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FACTORIZATION IN HIGH ENERGY HADRON COLLISIONS *)

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A B S T R A C T

The concept of factorization is discussed for elastic diffraction scattering and diffraction dissociation of hadrons at high energy. In addition to the usual definition in terms of the t -channel, a natural definition of factorization in the s -channel is proposed and compared with the former. It is shown that s -channel factorization of all diffractive processes is consistent with the assumption that elastic scattering is identical to the shadow of the diffraction dissociation processes.

*) Dedicated to the memory of Amos deShalit as a tribute to his personality and work.

1. INTRODUCTION

Good and Walker's idea that diffraction dissociation may play an important role in high energy hadron collisions [1] has recently received strong support from a detailed analysis of 3,4 and 5-body final states in pion-proton collisions. Thus, the processes $\pi p \rightarrow 2\pi N$ ($N = \text{nucleon}$) and $\pi p \rightarrow 2\pi\Delta$ (1238 MeV) at 8 and 16 GeV/c are found [2,3] to be strongly dominated by the dissociations of the proton into πN and $\pi\Delta$ respectively

$$\pi p \rightarrow \pi(\pi N), \quad \pi p \rightarrow \pi(\pi\Delta)$$

In the 4-body reaction $\pi^- p \rightarrow 2\pi^- \pi^+ p$ at 11 and 16 GeV/c, one found [4,5] marked dominance of

$$\pi^- p \rightarrow (2\pi^- \pi^+) p, \quad \pi^- p \rightarrow \pi^- (\pi^- \pi^+ p)$$

the dissociation of the π^- into $2\pi^- \pi^+$ being stronger than the one of p into $\pi^- \pi^+ p$. In all cases, the mass range of the dissociated system, although not extending very high, covers several known resonances and these are not prominently visible above what appears to be an appreciable background. The 5-body reaction $\pi^- p \rightarrow 2\pi^- \pi^+ \pi^0 p$ at 16 GeV/c has revealed [5] not only the proton dissociation

$$\pi^- p \rightarrow \pi^- (\pi^- \pi^+ \pi^0 p)$$

as a marked maximum in phase space, but also a weaker maximum corresponding to the double dissociation

$$\pi^- p \rightarrow (2\pi^- \pi^+) (\pi^0 p)$$

All dissociations mentioned so far can proceed without exchange of quantum numbers and are therefore expected to be diffractive. In addition, the 5-body reaction just mentioned shows an unexpected and very strong maximum corresponding to the pion dissociation

$$\pi^- p \rightarrow (2\pi^- \pi^+ \pi^0) p$$

It cannot be diffractive since $(2\pi^- \pi^+ \pi^0)$ has positive G-parity. At 11 GeV/c this process is even stronger and tends to overshadow the two diffraction dissociations.

The technique revealing these single and double dissociations will not be reviewed here. It is based on the smallness and approximate constancy of transverse momenta and it makes use of appropriately weighted distributions in longitudinal phase space, thereby deriving from the bubble chamber data highly differential information on the collision amplitude.

These indications for copious and frequent occurrence of diffraction dissociation raise the possibility of testing the well known conjecture of factorization. The processes mentioned above have cross sections of mb order, so that good hope exists for quantitative studies of a problem which was considered until now only for the 2-body diffraction dissociation processes $pp \rightarrow pN^*$, $\pi p \rightarrow \pi N^*$ where N^* is a $I = \frac{1}{2}$ nucleon resonance. In the latter case, the cross sections are much smaller, but factorization could nevertheless be qualitatively tested and was found to hold within errors [6].

In preparation for more quantitative experimental work, we devote this note to the question of the precise mathematical definition to be adopted for the factorization property. Two distinct definitions present themselves as natural conjectures, the usual one expressed in terms of the crossed or t-channel ($t = -$ momentum transfer squared), the other in terms

of the impact parameter representation in the direct or s-channel ($s = \text{c.m. energy squared}$). We have no theory to guide our choice between these alternative formulations and feel that both deserve to be tested experimentally. They are discussed in sections 2 and 3, while section 4 shows that s-channel factorization is consistent with elastic scattering being the shadow of diffraction dissociation.

2. CROSSED CHANNEL FACTORIZATION

We consider the hadronic reaction

$$A + B \rightarrow A' + B'$$

A' is identical to A or it is a resonant or non-resonant system of hadrons produced by diffraction dissociation of A , and similarly for B and B' .

