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A PARTON THEORY BASED ON THE DUAL RESONANCE MODEL

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A B S T R A C T

We consider the consequences of replacing the continuous string model of hadrons by an infinite but discrete chain of partons. It is shown that such a replacement leads to a sensible theory of form factors and deep inelastic electroproduction cross-sections. Experimental data concerning these quantities are used to estimate the departure from the continuum limit. We find that a hadron is probably a fairly loosely knit structure.

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I. INTRODUCTION

It has been known for some time that the dual theory of hadrons is the result of a continuous string model ¹⁾⁻³⁾. Although this model may be internally self-consistent, it has thus far not provided a reasonable explanation of the electromagnetic and weak properties of hadrons. In particular, the quantization of such systems introduces infinite zero point fluctuations in the space-time co-ordinates of the system which in turn cause the particles to have infinite extension ¹⁾. The duality property is also a consequence of infinite zero point fluctuations in the positions of the ends of the hadronic string. Therefore if we wish to retain duality, we must place the charges of hadrons in the interior of the string where it is possible to control the fluctuations without ruining duality. Before explaining how this can occur, let us review the theory of hadronic strings.

When a string moves through space-time, it sweeps out a two-dimensional world sheet ^{2),3)}. We view the sheet as a simply connected region of the complex plane such as a disc ²⁾ or a strip ⁴⁾. The infinite strip $0 \leq \theta \leq \pi$, $-\infty \leq \lambda \leq \infty$ is particularly useful for applying the ordinary rules of quantum mechanics while the disc gives a more symmetric representation of the theory. The dynamical degrees of freedom $X_\mu(\theta, \lambda)$ defining the four-dimensional space-time location of the point (θ, λ) satisfy

$$\frac{\partial^2 X_\mu}{\partial \lambda^2} + \frac{\partial^2 X_\mu}{\partial \theta^2} = 0 \quad (1)$$

and

$$\frac{\partial X_\mu}{\partial \theta} \Big|_{\theta=0, \pi} = 0 \quad (2)$$

The variable X_μ is expandable in the form

$$X_M = X^{C.M.} + i\sqrt{2} \sum_{l=1}^{\infty} \frac{a^+(l)e^{l\lambda} - a^-(l)e^{-l\lambda}}{\sqrt{l}} \cos l\theta \quad (3)$$

where $a^+(l)$ and $a^-(l)$ are ordinary harmonic oscillator creation and destruction operators and $X^{C.M.}$ is the centre-of-mass defined to be conjugate to the total four-momentum. The zero point fluctuations are easily seen if one computes the ground state value of

$$(\Delta X)^2 = \langle [X_M(\theta, \lambda) - X_M^{C.M.}]^2 \rangle \quad (4)$$

The result is

$$(\Delta X)^2 = \sum_{l=1}^{\infty} \frac{(\cos l\theta)^2}{l} \quad (5)$$

which is logarithmically divergent.

It is possible that the world sheet may really be a dense network of propagators and vertices and this is the view-point of this article ^{2),5),6)}. When drawn on the strip according to rules given in the following section, the diagram will have a certain density of propagators, $\rho(\theta, \lambda)$ at each point. Thus a cut through the sheet will see a hadron composed of partons instead of a continuous string.

To describe this system, we make a slight modification of Eq. (3) which is well known to solid state physicists. The modification consists of cutting off the normal mode expansion at a value $l(\theta, \lambda) = \pi\rho^{\frac{1}{2}}$ corresponding to a wavelength comparable to the separation between partons. We then write

$$X_M = X^{C.M.} + i\sqrt{2} \sum_{l=1}^{\pi\rho^{\frac{1}{2}}} \frac{a^+e^{l\lambda} - a^-e^{-l\lambda}}{\sqrt{l}} \cos l\theta \quad (6)$$

If we now calculate $\langle X^2 \rangle$ we find

$$(\Delta X)^2 = 2 \int_0^{\pi \rho^{1/2}} \frac{(\cos \theta)^2}{\rho} \sim \log \rho(\theta, \lambda) \quad (7)$$

Therefore by choosing $\rho(\theta, \lambda)$ infinite at $\theta=0$ and π and finite elsewhere, we can maintain infinite fluctuations at the edge of the hadronic string and therefore have duality while making the fluctuations finite in the interior. We shall see that if the charge of a hadron is distributed over the interior of the hadron by charge carrying parton lines that the above assumption produces a reasonable form factor. Furthermore, since the electromagnetic size of the particle is controlled by ρ the known experimental root mean square charge radius puts constraints on ρ .

A second argument in favour of a finite value for ρ comes from the parton theory of deep inelastic electroproduction⁶⁾⁻⁸⁾. The structure functions in this theory are directly related to the average longitudinal momentum distribution of partons in the infinite momentum frame. Experiment, interpreted in this manner indicates that as much as 1/3 of the hadron momentum is likely to be found on a single parton. As we shall see if the density of propagators becomes infinite on the strip each propagator will carry only an infinitesimal fraction of the momentum in disagreement with experiment.

