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COMPLEX REGGE POLES AND LINE REVERSED REACTIONS

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ABSTRACT

The theory of complex Regge poles is invoked to explain the experimental data on the ratio of cross-sections for line reversed pairs of hypercharge exchange reactions. The data behaves as should be expected theoretically.

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Two-body exotic amplitudes contain no resonances, and are predominantly real. The amplitude for a process connected to one of these by "line reversal" is not exotic, does contain resonances, and is complex. Let us refer to such a pair of processes as the exotic (E) and non-exotic (N) reactions, respectively.

It appears to be an experimental fact that the ratio $R(s,t)$ of exotic to non-exotic differential cross-sections for such pairs of pseudoscalar meson-baryon scattering processes is consistently larger than one¹⁾. The ratio seems to decrease towards one as the energy increases for charge exchange reactions; however, for hypercharge exchange it is apparently more nearly energy independent¹⁾. Further, for a given energy, the ratio becomes significantly larger at larger momentum transfers.

Duality conventionally arranges that an exotic amplitude is made real through exchange degeneracy: if even and odd signature trajectories having the same quantum numbers are equal, and their residues are equal too ("strong" exchange degeneracy), then the contribution of a pair of trajectories to an exotic amplitude is

$$\begin{aligned} A_E &\sim \beta(t) \left(\frac{1 + e^{-i\pi\alpha(t)}}{2} \right) s^{\alpha(t)} + \beta(t) \left(\frac{1 - e^{-i\pi\alpha(t)}}{2} \right) s^{\alpha(t)} \\ &= \beta(t) s^{\alpha(t)} \end{aligned} \quad (1)$$

which is purely real. The contribution of the same pair to the corresponding line reversed, or non-exotic, amplitude is, however,

$$\begin{aligned} A_N &\sim \beta(t) \left(\frac{1 + e^{-i\pi\alpha(t)}}{2} \right) s^{\alpha(t)} - \beta(t) \left(\frac{1 - e^{-i\pi\alpha(t)}}{2} \right) s^{\alpha(t)} \\ &= \beta(t) s^{\alpha(t)} e^{-i\pi\alpha(t)} \end{aligned} \quad (2)$$

and is said to have a "rotating phase".

Evidently, if at high energy a single such pair of Regge poles dominates both the exotic and non-exotic amplitudes, we must have $R(s,t) = 1$. For charge exchange, it seems indeed to be the case that at sufficiently high s , this conclusion is confirmed. For hypercharge exchange, however, the experimental situation suggests that R can become one only at a much higher s , if at all, and we must ask why.

One possible explanation is to observe that duality is not expected to be exact. Therefore the exchange degeneracy predicted by it should not be exact either. The most trivial way of breaking exchange degeneracy is to retain it for the trajectories but give it up for the residues ("weak" exchange degeneracy). This, however, is not sufficient for our purposes; we still have $R(s,t) = 1$. It is therefore necessary to break trajectory degeneracy as well²⁾.

The drawback to this explanation is that even strong exchange degeneracy seems to be supported to a rather high accuracy experimentally³⁾; it would therefore seem desirable to try to retain it to the greatest degree possible.

Another possible resolution of the difficulty is to invoke Regge cuts, and thereby deny the dominance of a single Regge-pole pair even at high energy. One may in this way hope to retain strong exchange degeneracy, even in the face of the experimental values of $R(s,t)$. However, again one's hopes are smashed. Absorptive cuts interfere destructively with their partner poles; hence adding absorptive cuts to an exchange degenerate pair of poles yields $R(s,t) < 1$. One way out is to reduce the strong exchange degeneracy to weak exchange degeneracy, and permit oneself to tamper with the residues. If one tampers to such a degree that the sign of one of the residues is in fact reversed, then the roles of A_E and A_N become interchanged: from Eqs. (1) and (2) we see that if the sign of the second residue is reversed, then A_E has the rotating phase while A_N is real. Now adding absorptive cuts will produce the desired $R(s,t) > 1$. The answer comes out right; one is, however, entitled to wonder whether such an extreme destruction of strong exchange degeneracy is really plausible.

Our feeling is that these attempts to understand $R(s,t)$ are not compelling. We should therefore like to suggest instead what seems to us to be a rather simple and natural explanation of the experimental situation, based on the idea of complex Regge poles^{5,6}).

