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ANALYTIC 4th ORDER CROSSED LADDER CONTRIBUTION
TO THE LAMB SHIFT

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A B S T R A C T

A systematic investigation is made of the work of Soto. The algebra is redone entirely. The integrals are tested numerically. Each dubious or litigious integral is recomputed analytically. The contribution to the 4th order slope at zero momentum transfer is found to be

$$F_1'(0)_{\text{cross}} = \frac{\alpha^2}{\pi^2} \left(-\frac{1181}{1728} + \frac{\pi^2}{72} + \frac{7}{12} \pi^2 \log 2 - \frac{7}{8} \zeta(3) - \frac{13}{36} \log \lambda^{-2} \right)$$

$$= \frac{\alpha^2}{\pi^2} \left(2.3925 - \frac{13}{36} \log \lambda^{-2} \right)$$

in the Feynman gauge.

Standard parametrization leads to an expression for the slope from the crossed ladder graph of the form

$$F_1'(0)_{\text{cross}} = \sum_{\ell} c_{\ell} I_{\ell}$$

where c_{ℓ} are rational numbers and I_{ℓ} are standard integrals. The sum contains around 250 terms. The index ℓ is a standard notation for a set of five indices

$$\ell = (k, m, n, r, s)$$

and the standard integrals are defined, for the non-infra-red terms by

$$I_{\ell} \equiv I(k, m, n, r, s) =$$

$$\int_0^1 dx \int_0^1 dy \int_0^1 dz \int_0^1 dw \frac{z^r w^s x^{n+2k-3-m} y^{n+k-2} \Delta^m}{\beta^n \Gamma^k},$$

$$\beta = 1 - xy(1 - wz + w^2 z^2) ; \Delta = 1 - x(1 - wz + w^2 z)$$

$$\Gamma = w^2 x \beta + y \Delta^2.$$

whenever necessary for the convergence of the integrals, over-all coefficients $R_1 = 1 - z$, $R_2 = 1 - w$, $R_3 = 1 - wz$ are factorized out.

Soto ¹⁾ has given tables of the coefficients ^{*)} and of the integrals. The investigation of these tables proceeded in the following way.

1. Coefficients c_{ℓ}

The γ algebra was carried out by means of SCHOONSCHIP ²⁾. We found it more convenient for numerical checks to use a different set of Feynman parameters than the ones used by Soto. However, they *) or combinations of them.

could be transformed into the Soto variables and in this way the coefficients C_ℓ were obtained. Roughly 75% agreed with the Tables. In the remaining 25%, some disagreed, some were missing, others were mixed up, etc.

2. Integrals I_ℓ

The integrals were divided into groups and were added up with the correct coefficients. The sum was compared with the result of a numerical integration of the same group of integrals. Whenever a deviation was found, a binary algebraic-numerical search was made in order to locate the integral ^{*)} that gave rise to the deviation. Finally, this integral was re-evaluated analytically. The following integrals were found to contain analytic errors. We list the correct values below

- 1) $I(2,1,1,1,3) = -\frac{1}{2}B + \zeta(3)$
- 2) $I(3,0,0,3,6) = \pi^2/32 - 1/8$
- 3) $I(3,1,1,3,4) = \pi^2/72 + 1/6$
- 4) $R_3 I(3,3,3,5,8) = (5753)/(1920)\pi^2 - 6B - 289/16$
- 5) $I(2,0,1,5,7) = (247)/(1152)\pi^2 - 97/48$

$$B = (\pi^2/2)\log 2 - 5/4 \zeta(3).$$

Finally, the correct integrals were all added up with the correct coefficients with the result

$$F'_1(0)_{\text{cross}} = \frac{\alpha^2}{\pi^2} \left(-\frac{13}{36} \log \lambda^{-2} - \frac{1181}{1728} + \frac{\pi^2}{72} + \frac{7}{12} \pi^2 \log 2 - \frac{7}{8} \zeta(3) \right) = \frac{\alpha^2}{\pi^2} \left(-\frac{13}{36} \log \lambda^{-2} + 2.3925 \right) \quad **)$$

*) It accidentally turned out that there was never more than one deviating integral within each group.

**) The infra-red part was already recomputed analytically in Ref. 3) and found in agreement with Soto's result.

This number was further checked numerically as one single integral with a precision of 10^{-4} . The result was

$$F_1'(0)_{\text{cross}}^{\text{numerically}} = \frac{\alpha^2}{\pi^2} \left(-\frac{13}{36} \text{Log } \lambda^{-2} + 2.39239 \pm 0.00018 \right)$$

The agreement is excellent and also agrees with Ref. 4).

The present investigation, although systematic, does not guarantee that the result is analytically correct. Not all integrals have been re-evaluated and diminutive errors of no practical consequence could conceivably have been overlooked. It must be nevertheless mentioned that all integrals contributing less than three standard deviations of the numerical integration have been either re-evaluated or numerically checked, as these might have escaped to the above investigation.

We wish to thank Dr. B. Lautrup for discussions and independent checks of some integrals.

REFERENCES

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