The constraints on ρ arising in this way are in quite satisfactory agreement with those coming from form factors. For this reason we feel that the continuum dual model is only a simplification of an essentially more complicated hadronic structure. It is very much like the nuclear liquid drop model and presumably has similar limitations.

II. STATISTICAL TREATMENT OF COMPLICATED FEYNMAN DIAGRAMS

Consider a planar Feynman diagram containing very many vertices and propagators such as the diagram in Fig. 1. The diagram need not be exactly planar but certainly must not contain any propagators which

cross a large number of others when drawn on a plane. We draw the diagram in a more or less arbitrary way on the u, v plane, taking care only to fill up a simply connected region. The i^{th} vertex is at the point u_i, v_i and in the co-ordinate representation is located at space-time point $X_M(u_i, v_i)$. The diagram is an integral over all $X_M(u_i, v_i)$. In this section, we shall show how to compute such diagrams in a simple statistical continuum approximation. Let us first write

$$A = \int I(X) \prod_i d^4 X(u_i, v_i) \quad (8)$$

where I is a function of all the X defined by the product of all propagators in the X representation. If the diagram is very dense we may think of $X(u_i, v_i)$ as a continuous function of u and v and replace the integral in Eq. (8) by a functional integral over all $X(u, v)$.

$$A \approx \int d^4 X(u, v) \cdot (X) \quad (9)$$

The integral is not computable in its present form but several authors have argued that since the mean momentum through the diagram must be distributed among so many lines each line carries a very small momentum. It may then be possible to replace the propagators $(p^2 + m^2)^{-1}$ by Gaussians $m^{-2} \exp[-p^2/m^2]$. Unfortunately the fluctuations about the mean momentum of a line do not decrease as the diagram density increases. In assuming the validity of Gaussian propagators when the mean momentum is small we are extrapolating far beyond what we can prove. It is not the purpose of this paper to attempt a proof but rather to show that the consequences are interesting enough to pursue the question further.

In co-ordinate space, the corresponding propagator is approximated by

$$\text{EXP} - [(X_i - X_j)^2 m^2/4] \quad (10)$$

Let us consider the contribution to $\mathcal{Z}(X)$ coming from propagators in a cell du, dv . A particular propagator extending from u, v to $u+\Delta u, v+\Delta v$ is given by

$$\exp - \left[\frac{m^2}{4} \left(\frac{\partial X}{\partial u} \Delta u + \frac{\partial X}{\partial v} \Delta v \right)^2 \right] \quad (11)$$

Let us suppose that the diagram is drawn in such a way that the mean values of $(\Delta u)^2$, $(\Delta v)^2$ and $\Delta u \Delta v$ exist in each small cell of u, v . It is then a well-known theorem that a change of variables can be made so that

$$\begin{aligned} (\Delta u)^2 &= (\Delta v)^2 \\ \Delta u \cdot \Delta v &= 0 \end{aligned} \quad (12)$$

in the mean. This theorem is true only in two dimensions and therefore cannot be used for diagrams which fill three or higher dimensional regions.

We now multiply the exponent by $\rho(u,v)dudv$, the number of propagators in $dudv$. The result for $\mathcal{Z}$ is then given by

$$\begin{aligned} \mathcal{Z} = \exp - \left\{ \frac{m^2}{4} \int du dv \rho(u,v) \langle \Delta u^2 \rangle \right. \\ \left. \cdot \left[\left(\frac{\partial X}{\partial u} \right)^2 + \left(\frac{\partial X}{\partial v} \right)^2 \right] \right\} \end{aligned} \quad (13)$$

Since the density of propagators must be inversely proportional to the square of the mean propagator length we can assume they cancel leaving a number of order of magnitude 1 depending on the details of the geometry. If we also choose $m^2 \sim 1$ then lumping constants together into $K \sim 1$

$$\mathcal{Z} = \exp - [K \int \mathcal{L} du dv] \quad (14)$$

where $\mathcal{L}$ is defined to be the conformally invariant Lagrange density

$$\mathcal{L} = \left(\frac{\partial X}{\partial u} \right)^2 + \left(\frac{\partial X}{\partial v} \right)^2 \quad (15)$$

The entire procedure by Eq. (9) is equivalent to a Feynman path integral. If the region on which the graph is drawn is the infinite strip, it defines the quantum mechanics of an elastic string propagating in the λ direction. Previously, we have shown how such a system leads to dual n point amplitudes ^{1),2),4),9)}. The actual functional integral gives a dual amplitude multiplied by a factor D^{-2} where D is a determinant which arises in the evaluation of path sums.