Four cases are possible

$A' = A$ and $B' = B$; elastic scattering

$A' \neq A$ and $B' = B$; single dissociation of A

$A' = A$ and $B' \neq B$; single dissociation of B

$A' \neq A$ and $B' \neq B$; double dissociation.

The most natural definition of factorization in the t-channel is the conventional one that the invariant collision amplitude T has the form

$$T = f(s, t) \gamma(t, x_A, x_A') \gamma(t, x_B, x_B') \quad (1)$$

s and t have the usual meaning in terms of four-momenta

$$s = (p_A + p_B)^2, \quad t = (p_A - p_{A'})^2 = (p_B - p_{B'})^2$$

$x_A, x_{A'}$ define the helicity of A and the full state of A' in a reference frame defined in terms of A, A' ; this is naturally chosen to be the Gottfried - Jackson frame, i.e. the rest frame of A' with z -axis in the direction of motion of A and y -axis normal to the collision plane, which is the plane containing $\vec{p}_A, \vec{p}_B$ ($\vec{p}$ = three-momenta in the frame considered). $x_B, x_{B'}$ are analogous variables for B, B' .

Eq. (1) is derived for exchange of a single Regge pole and expresses that in absence of degeneracy the residue factorizes. The factor $f(s, t)$ then has the form

$$f(s, t) = -\exp(-i \frac{\pi}{2} \alpha(t)) s^{\alpha(t)} \quad (2)$$

$$\alpha(t) \approx 1 \quad \text{for small } t \quad (\text{Pomeranchuk trajectory})$$

More complicated s -dependences are expected, however, because the "Pomeron", the object exchanged in diffractive processes, is presumably much more involved than a single pole. These complications remove the theoretical grounds for the exact factorization embodied in (1), which is therefore expected to have at most an approximate validity.

As far as γ is concerned, it can be taken real and the data show that it must drop rapidly with increasing $|t|$. A parametrization

$$\gamma(t, x_A, x_{A'}) = \gamma_0 \exp(a_0 t) \quad (3)$$

seems appropriate for describing the small $|t|$ region. γ_0 and a_0 are functions of x_A, x'_A , about which our information is very incomplete. There is a systematic decrease of a_0 for increasing mass $m_{A'}$ of the dissociated system A' , with additional fine structure related to resonances; the evidence has recently been summarized by Satz [7]. The general trend of this effect is understandable in a Pomeron exchange model. If P denotes the Pomeron, it means that the dependence of the virtual process $P + A \rightarrow A'$ on the "Pomeron mass" $(-t)^{\frac{1}{2}}$ decreases for increasing $m_{A'}$, as is natural since $m_{A'}$ is the energy of $P + A \rightarrow A'$ in its own rest frame.

Using (1) one obtains

$$d\sigma/dt = s^{-2} |f\gamma_{AA'}(t) \gamma_{BB'}(t)|^2 \quad (4)$$

with the notation

$$\gamma_{AA'}(t) = \gamma(t, x_A, x'_A), \quad \gamma_{BB'}(t) = \gamma(t, x_B, x'_B) \quad (5)$$

Eq. (4) holds in principle for given x and x' , whereas the measured $d\sigma/dt$ refers in practice to sums over most of the x, x' variables. Such sums, however, do not affect the factorization property of (4). This property should best be tested for given t values (whereby one should remember that at any finite s some neighbourhood of $t = 0$ is unattainable in dissociation, i.e. when $m_{A'} > m_A$ and/or $m_{B'} > m_B$). If one wants to test t -channel factorization with cross sections integrated over t , the slopes of $d\sigma/dt$ in t must be taken into account by means of eq. (3) and its analogon for B, B' . This has been done by Freund [6] for elastic scattering and diffraction dissociation of the proton in the reactions $\pi p \rightarrow \pi B'$, $pp \rightarrow pB'$, $B' = p$, $N(1400)$ and $N^*(1688)$. For each B' , the slopes turn out to be approximately the same in the πp and pp cases, so that they disappear from the $\sigma(\pi p \rightarrow \pi B')/\sigma(pp \rightarrow pB')$ ratio.

