pi/2)^{-\lambda-1} \quad (6)$$

The first two terms are due to the leading Regge poles P, P' and the intercepts are fixed at $\alpha_p = 1$ and $\alpha_{p'} = 0.5$ respectively. The last term arises from a leading Regge cut ($\alpha_c = 1$) contribution of the form

$$\int_{-\infty}^{\alpha_c} d\alpha \, g(\alpha) \omega^\alpha \exp(-i\pi\alpha/2) \quad (7)$$

with the spectral function chosen to be $g(\alpha) = (\alpha_c - \alpha)^\lambda \exp[\lambda(\alpha_c - \alpha)]$. To satisfy the theoretical constraint ¹⁸⁾ that $g(\alpha_c) = 0$ we require $\lambda > 0$. In order to facilitate comparison with Barger and Phillips ⁹⁾ we fix $\lambda = 0.1$.

A four parameter ($\gamma_p, \gamma_{p'}, \gamma_c$ and β) χ^2 fit is performed to values of σ_1 and α_1 which are obtained from the available πN data ^{1, 19-21)} above 8 GeV/c ^{*)}. Two solutions are found with similar minimum values of χ^2 . The corresponding values of the parameters are given in the Table and the resulting fits to the data are shown in the Figure. Solution A is similar to that obtained by Barger and Phillips ⁹⁾ (their parameters are $4\pi\gamma$ due to the difference in normalization).

*) The results obtained for the symmetric amplitude T_1 are essentially unchanged if instead the experimental values of $\sigma_\pm$ and $\alpha_\pm$ are fitted with (in addition to Eq. (6) the amplitude T_2 is parametrized in terms of ρ Regge exchange).

T A B L E

	γ_p	$\gamma_{p'}$	γ_c	b	χ^2	$\sigma_1(\infty)$
Solution A	6.8	11.9	-23.3	-1.8	63	33 mb
Solution B	4.9	3.4	1.15	4.5	53	24 mb
Poles alone	4.4	3.8	---	---	143	22 mb

The parameter corresponding to the two solutions in units $\hbar = c = \text{GeV} = 1$. For comparison we also show the best-fit solution if the cut form is omitted.

From physical arguments ^{*)} we expect the cut contributions to destructively interfere with the Regge pole contributions and to give a negative contribution to σ_1 . This is true for Solution A. Due to the value of b it is also true for Solution B in the region of the data but for higher energies ($\ln \omega > b$) the position is reversed. Strictly speaking for the integral (7) to exist for $\omega > \omega_0$ we would require $b < \ln \omega_0$ - however, this probably should not be regarded a stringent requirement since it comes from the behaviour of $\rho(\alpha)$ as $\alpha \rightarrow -\infty$. Thus, although the solutions cannot be distinguished by the data there is some evidence that Solution A is to be preferred.

b) Application of the Asymptotic Sum Rule

In order to use sum rule (5) to distinguish between the two solutions we choose a to be 8 GeV, so that the value of the right-hand side (RHS) can be computed from the high energy

^{*)} References given in Ref. 10) [see also Ref. 22)]; for a discussion in terms of the absorption model see, e.g., Ref. 23).

parametrizations. The LHS is calculated from the total cross-section data below 8 GeV and the values of the real part $\mathcal{D}_1(\omega)$ obtained from conventional forward dispersion relations ^{*)}. An advantage of the form (5) of the sum rule is that the LHS only contains $\mathcal{D}_1$ in the denominator, and here it is dominated by the contribution from A_1 ^{**)} . Calculating the LHS of Eq. (5) in this way we find

$$LHS = 0.50 \pm 0.02 \text{ GeV}^2,$$

whereas, using in turn the two high energy parametrizations, we have

$$RHS = \begin{cases} 0.52 \text{ GeV}^2 & \text{for Solution A,} \\ 0.28 \text{ GeV}^2 & \text{for Solution B.} \end{cases}$$

Thus the sum rule rules out Solution B and agrees very well with the physically favoured Solution A.

Since the derivation of the sum rule relies on the positivity of σ_1 it is possible to increase its sensitivity as follows. From the total cross-sections data we observe that $\sigma_1(\mu) < \sigma_1(\omega)$ for $\mu < \omega < 65 \text{ GeV}$. Assuming that this inequality remains true for all higher energies, we see that the sum rule is equally valid if $\sigma_1(\omega)$ is everywhere replaced by

*) We are indebted to Dr. R. Strauss (Karlsruhe) for providing us with a recent compilation of his values for $\mathcal{D}_\pm$ and $A_\pm$.

**) This remark is, of course, not true near threshold but here the integrand is very small since $A_1(\omega)$ vanishes like $(\omega^2 - \mu^2)^{\frac{1}{2}}$ as $\omega \rightarrow \mu$.