Several points are worth noting :

- 1) the cancellation of the density function ρ against $(\Delta u)^2$ occurs only in two dimensions;
- 2) since every channel contains very many partons, the parton thresholds for such graphs are very distant;
- 3) the parton mass must be of order 1 GeV in order to obtain agreement with known hadronic level spacings ⁵⁾;
- 4) when working on the strip Eq. (3) defines the Fourier resolution of X . The normal modes labelled ℓ define certain collective excitations of the system of partons composing a hadron. It is obvious that for finite density there is no meaning to contributions coming from modes with wavelength less than the mean parton spacing;
- 5) the basic symmetry of the theory is conformal transformations which transform the diagram preserving Eq. (12). If we keep fixed the region occupied by the graph, the conformal group reduces to the Möbius transformations or a group isomorphic to it.
- 6) the determinantal factor D^{-2} is the only thing in the final result which depends on the details of the graph.

III. PROPERTIES OF THE PARTON DISTRIBUTION

We begin our discussion of the parton distribution by computing the mean value of a parton momentum in a ground state hadron and then the fluctuations about this mean. We also calculate the mean square distance between a pair of nearest neighbour partons. We find that these last two quantities do not have an appreciable dependence on the density ρ . From this we learn that the density of the diagram and of the hadron in real space-time is not dependent on the density on the formal space of the strip or disc.

We have previously shown ^{2),4)} that the quantities $\partial X / \partial \theta$ and $\partial X / \partial \lambda$ define a two-dimensional flux of momentum on the world sheet. The two components of flux are given by

$$\left. \begin{aligned} p_{\lambda}(\theta, \lambda) &= \frac{P_{C.M.}}{\pi} + \frac{1}{\sqrt{2} \pi} \sum_i^{\pi \rho^{1/2}} (a^+ + a^-) \sqrt{2} \cos l\theta \\ p_{\theta} &= \frac{1}{\sqrt{2} \pi} \sum_i^{\pi \rho^{1/2}} (a^+ - a^-) \sqrt{2} \sin l\theta \end{aligned} \right\} \quad (16)$$

where $P_{C.M.}$ is the total four-momentum conjugate to $X_{C.M.}$.

The momentum on a parton line which is oriented along the unit two-vector $(n_{\lambda}, n_{\theta})$ is given by

$$k = \rho^{-1/2} p \cdot \hat{n} \quad (17)$$

From this we conclude that the mean momentum on a line at position θ and orientation $\hat{n}$ in the ground state is

$$\langle k \rangle = \frac{P_{C.M.}}{\pi \rho^{1/2}} n_{\lambda} \quad (18)$$

The fluctuations about this value are given by

$$\frac{1}{2\pi^2\rho} \langle 0 | \left[\sum_{\ell} (a^+ + a^-) \sqrt{\ell} \cos \ell\theta \right]^2 | 0 \rangle \quad (19)$$

$$= \frac{1}{2\pi^2\rho} \sum_{\ell} \ell (\cos \ell\theta)^2 \quad (20)$$

The sum in Eq. (20) is to be cut off when $\ell > \pi\rho^{\frac{1}{2}}$. To obtain an estimate, we can approximate $\cos^2$ by $\frac{1}{2}$ to obtain

$$\frac{\rho^{-1}}{4\pi^2} \sum_1^{\pi\rho^{\frac{1}{2}}} \ell \sim \frac{1}{8} \quad (21)$$

The interesting thing about this estimate is that the fluctuations in k are essentially independent of ρ .

In a similar manner we may try to evaluate the mean square distance between neighbouring partons. Using Eq. (6) we obtain

$$\begin{aligned} \langle (X_i - X_{i+1})^2 \rangle &= \rho^{-1} \sum 2\ell (\sin \ell\theta)^2 \\ &\sim \rho^{-1} \sum_1^{\pi\rho^{\frac{1}{2}}} \ell \end{aligned} \quad (22)$$

or

$$\sum_{n=1}^4 \Delta X_n \Delta X_n = \frac{1}{2} \times 4 = 2$$

which is again independent of ρ . Thus it is clear that the density function ρ does not measure the density of the diagrams in real space.

We shall find that there are important aspects of the electro-magnetic and weak couplings of hadrons which are quite sensitive to ρ . A crucial step in the development of our parton model is the

choice of a density function ρ . It is our opinion that our choice of density can be justified from an examination of which diagrams maximize the determinant which multiplies the dual amplitude when the path sum is performed. Since this determinant does not depend at all on the external lines the dominant density function will be the same for all graphs and may be computed by consideration of vacuum diagrams.

To guess the form that the density should have, we will rely on a symmetry argument. The symmetry is the conformal invariance of the action integral in Eq. (14) under conformal mappings which keep the region fixed. Now let us suppose we have a vacuum graph with a density ρ which is among the dominant diagrams. A conformal mapping of the diagram maps it into a new diagram with a density ρ' . Therefore if there is anything unique about the density of dominant graphs we must assume $\rho = \rho'$.