3. DIRECT CHANNEL FACTORIZATION

Present views on diffractive processes have returned to a considerable degree to the conventional, pre-Regge interpretation in terms of the s-channel. Quite recently, in addition, the spin properties of these processes have been partly elucidated, and the evidence [8] is in favour of conservation of s-channel helicities (these are the helicities defined in the c.m. system of $A + B \rightarrow A' + B'$). It is therefore tempting to look for a natural definition of factorization in the s-channel. Such a definition is readily found in the impact parameter representation (which is the high energy approximation to the orbital partial wave expansion). We write the collision amplitude of $A + B \rightarrow A' + B'$ in this representation

$$T(s,t) = 2\pi^{\frac{1}{2}}is \int_0^{\infty} h(b) J_0[b(-t)^{\frac{1}{2}}] b db \quad (6)$$

with h real, corresponding to a fixed Pomeranchuk pole. A more complicated s-dependence as in eq. (2) can of course also be put in the form (6), but h is then no longer real; for a linear trajectory $\alpha(t)$, h has a logarithmic dependence on s . We limit ourselves henceforth to the special case (6) with real h .

We propose the s-channel factorization property to be defined by

$$h(b) = \eta(b, y_A, y_A') \eta(b, y_B, y_B') \quad (7)$$

η being real; y_A is the s-channel helicity of A , whereas y_A' describes all variables needed to specify the state of A' in its helicity frame (rest frame of A' with z-axis opposite to the direction of motion of B' and y-axis normal to the collision plane). y_B, y_B' are similarly defined for B, B' .

The observed exponential shape of $d\sigma/dt$ in t at small t is obtained in the present case by adopting a gaussian b -dependence of η

$$\eta(b, x_A, x_{A'}) = \eta_0 \exp(-c_0 b^2) \quad (8)$$

with η_0, c_0 functions of $y_A, y_{A'}$. The effect described after eq. (3) now corresponds to an increase of $c_0(y_A, y_{A'})$ with increasing mass $m_{A'}$ of the dissociated system A' .

This property as well as the factorization (7) itself have a particularly intuitive interpretation in the s -channel. Eq. (7) essentially means that the probability amplitudes for A to remain A or get dissociated into any possible A' in the course of the collision with B is independent of what happens to B , except for an overall strength factor. As to the $m_{A'}$ -dependence of c_0 , dissociation of A into a system A' of high $m_{A'}$ is naturally expected to be easier the more central the collision, i.e. the smaller the impact parameter b ; hence the larger value of $c_0(x_A, x_{A'})$ for larger $m_{A'}$.

Experimental tests of the s -channel factorization property (7) are more difficult to carry out than in the t -channel case. We discuss them in the gaussian approximation (8) which gives

$$T(s, t) = \frac{1}{2} \text{ is } \eta_{AA'}^0 \eta_{BB'}^0 (c_{AA'} + c_{BB'})^{-1} \exp \left[(c_{AA'} + c_{BB'})^{-1} t/4 \right] \quad (9)$$

with the abbreviations

$$\eta_{AA'}^0 = \eta_0(y_A, y_{A'}), \quad c_{AA'} = c_0(y_A, y_{A'})$$

and similarly for B, B' . The cross section is

$$d\sigma/dt = s^{-2} |T|^2 \quad (10)$$

The difficulty is that in general no simple property results after a summation has been carried out over some of the y, y' variables; it originates from the occurrence of the expression $(c_{AA'} + c_{BB'})^{-1}$. But if the summation runs over a class of collisions such that $c_{AA'} + c_{BB'}$ would be approximately constant, (9) and (10) give factorization of $d\sigma/dt$ in the same fashion as (4).

The distinction between the two definitions of factorization centers therefore on the way the slope of $d\sigma/dt$ in t depends on the collision. Writing

$$d\sigma/dt = v \exp(wt),$$

we obtain from s-channel factorization an additivity property for w^{-1}

$$w^{-1} = 2c_{AA'} + 2c_{BB'}, \quad (11)$$

whereas the s-channel definition gives additivity for w

$$w = 2a_{AA'} + 2a_{BB'}, \quad (12)$$

We here have used (3), and the notation $a_{AA'}$ stands for $a_o(x_A, x_A')$ with a similar meaning for $a_{BB'}$.

Experiment has revealed considerable variations of the slope w , especially with the masses $m_{A'}$, $m_{B'}$. It may therefore become possible to study these variations in some detail and test which of eqs. (11) and (12) is compatible with experiment. The cases $\pi p \rightarrow \pi B'$, $pp \rightarrow pB'$ studied by Freund [6] do not allow to make a distinction. Indeed, as mentioned before, w has approximately the same value for the two reactions producing a given B' , so that the slopes disappear from the $\pi p/pp$ ratios, and our two definitions are equivalent.