The two hypercharge exchange processes to be discussed are Σ production and Λ production; specifically the exotic reactions $K^- p \rightarrow \pi^- \Sigma^+$ and $K^- n \rightarrow \pi^- \Lambda$ together with their line reversed non-exotic partners $\pi^+ p \rightarrow K^+ \Sigma^+$ and $\pi^- p \rightarrow K^0 \Lambda$. There are, for each of these, two scalar amplitudes which we take to be the conventional A' and B amplitudes.

The quantities of final interest to us will be the polarization, and the two exotic to non-exotic cross-section ratios:

$$R_{\Sigma}(s,t) \equiv \frac{d\sigma/dt (K^- p \rightarrow \pi^- \Sigma^+)}{d\sigma/dt (\pi^+ p \rightarrow K^+ \Sigma^+)} \quad (3)$$

and

$$R_{\Lambda}(s,t) = \frac{d\sigma/dt (K^- n \rightarrow \pi^- \Lambda)}{d\sigma/dt (\pi^- p \rightarrow K^0 \Lambda)} \quad (4)$$

The theory of complex Regge poles⁵⁾ suggests that we parametrize the exotic A' and B amplitudes as follows

$$A'_E(s,t) = \frac{1}{\pi} \int_{-\infty}^{\alpha_c} \frac{\text{Im } f_A(t,\ell) (s/s_0)^\ell}{(\ell - \alpha_R)^2 + \alpha_I^2} d\ell \quad (5)$$

$$B_E(s,t) = \frac{1}{\pi} \int_{-\infty}^{\alpha_c} \frac{\ell \text{Im } f_B(t,\ell) (s/s_0)^{\ell-1}}{(\ell - \alpha_R)^2 + \alpha_I^2} d\ell \quad (6)$$

(The factor ℓ in the B amplitude may be thought of as associated with the P'_ℓ appearing in the partial wave expansion for B . The phenomenological motivation for it is a break in the slope of the hypercharge exchange cross-section near $\alpha_R = 0$.)

$$A'_N(s, t) = \frac{1}{\pi} \int_{-\infty}^{\alpha_c} \frac{\text{Im } f_A(t, \ell) (s/s_0)^\ell e^{-i\pi\ell}}{(\ell - \alpha_R)^2 + \alpha_I^2} d\ell \quad (7)$$

$$B_N(s, t) = \frac{1}{\pi} \int_{-\infty}^{\alpha_c} \frac{\ell \text{Im } f_B(t, \ell) (s/s_0)^{\ell-1} e^{-i\pi\ell}}{(\ell - \alpha_R)^2 + \alpha_I^2} d\ell \quad (8)$$

The crucial assumption, taken from duality and exchange degeneracy, is that the functions $\text{Im } f$ appearing in both the E and N amplitudes are identical; as in the case of a single real pole, the exchange degenerate pair of complex poles (representing, of course, poles together with a Regge cut with branch point at α_c) combine in the exotic reactions to produce a purely real amplitude, and to produce a pure "rotating phase" in the non-exotic amplitudes.

In order to extract information from Eqs. (5) to (8), some simplifying assumption is needed; as usual^{5,6)} we assume the weight functions $\text{Im } f$ to be smoothly varying in ℓ over a region of order α_I around $\ell = \alpha_R$. Then we may extract the weight functions, evaluated at $\ell = \alpha_R$, from the integrals and thereby obtain

$$\begin{aligned} A'_E &= \frac{\text{Im } f_A(t, \alpha_R)}{2\pi i \alpha_I} \{ s^{\alpha_+} \text{Ei}((\alpha_c - \alpha_+) \log s/s_0) \\ &\quad - s^{\alpha_-} \text{Ei}((\alpha_c - \alpha_-) \log s/s_0) \} \\ B_E &= \frac{\text{Im } f_B(t, \alpha_R)}{2\pi i \alpha_I} (s/s_0) \{ \alpha_+ s^{\alpha_+} \text{Ei}((\alpha_c - \alpha_+) \log s/s_0) \\ &\quad - \alpha_- s^{\alpha_-} \text{Ei}((\alpha_c - \alpha_-) \log s/s_0) \} \end{aligned} \quad (9)$$

and

$$\begin{aligned}
 A'_N &= \frac{\text{Im } f_A(t, \alpha_R)}{2\pi i \alpha_I} s^{\alpha_+} \text{Ei}((\alpha_c - \alpha_+)(\log s/s_0 - i\pi)) \\
 &\quad - s^{\alpha_-} \text{Ei}((\alpha_c - \alpha_-)(\log s/s_0 - i\pi)) \} \\
 B_N &= \frac{\text{Im } f_B(t, \alpha_R)}{2\pi i \alpha_I} (s/s_0) \{ \alpha_+ s^{\alpha_+} \text{Ei}((\alpha_c - \alpha_+)(\log s/s_0 - i\pi)) \\
 &\quad - \alpha_- s^{\alpha_-} \text{Ei}((\alpha_c - \alpha_-)(\log s/s_0 - i\pi)) \} ,
 \end{aligned} \tag{10}$$

where we write $\alpha_{\pm} = \alpha_R \pm i\alpha_I$.