If we work with the disc on the complex W plane, then the conformal mappings to be considered are

$$W' = e^{i\beta} \frac{W + a}{a^* W + 1}$$

and one easily finds that the only conformally invariant density functions are of the form

$$\rho_0 / (1 - W^* W)^2 \quad (23)$$

If instead we work with the strip on the λ, θ plane, the density in Eq. (23) becomes

$$\rho(\theta, \lambda) = \frac{1}{\lambda_0^2 (\sin \theta)^2} \quad (24)$$

The parameter λ_0 roughly measures the average propagator length near the middle. As we approach the edges ($\theta = 0, \pi$) the density diverges.

To examine the structure of a hadron, a cut through the diagram along a line $\lambda = \text{constant}$ may be made. The number of parton crossing the cut in a length $d\theta$ is $\rho^{\frac{1}{2}}$ so that the total number of partons in a hadron is the logarithmically divergent integral

$$N = \frac{1}{\lambda_0} \int_0^\pi (\sin \theta)^{-1} d\theta \quad (25)$$

So far, we have considered diagrams which are vacuum-like. By this we mean that the internal structure of the diagram is identical to a vacuum graph. If, for example, a charge entered the diagram at some boundary point and left another, a net flow of charge would occur on the world sheet, thus making it internally dissimilar to a vacuum diagram. We shall assume that the extra charge is carried by a small number of lines walking somewhat randomly through the vacuum diagram. The basic density of the vacuum material we assume unchanged.

Consider, for example, the history of a singly charged hadron laid out on the λ, θ strip. We may meaningfully ask for the average charge crossing a $\lambda = \text{constant}$ surface per unit θ at a given point. For symmetry reasons, the result must be a function of θ alone which we denote $\sigma(\theta)$. Now the total number of lines crossing the $\lambda = \text{constant}$ line is $1/(\lambda_0 \sin \theta)$. It would not be correct to make the flow of charge proportional to this, since the total charge crossing $\lambda = \text{constant}$ would not be normalizable. Indeed, since there is a logarithmically divergent number of low momentum partons near the edges, it must be assumed that the mean charge flow per parton in this region is essentially zero, so that the existence of such a flow is unimportant to the properties of diagrams near the boundaries.

IV. HADRONIC FORM FACTORS

For simplicity we first consider a scalar form factor for a scalar photon which can couple to any parton line in a diagram. The amplitude for the absorption of a momentum q at the point θ is

$$F_i(g^2) = \langle i | e^{i g X(\theta)} | i \rangle \quad (26)$$

For a ground state to ground state transition

$$F_0(g^2) = \langle 0 | e^{i g X(\theta)} | 0 \rangle \quad (27)$$

Equation (3), together with some elementary properties of creation and annihilation operators gives

$$F_0(g^2) = e^{-g^2 \sum_l \frac{(\cos l\theta)^2}{l}} \quad (28)$$

To obtain the form factor, this amplitude must be coherently summed or integrated over θ with a weight describing the conformally invariant density of propagators to which the photon can couple.

$$F(g^2) = \int \frac{d\theta}{(\sin \theta)^2} F_0(g^2) \quad (29)$$

The sum in the exponent is logarithmically divergent and should be cut off at $l = \pi \rho^{\frac{1}{2}}$. It is given by

$$\frac{1}{4} \sum_l \left(\frac{2}{l} + \frac{e^{2il\theta}}{l} + \frac{e^{-2il\theta}}{l} \right) \quad (30)$$

The last two terms define the convergent sums

$$\begin{aligned} & -\frac{1}{4} \log (1 - e^{2i\theta})(1 - e^{-2i\theta}) \\ & = -\frac{1}{2} \log (2 \sin \theta) \end{aligned}$$

Since these terms converge, they need not be cut off. The first term of Eq. (30) must be cut off at $\pi/\lambda_0(\sin\theta)^{-1}$. The sum is well approximated by $\frac{1}{2}\log[\pi/\lambda_0\sin\theta]$. Combining all terms gives

$$\sum_l \frac{(\cos l\theta)^2}{l} \sim -\frac{1}{2} \log \frac{2\lambda_0}{\pi} - \log \sin\theta \quad (31)$$

which may be substituted into Eq. (29) giving

$$F(q^2) = \int d\theta (\sin\theta)^{q^2-2} \left(\frac{2\lambda_0}{\pi}\right)^{q^2/2} \quad (32)$$

$$= \left(\frac{2\lambda_0}{\pi}\right)^{q^2/2} \int_0^1 v^{\frac{q^2-3}{2}} (1-v)^{1/2}$$

$$= \left(\frac{2\lambda_0}{\pi}\right)^{q^2/2} \frac{\Gamma(\frac{q^2}{2} - \frac{1}{2}) \Gamma(-\frac{1}{2})}{\Gamma(\frac{q^2}{2} - 1)} \quad (33)$$

The region $\sin\theta=0$ makes a series of poles in q^2 beginning at $q^2=1$. This is consistent with the basic trajectory height which occurs in the model without quantum numbers. We also note that poles are spaced by two-units.

