QCD at finite Baryon Density: Instantons and Color Superconductivity

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Abstract

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1. Introduction

This meeting is concerned with summarizing our understanding of hadronic matter under extreme conditions. While important progress has been made with respect to the situation at finite temperature and zero baryon density, the problem of cold dense matter is much less understood. From the point of view of lattice QCD this is due to the “sign problem”, the fact that at non-zero chemical potential the fermionic determinant is complex. This means that Monte Carlo simulations based on straightforward importance sampling are not possible. Also, the possible phase structure of dense matter is very rich. In addition to the nuclear and quark matter phases, there may very well exist intermediate phases containing pion or kaon condensates, strange matter droplets or density isomers.

We should also emphasize that even though RHIC and LHC are mainly designed to explore the high temperature, low baryon density regime, the problem of cold dense matter is not entirely academic. First, understanding the equation of state of cold matter is of course essential for unravelling the structure of neutron stars. Second, even in ultra-relativistic heavy ion collisions the conditions in the target and projectile fragmentation region should correspond to rather cold, dense matter. And finally, systematically exploring the regime of highest baryon densities achieved somewhere between SIS/GSI, AGS and SPS energies should remain a priority.

2. Diquark Condensation

Naively, asymptotic freedom implies that cold quark matter is a nearly ideal Fermi gas of quarks. This system would behave quite similar to a cold abelian plasma, with color fields Debye-screened at momenta $p < M_D \sim g\mu$, collective plasma excitations with energy $\omega_P \sim g\mu$, etc. However, since the Coulomb interaction between quarks of different colors is attractive, we expect the formation of Cooper pairs near the Fermi surface and cold quark matter should behave as a (color) superconductor. This phenomenon was

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pointed out a long time ago [1] (although this work seems to have been largely forgotten). The magnitude of the energy gap Δ and the critical temperature T_c were estimated to be less than 1 MeV.

In this contribution we show that non-perturbative effects can lead to diquark condensates with Δ, T_c about two orders of magnitude bigger. Also, as distinct from weak-coupling superconductors, these pair correlations continue to have important effects on the equation of state even above the critical temperature or below the critical density for superconductivity. The non-perturbative effects we are taking into account are induced by instantons. For two flavors (up and down) the $(\bar{q}q)$ interaction generated by instantons is given by [2]

$$\mathcal{L} = G \frac{1}{8N_c^2} [(\bar{\psi}\tau^-\psi)^2 + (\bar{\psi}\tau^-\gamma_5\psi)^2], \quad (1)$$

where we have added the interaction in the direct and exchange channels and dropped color octet terms. N_c is the number of colors and $\tau^- = (\vec{\tau}, i)$ is an isospin matrix. We will specify the coupling constant G below. There is extensive evidence for the importance of this interaction from (i) phenomenological studies of current correlation functions in QCD, (ii) the success of hadronic spectroscopy in the instanton liquid model, and (iii) studies of instantons and their effects on the lattice, see [3] for a review of these issues.

The result (1) can be Fierz-rearranged into a (qq) interaction. We find

$$\mathcal{L} = G \left\{ -\frac{1}{16N_c(N_c-1)} [(\psi^T C \tau_2 \lambda_A^n \psi)(\bar{\psi} \tau_2 \lambda_A^n C \bar{\psi}^T) + (\psi^T C \tau_2 \lambda_A^n \gamma_5 \psi)(\bar{\psi} \tau_2 \lambda_A^n \gamma_5 C \bar{\psi}^T)] \right. \\ \left. + \frac{1}{32N_c(N_c+1)} (\psi^T C \tau_2 \lambda_S^n \sigma_{\mu\nu} \psi)(\bar{\psi} \tau_2 \lambda_S^n \sigma_{\mu\nu} C \bar{\psi}^T) \right\} \quad (2)$$

Here, C is the charge conjugation matrix, τ_2 is the anti-symmetric Pauli matrix, $\lambda_{A,S}$ are the anti-symmetric (color $\bar{3}$) and symmetric (color 6) color generators. The effective lagrangian (2) provides a strong attractive interaction between an up and a down quark with anti-parallel spins ($J^P = 0^+$) in the color anti-triplet channel, and a repulsive interaction in the 0^- channel. A current with 0^+ diquark quantum numbers is given by $\epsilon_{abc} u_b^T C \gamma_5 d_c$.

This interaction plays an important role in the structure of baryons. Coupling a third quark to the 0^+ diquark current gives a color singlet current $\eta = \epsilon_{abc}(u_a^T C \gamma_5 d_b)u_c$ with the quantum numbers of the nucleon. Lattice calculations have shown that the overlap of the nucleon with this current is large, much bigger than the overlap with the current built from 0^- diquarks. Also, we have calculated the mass of the nucleon in the instanton liquid and obtained very good agreement with experiment. Since there is no confinement in the instanton model, one can compare the diquark mass to the two constituent quark threshold. The result is a deeply bound scalar diquark $2m_q - m_{Sdq} \simeq 200 - 300$ MeV, whereas all other channels (vectors and axial-vectors, color 6 diquarks, etc.) are at most weakly bound.