The quantities R and P, calculated from these expressions, depend on two parameters (one of which can, in principle, depend on t) which we may take to be s_0 and

$$b(t) \equiv s_0 \frac{\text{Im } f_B(t, \alpha_R)}{\text{Im } f_A(t, \alpha_R)} , \tag{11}$$

as well as on the t-dependent trajectory functions $\alpha_R(t)$ and $\alpha_I(t)$. We shall choose α_R to be the normal K^* and K^{**} trajectory: $\alpha_R = 0.4 + t$.

Representative results for R are shown in Table 1, for various values of the parameters. It can be seen that R is quite insensitive to the value chosen for b. The choices $\alpha_I = 0.2$ and $\alpha_I = 0.3$ reflect the fact that the reduction in the non-exotic cross-section comes from the phase factor $e^{i\pi\ell}$, and this is significant only if $\alpha_I \sim 1/\pi$. Smaller values of α_I will result in an R near unity.

Finally, we show curves for $R(s, t)$ as a function of t, together with experimental data, for Σ production in Fig. 1a and for Λ production in Fig. 1b. The fits are shown for two models of $\alpha_I(t)$, namely $\alpha_I = 0.6 \sqrt{-t}$ and $\alpha_I = 0.2 (1 - 2t)$. [Roughly speaking, these may be thought of as corresponding to square root and logarithmic cut models, respectively⁶⁾]. The other parameters are taken as $s_0 = 2$ and $b = 10$; these are both quite reasonable values.

Figure 2 shows the predicted polarization at 5.5 GeV for $\pi^+ p \rightarrow K^+ \Sigma^+$. Curves are shown for two values of b , $b = 10$ and $b = 20$, to illustrate the sensitivity to this parameter. The polarization is totally insensitive to the detailed form for α_I ; the curves for $\alpha_I = 0.6 \sqrt{-t}$ and $\alpha_I = 0.2 (1 - 2t)$ lie on top of one another. The 10 and 14 GeV polarization data have the same general features, but rather poor statistics. For the exotic process, of course, A' and B remain real if the unperturbed pole contributions (i.e. those in the limit $\alpha_I \rightarrow 0$) are purely real. But we see from Eq. (9) that a small imaginary part in the unperturbed pole contributions can generate a large imaginary part in the resulting B amplitude in the region $\alpha_R \sim 0$, and hence a large polarization in that neighbourhood. However, a quantitative fit along this line may not be worth while at present, since the polarization data is limited to the 3-3.9 GeV range.

While the experimental data is not of overpowering accuracy, quite reasonable fits are obtained. But perhaps more important, the values of R are all automatically greater than one, and the increase of R with $-t$ away from $t = 0$ is also obtained naturally, as is the near independence of R of the energy. These qualitative features do not depend on any detailed choice of parameters, but are inherent in Eqs. (9) and (10). Altogether, it is fair to say that, from the point of view of complex poles, there is nothing surprising in the existing hypercharge exchange data: indeed, the observed results are just what we should expect.

As the energy increases to very high values, the approximation of extracting $\text{Im } f(t, \ell)$ at $\ell = \alpha_R$ fails, and the integrals in Eqs. (5) to (8) become dominated by the upper end of the branch cut. Therefore, eventually, we may expect R to approach one. How soon this should occur depends on the size of α_I ; we have found a rather large value.

Finally, a comment about charge exchange. Here the ratio R tends toward one much more swiftly as the energy increases. From the point of view of complex poles, all this tells us is that α_I for the ρ - A_2 complex pair is considerably less than that for the K^*-K^{**} complex pair. We already have independent information regarding the size of α_I for the P from fits to π -N CEX data⁶⁾, and these fits do in fact suggest a noticeably smaller value of α_I .