The essential singularity $(2\lambda_0/\pi)^{q^2/2}$ originates from the Gaussian approximation which cannot be trusted when the photon brings in a large momentum to the propagators of the diagram. Presumably, the essential singularity is replacing parton thresholds in the form factor.

To compute an electromagnetic form factor, we must replace the factor $d\theta/\sin^2\theta$ by a corresponding factor which weights the integral according to the charge flow in the diagram. The relevant factor is $\sigma(\theta)d\theta$. This reflects the fact that the photon does not see the neutral vacuum sea of partons but rather samples the extra charge flowing through. Hence we get

$$F(q^2) = \int \sigma(\theta) (\sin \theta)^{q^2} d\theta \left(\frac{2\lambda_0}{\pi} \right)^{q^2/2}$$

Experimentally, the position of the first pole in the form factor is at $q^2 = -m^2 = -\frac{1}{2}$. In order to obtain such a pole $\sigma(\theta)$ must behave like $(\sin \theta)^{-\frac{1}{2}}$ near $\theta = 0$ or π . We take as a simple model $\sigma = (\sin \theta)^{-\frac{1}{2}}$ to get

$$F(q^2) = \left(\frac{2\lambda_0}{\pi} \right)^{q^2/2} \frac{\Gamma(\frac{q^2}{2} - \frac{1}{4}) \Gamma(-\frac{1}{2})}{\Gamma(\frac{q^2}{2} - \frac{3}{4})} \quad (34)$$

Comparison with experiment should of course only be attempted near $q^2 = 0$ due to the limitations of the Gaussian approximation. Comparison is best done in terms of the logarithmic derivative of F at $q^2 = 0$. Numerical evaluation of the logarithmic derivative of the β function gives about 3 which accounts for essentially all of the experimental value. Hence

$$\frac{2\lambda_0}{\pi} \sim 1 \quad (35)$$

Thus on a strip of width π the mean propagator length at the middle is of order $\pi/2$. The propagator lengths of course decrease like $\sin \theta$, nearer to the edges. A better fit over a wider region of q^2 can be secured with λ_0 slightly smaller at about 1 instead of $\pi/2$. It can be concluded that sizable corrections to the infinite density limit are necessary.

Transition form factors can also be computed by the same method. Before doing so we shall compute such form factors for a simple four-dimensional harmonic oscillator ¹⁰⁾. The vertex as always is of the form e^{iqX} with

$$X = i\sqrt{2} (a^+ - a^-) \quad (36)$$

The transition to an arbitrary coherent state $|\alpha\rangle = e^{a^+\alpha} |0\rangle$ is given by

$$\langle 0 | e^{i g X} | \alpha \rangle = e^{g^2 + g\alpha} \quad (37)$$

To compute the transition to a state on the leading trajectory, we introduce a polarization tensor $\epsilon_{\mu,\nu}, \dots$ with J indices. This is to be contracted with

$$\left(\frac{\partial}{\partial \alpha_\mu} \frac{\partial}{\partial \alpha_\nu} \dots \right) \langle 0 | e^{i g X} | \alpha \rangle \Big|_{\alpha=0} \quad (38)$$

or

$$(g_\mu g_\nu \dots) e^{-g^2} \quad (39)$$

Experimentally the form factor connecting the ground state to the state J seems to behave more like

$$(\epsilon_{\mu,\nu}, \dots) (g_\mu g_\nu \dots) \frac{f(g^2)}{|g|^J} \quad (40)$$

This can be deduced from our model although the weight of the evidence coming from this agreement is not large because it involves large q^2 . To derive it we replace Eq. (38) by

$$\left(\frac{\partial}{\partial \alpha_\mu} \dots \right) \int \langle 0 | e^{i g X(\theta)} | \alpha \rangle (\sin \theta)^{1/2} d\theta \quad (41)$$

where $|q\rangle$ is a state which is unexcited in all but the first mode and is coherent for $l=1$. The reason for this is that the leading trajectory is excited only in the first mode. The result is approximated by

$$(\partial_{\alpha_1} \partial_{\alpha_2} \dots) \left(\frac{2\lambda_0}{\pi} \right)^{g^2/2} \int d\theta e^{\sqrt{2} g \cdot \alpha \cos \theta} \cdot (\sin \theta)^{g^2 - 1/2} \quad (42)$$

$$= \sqrt{2} (g_\mu g_\nu \dots) \left(\frac{2\lambda_0}{\pi} \right)^{g^2/2} \int d\theta (\sin \theta)^{g^2 - 1/2} \cdot (\cos \theta)^J \quad (43)$$

The integral is

$$\int du u^{g^2 - 1/2} (1-u^2)^{\frac{J}{2} - \frac{1}{2}} \quad (44)$$

which behaves like

$$(g^2)^{-1/2} (g^2)^{-J/2} \quad (45)$$

for large q^2 , agreeing with Eq. (40). Again we must emphasize that this result is not at all conclusive since it involves large q^2 .

The limit $\lambda_0 \rightarrow 0$ which defines the pure dual theory is very bad from the view-point of form factors since the factor $\lambda_0^{q^2/2}$ causes the form factor to be zero unless $q^2 = 0$.