4. FACTORIZATION AND UNITARITY CONDITION IN THE DIRECT CHANNEL

It is interesting to consider the effect of s-channel factorization in the unitarity condition for elastic $A + B$ scattering at very high energy. With neglect of spins, this condition applied to (6) for the elastic case gives [9]

$$h_{el}(b) = [h_{el}(b)]^2 + G(b) \quad (13)$$

h_{el} refers to $A + B \rightarrow A + B$ and $G(b)$ is the overlap function which describes the elastic shadow scattering due to all inelastic channels open to the $A + B$ collisions. G is a sum of positive terms, one for each of these channels. The term corresponding to a diffraction dissociation process $A + B \rightarrow A' + B'$ of amplitude $h(b)$ is simply

$$[h(b)]^2 = [\eta_{AA'}(b) \eta_{BB'}(b)]^2$$

where (7) has been used with an obvious change of notation. If $A' = A$ and $B' = B$, $h(b)$ becomes $h_{el}(b)$, and (13) can be written as

$$\eta_{AA}(b) \cdot \eta_{BB}(b) = \left(\sum_{A'} [\eta_{AA'}(b)]^2 \right) \cdot \left(\sum_{B'} [\eta_{BB'}(b)]^2 \right) + G_{nd}(b) \quad (14)$$

The sum $\sum_{A'}$ includes the term $A' = A$, and similarly for $\sum_{B'}$. The quantity G_{nd} is the overlap function for channels which are not of diffraction dissociation type.

It may be that the latter channels become unimportant at high energy, so that G_{nd} could be negligible in eq. (14). This relation then shows the consistency of s-channel factorization in the elastic and dissociation channels even when the former is the shadow scattering of the latter. When applied to $A + A$ scattering, it reduces to the equation

$$\eta_{AA}(b) = \sum_{A'} [\eta_{AA'}(b)]^2 \quad (15)$$

because $\eta_{AA}(b)$ can always be taken positive. Eq. (15) should hold for any hadron A , the sum running over $A' = A$ as well as over all systems A' into which A can be diffractively dissociated. Shadow scattering of hadron A is now related by unitarity to its diffraction dissociation, independently of the hadron B which scatters or dissociates it and independently of what happens to B . Note that when G_{nd} is not negligible, eq. (15) is replaced by the inequality

$$\eta_{AA}(b) > \sum_{A'} [\eta_{AA'}(b)]^2 \quad (16)$$

This follows from (14) for $B = A$ and from the positivity of G_{nd} .

REFERENCES

- 1 M.L. Good and W.D. Walker, Phys.Rev. 120 (1960) 1857.
- 2 A. Białas, A. Eskreys, W. Kittel, S. Pokorski, J.D. Tuominiemi and L. Van Hove, Nucl.Phys. B11 (1969) 479.
- 3 J. Bartsch, K. Boesebeck, M. Deutschmann, R. Schulte, R. Speth, H. Böttcher, H.H. Kaufmann, S. Nowak, K. Böckmann, W. Johnssen, M. Rost, K. Sternberger, K. Barnham, V.T. Cocconi, P.F. Dalpiaz, R. Ely, J.D. Hansen, W. Kittel, K.P.C. Lindblom, D.R.O. Morrison, H. Tøfte, S. Brandt, V. Lüth, T.P. Shah, N.C. Barford, M.J. Counihan, D.P. Dallman, G.A. Grammatikakis, B. Buschbeck, C. Gottfried, M. Markytan, G. Otter and P. Schmid, Nucl.Phys. B19 (1970) 381.
- 4 L. Van Hove, Some Trends and Problems in the Study of High Multiplicity Hadronic Interactions, Colloquium on High Multiplicity Hadronic Interactions, Paris, 13 - 15 May, 1970. CERN preprint TH.1178.
- 5 W. Kittel, S. Ratti and L. Van Hove (Aachen - Berlin - Bonn - CERN - Heidelberg Collaboration and Genova - Hamburg - Milano - Saclay Collaboration), Single and Double Dissociation in π^-p Collisions, Paper submitted to the XVth International Conference on High Energy Physics, Kiev, 26 August - 4 September, 1970.
- 6 P.G.O. Freund, Phys.Rev.Lett. 21 (1968) 1375.
- 7 H. Satz, On the Mass Dependence of Momentum Transfer Distributions in Diffraction Dissociation, CERN preprint TH.1175 (1970), to appear in Phys.Letters B.
- 8 F.J. Gilman, J. Pumplin, A. Schwimmer, L. Stodolsky, Phys.Letters 31B (1970) 387.
- 9 L. Van Hove, Rev.Mod.Phys. 36 (1964) 655.