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Table 1

Predicted ratio for Σ production at 15 GeV/c laboratory momentum

R in GeV ²	b = 0				b = 10 GeV				b = 20 GeV			
	$r_0 = 2 \text{ GeV}^2$		$r_0 = 4 \text{ GeV}^2$		$r_0 = 2 \text{ GeV}^2$		$r_0 = 4 \text{ GeV}^2$		$r_0 = 2 \text{ GeV}^2$		$r_0 = 4 \text{ GeV}^2$	
	$\alpha_I=0.2$	$\alpha_I=0.3$	$\alpha_I=0.2$	$\alpha_I=0.3$	$\alpha_I=0.2$	$\alpha_I=0.3$	$\alpha_I=0.2$	$\alpha_I=0.3$	$\alpha_I=0.2$	$\alpha_I=0.3$	$\alpha_I=0.2$	$\alpha_I=0.3$
0	1.26	1.36	1.34	1.49	1.27	1.37	1.34	1.49	1.28	1.38	1.36	1.50
-0.2	1.32	1.44	1.40	1.57	1.30	1.41	1.37	1.52	1.26	1.34	1.31	1.42
-0.4	1.51	1.66	1.58	1.81	1.47	1.58	1.54	1.71	1.37	1.41	1.41	1.49
-0.6	1.87	2.07	1.92	2.26	1.75	1.89	1.80	2.05	1.47	1.49	1.55	1.62

REFERENCES AND FOOTNOTES

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- 2) P. Anvil et al., Phys. Letters 31 B, 303 (1970).
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- 4) C. Meyer et al., Nuclear Phys. B23, 99 (1970).
This reversal of the relative sign of the K^* and K^{**} residues is actually carried out in the A' amplitude for Σ production -- the result is to make the A' amplitude for $\pi^+p \rightarrow K^+\Sigma^+$ real and that for $K^-p \rightarrow \pi^-\Sigma^+$ have a rotating phase. However, even this extravagance fails to produce an $R > 1$ for Λ production (the ratio $K^-n \rightarrow \pi^-\Lambda$ to $\pi^-p \rightarrow K^0\Lambda$). This can be seen from SU(3): once the ρ and A_2 residues are made equal, then the K^* and K^{**} residues can be made nearly equal and opposite for either Σ production or for Λ production but not for both.
- 5) J. Ball, G. Marchesini and F. Zachariasen, Phys. Letters 31 B, 583 (1970).
P. Kaus and F. Zachariasen, Phys. Rev. D1, 2962 (1970).
- 6) B.R. Desai, P. Kaus, R. Park and F. Zachariasen, Phys. Rev. Letters 25, 1389 (1970).
- 7) Data on $K^-p \rightarrow \pi^-\Sigma^+$ at 5.5 GeV are from J.S. Loos et al., Phys. Rev. 173, 1330 (1968); and at 16 GeV from D. Birnbaum et al., Phys. Letters 13 B, 484 (1970).
Data on $\pi^+p \rightarrow K^+\Sigma^+$ at 5 GeV are from S.M. Pruss et al., Phys. Rev. Letters 23, 189 (1969); and at 14 GeV from A. Bashian et al., reported by J.V. Allaby at the 15th Int. Conf. on High-Energy Physics at Kiev, September 1970). (Note that the forward-most points reported here are considerably larger than those in preliminary reports of the same experiment: A. Bashian et al., Proc. Conf. on High-Energy Processes, Stonly Brook (1969), p. 75. We wish to thank Dr. G.C. Fischer for clarifying this point for us.)
- 8) Data on $K^-n \rightarrow \pi^-\Lambda$ at 3.9 GeV are from D.J. Crennell et al., Phys. Rev. Letters 23, 1347 (1969).
Data on $\pi^-p \rightarrow K^0\Lambda$ at 4 GeV are from O.I. Dahl et al., Phys. Rev. 163, 1430 (1967).

Figure captions

- Fig. 1a : $R(s,t)$ for Σ production in the range 5 to 5.5 GeV.
(The experimental data is from Ref. 7).
- 1b : $R(s,t)$ for Σ production in the range 14 to 16 GeV.
(The experimental data is from Ref. 9).
- 1c : $R(s,t)$ for Λ production in the range 3.9 to 4.0 GeV.
(The experimental data is from Ref. 8).
- Fig. 2 : Polarization $P(s,t)$ in $\pi^+ p \rightarrow K^+ \Sigma^+$ at 5.5 GeV. (The experimental data is from Loos et al., Ref. 7.) The curves are indistinguishable for $\alpha_I = 0.6\sqrt{-t}$ and $\alpha_I = 0.2(1 - 2t)$.

FIG. 1a

Ratio R_Σ for Σ production at 14–16 GeV/c

FIG.1c