V. DEEP INELASTIC SCATTERING

Generally it is not easy to make reliable computations of deep inelastic electroproduction into definite final states because of the large momentum deposited in the graph by the photon. The total cross-section for $\gamma + P \rightarrow P + \text{anything}$ is possible however. We begin by computing the off-shell forward Compton amplitude for a photon of four-momentum q on a target hadron of momentum P . The dominant graphs in the deep inelastic limit ⁶⁾⁻⁸⁾ ($q^2 \rightarrow \infty$, $P \cdot q \rightarrow -\infty$, $-q^2/2P \cdot q = X$) are those in which the initial and final photon couple to the same parton line as in Fig. 2. Hence only one propagator is required to carry large momentum and it is not difficult to treat it correctly.

The expression for the momentum on a parton is

$$k = p(\theta, \lambda) \cdot \hat{n} \quad p^{-1/2} \quad (46)$$

where $\hat{n}$ is a unit two-vector in the direction of the propagator and p is a two-vector of momentum flux defined by

$$p_\lambda = \frac{i}{2\pi} \frac{\partial X_M}{\partial \lambda}, \quad p_\theta = \frac{i}{2\pi} \frac{\partial X_M}{\partial \theta} \quad (47)$$

Let the propagator to which the photons couple be at θ oriented at an angle ϕ with respect to the λ axis. The Compton amplitude for such a diagram is

$$e^2 \langle 0 | \frac{(2k+g)_\mu (2k+g)_\nu}{(k+g)^2 + m^2} | 0 \rangle \quad (48)$$

where e is the parton charge. We have ignored a contact term required for current conservation. Since it is real, it does not contribute to the absorptive part of the diagram which is our goal.

The expectation value of k in the ground state is easily seen to be

$$\frac{p\lambda_0}{\pi} \sin \theta \cos \phi \quad (49)$$

In addition k has a quantum mechanically fluctuating part which we denote by f . Equation (48) may be written

$$\left\langle \frac{(2\langle k \rangle + g + f)_\mu (2\langle k \rangle + g + f)_\nu}{(\langle k \rangle + g + f)^2 + m^2} \right\rangle e^2 \quad (50)$$

This may be evaluated in a variety of ways. The simplest is to use the identity

$$\frac{a_\mu a_\nu}{B} = \frac{\partial^2}{\partial \sigma_\mu \partial \sigma_\nu} \int_0^\infty d\lambda e^{a \cdot \sigma + B\lambda} \quad (51)$$

giving

$$\frac{\partial^2}{\partial \sigma_\mu \partial \sigma_\nu} \int_0^\infty d\lambda \left\langle e^{\sigma(2\langle k \rangle + g + f) + \lambda(\langle k \rangle + g + f)^2 + \lambda m^2} \right\rangle$$

Using Eq. (49) and taking the deep inelastic limit, we get

$$\frac{1}{g^2} \frac{\partial^2}{\partial \sigma_\mu \partial \sigma_\nu} \int_0^\infty dL e^{\sigma \cdot (2\langle k \rangle + g) + L(1 - \frac{\lambda_0}{2\pi X} \sin \theta \cos \phi)} \cdot \langle 0 | e^{\sigma \cdot f + \frac{L}{g^2} (\langle k \rangle f + g f + f^2 + m^2 + \langle k^2 \rangle)} | 0 \rangle \quad (52)$$

where we have changed variables to $L = g^2 \lambda$.

If we now take the deep inelastic limit, we find the last factor becomes just

$$\langle 0 | e^{\sigma \cdot f} | 0 \rangle = e^{K \sigma_\mu \sigma_\nu}$$

where K is a constant depending on θ , ϕ and λ_0 . Performing the derivatives gives

$$\{ K g_{\mu\nu} + [2\langle k \rangle + g]_\mu [2\langle k \rangle + g]_\nu \}$$

$$\frac{1}{g^2 \left[1 - \frac{\lambda_0}{2\pi x} \sin\theta \cos\phi \right]}$$

The structure function $2W_2$ which is most easily compared with data involves the term $[2\langle k \rangle + g]_\mu [2\langle k \rangle + g]_\nu$. Retaining only this part we then obtain

$$e^2 \frac{[2P \frac{\lambda_0}{\pi} \sin\theta \cos\phi + g]_\mu [2P \frac{\lambda_0}{\pi} \sin\theta \cos\phi + g]_\nu}{g^2 + 2P \cdot g \frac{\lambda_0}{\pi} \sin\theta \cos\phi} \quad (53)$$

which must be integrated over θ with a weight which measures the density of charged propagators. This is taken to be some fraction, s , of all propagators. The factor is

$$\frac{s}{\lambda_0^2 \sin^2\theta} d\theta$$

We also integrate over ϕ to account for a statistical collection of graphs with propagators pointing in every direction at each point.

