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THE α^2 RADIATIVE LAMB SHIFT

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A B S T R A C T

The fourth order crossed electromagnetic vertex is examined. A very high precision numerical evaluation of the slope at $q^2 = 0$ is performed to yield

$$m^2 \frac{\partial^2 F_{1\text{cross}}(q^2=0)}{\partial q^2} = \frac{\alpha^2}{\pi^2} \left(-\frac{13}{36} \log \lambda^{-2} + 2.3756 \pm 0.0019 \right)$$

in the Feynman gauge.

This completes the fourth order radiative shift which contributes in H and D by $(0.427 \pm 0.002)\text{MHz}$. An uncertainty of this order can be neglected in the scattering approximation and is of no significance for physical applications.

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The computer evaluation of the slope at $q^2 = 0$ of the electron Dirac form factor by Appelquist and Brodsky ¹⁾ revealed two substantial differences with the previous calculations ^{2),3)}.

Of the five diagrams contributing in fourth order, the non-infra-red part of the corner diagram and that of the crossed diagram disagreed. The discrepancy of nearly 20% in the corner graph has been cleared up by a new analytical calculation ⁴⁾ which confirms the value of Ref. 1).

For the crossed diagram, a 2% difference still survives. The value of Ref. 1) is $(2.37 \pm 0.02) \alpha^2/\pi^2$; that of Ref. 3) is $2.314 \alpha^2/\pi^2$. A high precision determination of this quantity is reported in this paper.

The following computation of the crossed slope is a by-product of an integral representation of the off-shell fourth order electromagnetic vertex, which will be used as a tool of calculation in some of the sixth order contributions to the electron anomalous magnetic moment. Two different parametrizations have been done by hand, the first one by an off-diagonal orthogonalization, the second one by a chain procedure with non-orthogonal denominator. This second way has proven to be much more adequate for subsequent numerical integration exhibiting a much better interval structure along the four axes of integration.

The Dirac algebra has been done fully by hand and then entirely checked by computer, using the algebraic program SCHOONSHIP ⁵⁾. Next, the infra-red contributions have been isolated, computed analytically and checked by least square fit to the computer numerical integration for $10^2 < \lambda^{-2} < 10^7$. This part contributes

$$\left(\xi(2) - \frac{29}{27} - \frac{13}{36} \log \lambda^2 \right) \frac{\alpha^2}{\pi^2} = \left(0.5709 - \frac{13}{36} \log \lambda^2 \right) \frac{\alpha^2}{\pi^2}$$

in agreement with Ref. 3).

The numerical four-variable integration has been performed using RIWIAD ⁶⁾. This subroutine has been exploited with successively higher precision requirements. With the first parametrization, the precision was stopped at 1% with the result $(2.38 \pm 0.02) \alpha^2/\pi^2$ and simply used as an additional order of magnitude check. With the second, a first low precision evaluation led to $(2.395 \pm 0.035) \alpha^2/\pi^2$. Increasing then the number of subvolumes and the number of iterations, a 0.12% result was thus obtained with the value $(2.375 \pm 0.003) \alpha^2/\pi^2$. The precision has finally been pushed up to 0.07% (80,000 subvolumes, 4,600,000 calls to the function), and the following value obtained :

$$m^2 \frac{\partial}{\partial q^2} F_1(q^2=0)_{\text{cross}} = \frac{\alpha^2}{\pi^2} \left(-\frac{13}{36} \text{Log } \lambda^{-2} + 2.3756 \pm 0.0019 \right) \quad (1)$$

The agreement with Ref. 1) is patent whereas Ref. 2) is far outside the error. One may now collect the results of the five terms contributing to the total fourth order radiative shift. This sums up to

$$m^2 \frac{\partial}{\partial q^2} F_1(q^2=0) = \frac{\alpha^2}{\pi^2} (0.452 \pm 0.002) \quad (2)$$

It is worth-while at this point mentioning that the fourth order radiative level shift of a bound electron-nucleus system is not exactly that given by the slope at $q^2 = 0$, Eq. (2). This is only an approximation known as scattering approximation ⁷⁾, and further $Z \alpha$ corrections must be accounted for if one wants an exact result. Therefore, the reached accuracy of 0.5%, being of order α , is fully satisfactory in this approximation and this part of the Lamb shift can be considered as settled. In H and D, its contribution is 0.43 MHz ; in He^+ it amounts to 6.88 MHz.

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REFERENCES

- 1) T. Appelquist and S.J. Brodsky - Phys.Rev.Letters 24, 562 (1970),
and SLAC-PUB-756 (1970).
- 2) J. Weneser, R. Bersohn and N.M. Kroll - Phys.Rev. 91, 1257 (1953).
- 3) M.F. Soto Jr. - Phys.Rev.Letters 17, 1153 (1966) ; Phys.Rev. A2,
734 (1970).
- 4) R. Barbieri, J.A. Mignaco and E. Remiddi - Nuovo Cimento Letters
2, 588 (1970).
- 5) M. Veltman - CERN Internal Report (1967).
- 6) "Riemann Integration With Interval Adjustment" - Original version
G. Sheppey, CERN ; present version by A.J. Dufner,
B. Lautrup and G. Sheppey.
- 7) R.L. Mills and N.M. Kroll - Phys.Rev. 98, 1489 (1955).