The possible role of diquark clusters in quark matter was discussed in [4]. It was noted that a loosely bound “third” quark in the nucleon may find a partner in dense matter. However, this effect is less important than Bose condensation. In order to illustrate this statement, let us study a schematic equation of state for quark-diquark matter [5]. For

Figure 1. Energy per baryon (in MeV) versus baryon charge density n_b (fm^{-3}).

Figure 2. The gap $\Delta(\mu, T=0)$ and critical temperature T_c as a function of the baryon density.

definiteness, we consider (udd) matter relevant for neutron stars. Since scalar diquarks are color anti-triplets, a Bose condensate will select a direction in color space. This means that $SU(3)$ gauge invariance is broken to $SU(2)$ by a Higgs mechanism.

Diquarks are favored over a quark Fermi gas due to both their binding $m_S < 2m_{eff}$ as well as their Bose character. However, even at $T = 0$ we can never have an infinite number of diquarks condenses in the $p = 0$ state: diquarks are composite objects and, like nucleons, their interaction should have a repulsive core. We take this into account by including a scattering length $a \simeq 0.3$ fm into the expression for the energy per quark in a diquark Bose gas with weakly repulsive interaction. The repulsion makes the pure diquark gas less favorable than an optimal Bose-Fermi mixture. Results of our calculations are shown in Fig. 1, in which matter consists of (i) a Bose gas of diquarks in chemical equilibrium with (ii) a Fermi gas of uncondensed quarks, color neutralized by an appropriate amount of (iii) quarks of the third color. For definiteness, we use the di-/quark masses of 500 MeV and 400 MeV, respectively. In order to account for the effects of confinement we have also included color strings in our equation of state.

If not for confinement, a diquark Bose condensate would be the ground state of hadronic matter. A diquark/quark/string mixture has a shallow (meta-stable) minimum leading to a mixed phase at $n \approx 0.5 - 1 \text{ fm}^{-3}$. This may very well be an artefact of our crude model. The important conclusion is that the quark-diquark mixture has a significantly lower total energy compared to quark matter. The critical temperature for Bose condensation can be roughly estimated from Einstein's ideal gas expression, $T_c = 3.31 n_{dq}^{2/3} / m$, which is about 100 MeV at the crossing point.

3. Gap equation

We now turn to the high density limit, with most of the quarks forming a Fermi gas and instanton-induced forces operating only near the Fermi surface. The width of this zone (the analogue of the Debye frequency in a phonon superconductor) is governed by the instanton form factor. The full problem of evaluating the size of the gap is quite involved.

Instead, we will consider a BCS-type gap equation where the form factor is represented by a fixed cutoff $\lambda \simeq 0.3\text{GeV}$. The gap equation in the scalar diquark channel is given by

$$1 = \frac{8}{(2\pi)^2} g_{eff}(\mu) \int_{p_F-\lambda}^{p_F+\lambda} p^2 dp \frac{\tanh(\epsilon_p(\Delta)/2T)}{\epsilon_p(\Delta)} \quad (3)$$

with $\epsilon_p(\Delta) = [(\omega_p - \mu)^2 + \Delta(\mu, T)^2]^{\frac{1}{2}}$, $\omega_p^2 = p^2 + m_q^2$ and $p_F^2 = \mu^2 - m_q^2$. For $\mu \geq \mu_c$ the Debye screening of the instanton fields becomes effective. This effect is included in the coupling constant $g_{eff}(\mu) \sim \exp[-N_f \rho^2 (\mu^2 - \mu_c^2) \theta(\mu - \mu_c)]$. For more details see [5]. Fig. 2 shows the gap $\Delta(\mu, T = 0)$ and the critical temperature for two different values of the critical chemical potential. At large μ the gap is strongly suppressed by screening effects, while at small μ it is reduced due to the decrease of the density of states at the Fermi surface (in this regime the approach discussed in Sec. 2 is more appropriate). The maximum gap is approximately linear in the critical density and reaches 50-100 MeV.

There are a number effects beyond scalar-isoscalar diquark condensation that are not included here. These include coexistence of $\langle \bar{q}q \rangle$ and $\langle qq \rangle$ condensates, strange diquark condensation, vector diquark condensation, possible color 6 condensates [6], etc. This list again illustrates the richness of the QCD phase diagram at finite baryon density.

4. Summary

To summarize, instantons lead to strong attraction in the scalar ud -diquark channel. If not for confinement, a Bose condensed diquark gas would be the ground state at low densities. The transition from nuclear matter to diquark-condensed matter starts at densities $n_b \simeq 0.7 - 1 \text{ fm}^{-3}$, as a phase in which both $\langle \bar{q}q \rangle$, $\langle qq \rangle$ are non-zero. An appreciable BCS-like gap builds up towards the chiral restoration line, with $T_c \simeq 50 \text{ MeV}$. At even larger densities, instantons are Debye-screened and $\langle qq \rangle$ is dominantly due to one gluon exchange.

Ultimately, we would like to be able to answer some of the questions posed in the last section by doing direct simulations of full QCD or the instanton liquid at finite baryon density. Recently, we have made some progress in performing simulations in the instanton liquid [7]. For two colors, there is no sign problem and one can observe chiral symmetry restoration and the formation of a diquark condensate. Real QCD with $N_c = 3$ is more difficult, but simulations at large chemical potential can be done. We observe chiral symmetry restoration due to the formation of instanton “polymers”. Measurements of different diquark condensates are in progress.

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