$\sigma_1(\omega) - \Delta$ provided $\Delta < \sigma_1(\omega)$. For instance, choosing $\Delta = 6$ mb we find that

$$LHS = 0.62 \pm 0.03 \text{ GeV}^2$$

as compared to

$$RHS = \begin{cases} 0.66 \text{ GeV}^2 & \text{for Solution A,} \\ 0.24 \text{ GeV}^2 & \text{for Solution B.} \end{cases}$$

As expected we see that the sensitivity of the sum rule test has been increased.

4. CONCLUSIONS

Although Solution A is an acceptable fit to both the data and the asymptotic sum rule, we do not claim that it gives the only possible asymptotic behaviour. Different parametric forms may give solutions with equally good fits to the available data and the sum rule. In this connection we note that the real part of the cut contribution dominates asymptotically over the real part due to the Regge poles and this again emphasizes the importance of measurements of $\alpha_{\pm}$ at high energies.

However, we have shown that the asymptotic sum rule can serve as a useful constraint and eliminate otherwise acceptable solutions. An advantage of the sum rule is that by far the largest contribution to the RHS comes from the term involving $\sigma_1(\infty)$ and that this is determined by a single parameter γ_p .

ACKNOWLEDGEMENTS

It is a pleasure to thank Drs. P. Landshoff, C. Michael, C. Schmid, L. Sertorio and G. Winbow for many useful discussions, and to thank Professor J. Prentki for the hospitality extended to us at CERN.

R E F E R E N C E S

- 1) J.V. Allaby, Yu.B. Bushnin, S.P. Denisov, A.N. Diddens, R.W. Dobinson, S.V. Donskov, G. Giacomelli, Yu.P. Gorin, A. Klovning, A.P. Petrukhin, Yu.D. Prokoshkin, R.S. Shuvalov, C.A. Ståhlbrandt and D.A. Stoyanova, Phys. Letters 30B, 500 (1969).
- 2) R.J. Eden, University of California Preprint, UCR-34P, 107-105, (1969).
- 3) A. Martin, "Remarks about the Pomeranchuk Theorem", CERN Preprint TH.1075 (1969).
- 4) W.S. Lam and T.N. Truong, Phys. Letters 31B, 307 (1970).
- 5) V. Barger and R.J.N. Phillips, Phys. Letters 31B, 643 (1970).
- 6) D. Horn and A. Yahil, Caltech Preprint CALT-66-238 (1970).
- 7) R. Arnowitt and P. Rotelli, Imperial College Preprint ICTP/69/14 (1970).
- 8) R. Wit, to be published in Nuovo Cimento.
- 9) V. Barger and R.J.N. Phillips, Phys. Rev. Letters 24, 291 (1970).
- 10) Cf. R.J.N. Phillips, Lectures on Regge phenomenology, Rutherford Laboratory Preprint, 1970.
- 11) R. Wit, Acta Phys. Polon. 24, 271 (1963).
- 12) N.N. Khuri and T. Kinoshita, Phys. Rev. 137, B720 (1965).
- 13) M. Olsson, Phys. Rev. 171, 1681 (1968).
- 14) R. Thews, High Energy Total Cross Sections and the Adler-Weisberger Relation, University of Rochester Preprint, UR-875-298 (1970).
- 15) D.L. Pursey and L. Sertorio, Phys. Rev. 155, 1591 (1967).
- 16) H. Lehmann, Nuclear Physics 29, 300 (1962).
- 17) G. Alberi, L. Bertocchi and P.J.R. Soper, Trieste Preprint IC/70/39.
- 18) J.B. Bronzan and C.E. Jones, Phys. Rev. 160, 1494 (1967).

- 19) K.J. Foley, R.S. Jones, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.D. Platner, S.A. Quarles and E.H. Willen, Phys. Rev. Letters 19, 330 (1967).
- 20) W. Galbraith, E.W. Jenkins, T.F. Kycia, B.A. Leontic, R.H. Phillips, A.L. Read and R. Rubinstein, Phys. Rev. 138, B913 (1965).
- 21) K.J. Foley, R.S. Jones, S.J. Lindenbaum, W.A. Love, S. Ozaki, E.D. Platner, S.A. Quarles and E.H. Willen, Phys. Rev. 181, 1775 (1969).
- 22) N.W. Dean, Phys. Rev 182, 1695 (1969).
- 23) F. Henyey, G.L. Kane, Jon Pumplin and M.H. Ross, Phys. Rev. 182, 1579 (1969).

FIGURE CAPTION

The fits to (a) the data for the total cross-section $\sigma_1 = \frac{1}{2}(\sigma_- + \sigma_+)$, (b) the real part of the symmetric amplitude as calculated from the data of Ref. 21). The curves denoted by A and B correspond to the solutions given in the Table.

● FOLEY et al.(21)