The absorptive part, A , of the amplitude is obtained by replacing the pole $(q^2 + 2qP(\lambda_0/\pi) \sin\theta \cos\phi)^{-1}$ by a δ function

$$\delta(g^2 + 2gP \frac{\lambda_0}{\pi} \sin\theta \cos\phi)$$

It is interesting that this δ function constraints $-q^2/2Pq$ or X to be equal to $\lambda_0 \sin\theta \cos\phi / \pi$. This is the fraction of hadron momentum contained in the non-fluctuating part of the parton momentum. Thus we have a similar situation to that of the parton model of Bjorken and Feynman where X is the fraction of longitudinal momentum carried by a parton. In order to account for the observed behaviour of $\sqrt{W_2}$ near $X \rightarrow 0$ Bjorken and Feynman postulate an infinite sea of very low momentum partons which Feynman calls "wee" partons. In our model, this sea is natural and comes from the divergence of ρ near the edges $\theta = 0, \pi$. Let us see how a density of low momentum partons produces the observed constancy of $\sqrt{W_2}$ near $X = 0$. Replacing the denominator by the δ function and integrating on θ gives

$$\frac{e^2 s}{2P \cdot g \lambda_0^2} \int_0^{\arccos \frac{X\pi}{\lambda_0}} d\phi \frac{\cos\phi \lambda_0^2 [2XP - g]_+ [2XP - g]_-}{X^2 \pi^2 \sqrt{1 - \frac{X^2 \pi^2}{\lambda_0^2 \cos^2\phi}}} \quad (54)$$

which identifies the structure function $\sqrt{W_2}$ as

$$\frac{e^2 s}{\pi^2} \int_0^{\arccos(X\pi/\lambda_0)} \cos\phi \left[1 - \frac{X^2 \pi^2}{\lambda_0^2 (\cos\phi)^2} \right]^{-1/2} d\phi \quad (55)$$

The X^2 in the denominator of Eq. (54) which came from the all important divergent density $1/\sin^2\theta$ combined with the factors

$$[-2XP + g]_+ [-2XP + g]_-$$

to give the usual kinematic factor multiplying $\sqrt{W_2}$. Equation (55) then goes to a constant as $X \rightarrow 0$.

A preliminary fit to the experimental value of $\sqrt{W_2}$ has been attempted by H.B. Nielsen and A. Kraemmer. In making the fit we note that Eq. (55) vanishes for $X > \lambda_0/\pi$. This is because no parton carries more momenta than the parton at $\theta = \pi/2$ which carries a fraction of λ_0/π of the total. If we identify this behaviour with the sharp experimental decrease near $X = 1/3$ we are led to estimate $\lambda_0 \sim 1$. This agrees nicely with our previous form factor determination.

Of course $\sqrt{W_2}$ does not become zero for $X > 1/3$ experimentally. It merely begins a sharp downward decrease. The portion of the curve between $X = 1/3$ and $X = 1$ not accounted for must be due to graphs with a much lower density than we have estimated. These effects are presumably outside the scope of our particular parton distribution. The effects which make the universal function go to a constant $X \rightarrow 0$ are the outstanding achievement of the model.

It should be noted that we have not included in our analysis the effect of extra charge flowing through the diagrams, so that the results are really applicable to uncharged hadrons. If the extra charge flows according to $\sigma(\theta) = (\sin\theta)^{-1/2}$ then an extra term appears in $\sqrt{W_2}$ which behaved like $X^{1/2}$ near $X = 0$. This extra term which is the difference between $\sqrt{W_2}$ for a charged and uncharged hadron should show up as a difference between $\sqrt{W_2}$ for protons and neutrons when $X \neq 0$.

Of some interest is the question of what kinds of states account for the inelasticity in our calculation. For this purpose, it is instructive to look at nuclear physics, or even better, solid state physics. When a parton in these theories is hit very hard, the dominant processes do not involve excitation of nice regular vibrations of the lattice but rather they involve an ejection of the parton out of its lattice position. If the impulse is hard enough, then we see the asymptotic process of ejection of the parton right out of the system. If it is somewhat less severe, we may excite a quasi-free single particle motion in which the parton moves freely through the lattice

with a sizable mean free path. Our guess is that to have scaling it is not necessary to entirely eject the parton but that it is enough to have the parton ejected move freely for a distance large in comparison with $1/\sqrt{s}$. In either case, we probably have to excite states which are outside the scope of the string model excitations. It is possible for example that states similar to single particle excitations exist in hadron dynamics, as well as the more collective strong vibrations.

VI. NATURE OF THE POMERON

In Feynman's parton theory the Pomeron is due to the exchange of very low momentum partons which in the present model are situated very near the edge of the strip. If we draw two strips side by side to represent the two hadrons in a collision, then the Pomeron is due to the exchange of many partons crossing from one edge to another as in Fig. 3. If we approximate this situation by one big dense planar diagram in Fig. 4, then it becomes a typical dual amplitude. The bridge connecting the two strips has no average flow of quantum numbers and it is in our opinion significant that the natural trajectory height of uncharged trajectories in dual theory is

$$\alpha(0) = 1$$

The difficulty with this interpretation is that the dual channel to the Pomeron must be exotic and therefore no Pomeron will be seen below the energy for forming exotic mesons.

If, on the other hand, the dominant graphs exchange two clumps of partons as in Fig. 5 then the dual channel contains pairs of ordinary mesons and the absorptive part will begin at lower energies. Phenomenologically, Freund and Harari have observed just such a connection between the t channel Pomeron exchange and the s channel multibody continuum. It is therefore tempting to identify such diagrams with the Pomeron. In fact, following the method of Kikkawa, Sakita and Virasoro, it is not hard to see that this graph has the behaviour

$S \alpha(t)$

with $\alpha(t)$ being the natural uncharged trajectory. This interpretation of the Pomeron differs from a previous one¹¹⁾ by myself and Frye in which the Pomeron had slope $\frac{1}{2}$ but an intercept of $1/3$. This previous model involved the exchange of a tube between hadrons which grows out of the interior and not the edge of a hadron.

The apparent inability of the earlier model to give $\alpha(0) = 1$ and the unphysical singularities which it gave rise to tend to favour the present model.

We can also see quite clearly why the charged trajectories have to be lower than the Pomeron. Since the flux of charges through the world sheet must be normalizable, the actual quantum numbers of hadrons are not as densely distributed near their edge as are the neutral wee partons.

Since in the dual theory the scattering of hadrons occurs by the temporary joining of edges, it is harder for quantum numbers to be exchanged than for the wee partons. In Feynman's language, high energy scattering happens only by wee parton exchange. Since there is an infinity of wee partons and only a finite amount of charge in a hadron, the exchange of charge among the wee partons is suppressed.

The nucleon presents a dilemma in our theory if we assume the partons which carry the quantum numbers are quarks. Let us suppose a nucleon history is a strip consisting of three quarks moving through a conformally invariant vacuum like sea. Now we know that it is not easy for the quark lines to cross, since such a possibility leads to exotic resonances. The fact that exotics are somewhat suppressed probably indicates that it costs a good deal of action to cross two quark lines on the sheet. This leads to the idea of a preferred quark in the middle, which is unavailable at the edge and cannot be involved in the poles of the electromagnetic form factor. But if the ρ meson is universally coupled to the isospin, all of the quarks must be available to form ρ mesons at the edge.

A similar dilemma occurs with respect to the Pomeron which seems to couple to the nucleon with a strength $\frac{3}{2}$ that of its coupling to the π meson. This requires that the nucleons have $\frac{3}{2}$ as many wee partons as the π meson. On the other hand, we have argued that the presence of a few extra quarks would make little difference to the structure of the wee sea.

To remedy these difficulties we would like to introduce a somewhat different nucleon structure. We propose a picture in which a central core of three quarks interacts with a cloud of meson strings. The dominant configuration might be that in which the antiquarks of three mesons bond with the core quarks forming three strings joined at the centre. Each string would have a free end, very many propagators distant from the core, and the dynamics of its associated wee sea would be similar to that of a meson's. This would allow each end to couple to a P meson or a Pomeron and would circumvent the dilemma.

R E F E R E N C E S

- 1) L. Susskind - "Structure of Hadrons Implied by Duality",
Phys.Rev. 1, 1182 (1970).
- 2) H.B. Nielsen - "An Almost Physical Interpretation of the Dual
N Point Function", NORDITA Preprint (1970).
- 3) Y. Nambu - University of Chicago Preprints (1969, 1970).
- 4) L. Susskind - "Dual Symmetric Theory of Hadrons and Space Time
Approach to Duality", to be published in Nuovo Cimento.
- 5) B. Sakita and M.A. Virasoro - Phys.Rev.Letters 24, 1146 (1970).;
H.B. Nielsen and P. Olesen - "A Parton View of Dual N Point
Functions", to be published in Phys.Letters.
- 6) J.D. Bjorken and E.A. Paschos - SLAC PUB 572 (1969).
- 7) S. Drell, D. Levy and T.M. Yang - to be published in Phys.Rev.
- 8) R.P. Feynman - unpublished work.
- 9) H.M. Chan - Phys.Letters 28B, 425 (1969) ;
K. Bardakçi and H. Ruegg - Phys.Letters 28B, 342 (1968) ;
C. Goebel and B. Sakita - Phys.Rev.Letters 22, 257 (1969) ;
Z. Koba and H.B. Nielsen - Nuclear Phys. B10, 633 (1969).
- 10) L. Susskind - Phys.Rev.Letters 23, 545 (1969).
- 11) G. Frye and L. Susskind - Phys.Letters 31B, 589 (1970) ;
C.S. Hsue, B. Sakita and M. Virasoro - Wisconsin Preprint (1970).

FIG.1

FIG. 2

FIG.3

FIG. 4

FIG. 5